B-meson and Bottomonium spectroscopy from radiatively improved NRQCD

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Intro

Recent improvements made in NRQCD and gluon actions

- $\mathcal{O}(\alpha_s)$ improved matching coefficients available for the first time
- Charm quarks now included in the sea
- Smaller gluon/sea quark discretisation errors
- High statistics - 16k correlators
- Get the spectrum right before doing decays & mixing

We use NRQCD for heavy-heavy and heavy-light

References:

- "Prediction of the bottomonium D-wave spectrum from full lattice QCD", J.Daldrop et al., Phys.Rev.Lett. 108 (2012) 102003.
- "The Upsilon spectrum and the determination of the lattice spacing from lattice QCD including charm quarks in the sea", R. Dowdall et al., Phys. Rev. D 85, 054509 (2012).
- "Precise B, B_s and B_c meson masses and hyperfine splittings from lattice QCD including charm quarks in the sea", R. Dowdall et al. In preparation.

Nonrelativistic QCD

- Effective field theory valid for small quark velocity v
- $\mathcal{O}(v^4)$ Hamiltonian including discretisation corrections:

$$
\begin{aligned}
H_0 &= -\frac{\Delta^{(2)}}{2m_b} \\
\delta H &= -c_1 \frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8m_b^2}\left(\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla\right) - c_3 \frac{1}{8m_b^2} \sigma \cdot \left(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla}\right) \\
&\quad - c_4 \frac{1}{2m_b} \sigma \cdot \tilde{\mathbf{B}} + c_5 \frac{a^2 \Delta^{(4)}}{24 m_b} - c_6 \frac{a(\Delta^{(2)})^2}{16 n m_b^2}
\end{aligned}
$$

- Wilson coeff. must be matched to QCD:
  - $c_i = 1$ at tree level by matching continuum NRQCD to QCD
- b quark mass am_b, $\tilde{\mathbf{E}}, \tilde{\mathbf{B}}$ improved clover chromo-electric/magnetic fields, $\Delta^{(2)}, \nabla, \Delta^{(4)}$ are lattice derivatives

Nonrelativistic QCD

Quark propagators generated by time evolution

$$G(\mathbf{x}, t+1) \quad = \quad \left(1 - \frac{\delta H}{2}\right)\left(1 - \frac{H_0}{2n}\right)^n U_t^\dagger(x)\left(1 - \frac{H_0}{2n}\right)^n \left(1 - \frac{\delta H}{2}\right) G(\vec{x}, t)$$

- n is a stability parameter, require $n > 3/(2am_b)$. We use $n = 4$.

Radiative corrections to c_i are the dominant systematic error in several quantities - e.g. hyperfine and radial splittings

- Matching coefficients in the action are a function of the cutoff am_b
- Short distance coefficients $\implies$ perturbative. They have the expansion:

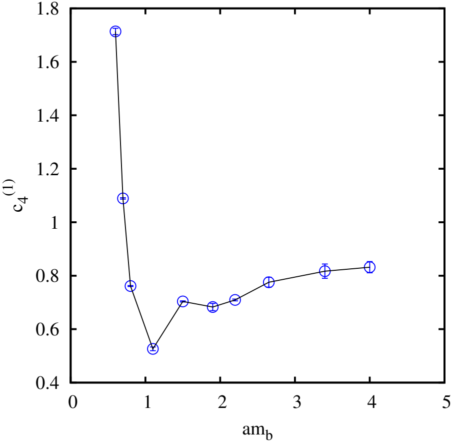
$$c_i = 1 + \alpha_s c_i^{(1)} + \mathcal{O}(\alpha_s^2)$$

- HPQCD have calculated most through $\mathcal{O}(\alpha_s)$
- We also consider matching to experiment

Perturbative improvement

- Kinetic terms are found by computing the NRQCD quark self-energy and ensuring the correct energy-momentum relation holds

- c_4 term controls size of the spin dependent splittings
- Calculated by matching the effective action in NRQCD to continuum QCD [T. Hammant et al (2011)]
- Plot shows c_4 is well behaved in the region we are working
- Diverges for $am_b < 1$ as expected



NRQCD

Advantages and disadvantages

Advantages:

- Very cheap numerically $\Longrightarrow$ high statistics
- Can extract excited states easily
- Effect of each term is well understood
- The same action can be used for onium and heavy-light systems
  - B-meson spectrum has no free parameters

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Disadvantages:

- Unphysical energy shift since am_b integrated out
  - $M_\Upsilon = 2m_b + (E_{\rm sim} - 2E_0)$
  - only calculate splittings or kinetic masses
- Non-renormalisable
  - requires matching or loss of predictive power for improved accuracy
- Coefficients diverge for $am_b \to 0 \implies$ cannot take $a \to 0$
  - This does not mean we cannot extract physical results!
- Current corrections and renormalisation

NRQCD - physical results

In a relativistic action, physical quantities are const. up to scaling violations

$$a\frac{d}{da}f(a) = \mathcal{O}(a^n(\ln a)^m)$$

So lattice data are fit to a form, e.g.

$$f(a) = f_{\rm phys}\left(1 + k_1(\Lambda a)^2 + k_2(\Lambda a)^4 + ...\right)$$

with $k_i = \mathcal{O}(1)$

- NRQCD action includes discretisation correction terms (c_5 and c_6)
- At tree level, lattice artifacts appear at $\mathcal{O}(a^4)$
- Radiative corrections to $c_5, c_6 \implies$ terms higher order in α_s

$$a\frac{d}{da}f(a) = \mathcal{O}(a^4, \alpha_s^2 a^2)$$

- Coefficients of $\alpha_s^2 a^2$ depend on the effective field theory cutoff am_b

NRQCD - physical results

- Results can depend on am_b as well as lattice spacing
- As long as we work with $am_b > 1$, dependence is mild
- Allow for this dependence with $\delta x_m = (am_b - 2.7)/1.5$, varies between ± 0.5

$$
\begin{aligned}
f(a, am_b) &= f_{\text{phys}}\Big[1 + c_1(\Lambda a)^2(1 + c_{1b}\delta x_m + c_{1bb}(\delta x_m)^2) \\
&\quad + c_2(\Lambda a)^4(1 + c_{2b}\delta x_m + c_{2bb}(\delta x_m)^2)\Big]
\end{aligned}
$$

- Priors are: 0.0(3) for a^2 terms, 0(1) for a^4, 0(1) for δx_m
- Obtain physical results just as with any other quark formalism
- Must include an additional error from am_b dependence in our error budget.
- Other terms such as sea quark mass dependence included as usual
- All data fit to a form of this kind

Ensembles

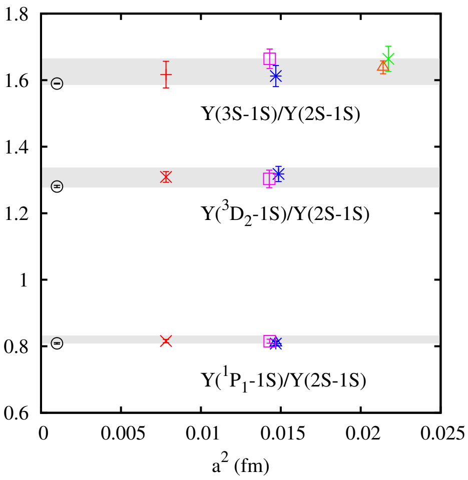
5 MILC ensembles including 2+1+1 flavours of HISQ sea quarks [Bazavov et al (2010)]

Set	β	$a(fm)$	am_l	am_s	am_c	$L/a \times T/a$
1	5.80	~ 0.15	0.013	0.065	0.838	16×48
2	5.80	~ 0.15	0.0064	0.064	0.828	24×48
3	6.00	~ 0.12	0.0102	0.0509	0.635	24×64
4	6.00	~ 0.12	0.00507	0.0507	0.628	32×64
5	6.30	~ 0.09	0.0074	0.037	0.440	32×96

- Tadpole and one-loop improved Lüscher-Weisz action
- Coefficients include gluonic loops and effect of N_f HISQ sea quarks
- $\sim$1000 cfgs in each ensemble
- Light quark masses of $m_l/m_s = 0.1, 0.2$
  (Pion masses of 220MeV - 315MeV)
- s and c quarks well tuned
- $M_\pi L > 4$ for all except set 1
- Lattice spacings fixed with $\Upsilon(2S - 1S)$ splitting

Bottomonium Results

Splitting Ratios



$$\frac{\Upsilon(3S-1S)}{\Upsilon(2S-1S)} = 1.621(36)$$
$$\text{Expmt} = 1.5896(12)$$

$$\frac{1^1P_1-\overline{1S}}{\Upsilon(2S-1S)} = 0.817(11)$$
$$\text{Expmt} = 0.8088(23)$$

$$\frac{1^3D_2-\overline{1S}}{\Upsilon(2S-1S)} = 1.307(30)$$
$$\text{Expmt} = 1.280(3)$$

Error budget %:

	R_P	R_S	R_D
stats/fitting	1.0	1.8	1.4
a-dependence	0.6	1.2	1.4
m_l-dependence	0.6	0.5	0.5
am_b-dependence	0.1	0.2	0.1
systematics	0.5	1.0	1.0
f. volume & tuning	0	0	0
EM/ η_b	0.2	0.2	0.2
Total	1.4	2.4	2.3

$b\bar{b}$ Spin independent systematic errors

Sources:

- Missing relativistic corrections from higher order v^6 terms
- Missing radiative corrections - mostly $\mathcal{O}(\alpha_s^2 v^4)$
  - These are estimated from a potential model
- Radiative corrections to discretisation correction terms $\mathcal{O}(\alpha_s^2 a^2)$
- Higher order discretisation errors $\mathcal{O}(a^4)$
  - Negligible - based on comparison of M_{Kin} for different p

Correction	relativistic	radiative	discretisation
Est. %age in $2S-1S$			
very coarse	0.5	0.6	0.5
coarse	0.5	0.5	0.3
fine	0.5	0.3	0.1
Est. %age in $1P-1S$			
very coarse	1.0	1.5	2.3
coarse	1.0	1.1	1.2
fine	1.0	0.6	0.4

- Finite volume effects negligible - large spatial volumes
- Estimates of annihilation, EM effects

P-wave spin splittings

P-wave spectrum $\chi_{b0}, \chi_{b1}, \chi_{b2}$ is used to non-perturbatively tune c_3, c_4

$$M(\chi_{b2}) - 3M(\chi_{b1}) + 2M(\chi_{b0}) \propto c_4^2, \qquad 5M(\chi_{b2}) - 3M(\chi_{b1}) - 2M(\chi_{b0}) \propto c_3$$

- Tree level coefficients give slightly incorrect splittings
- Tuned c_4 agrees well with perturbative calculation $\implies \mathcal{O}(\alpha_s^2)$ corrections small
- P-wave hyperfine splitting is zero

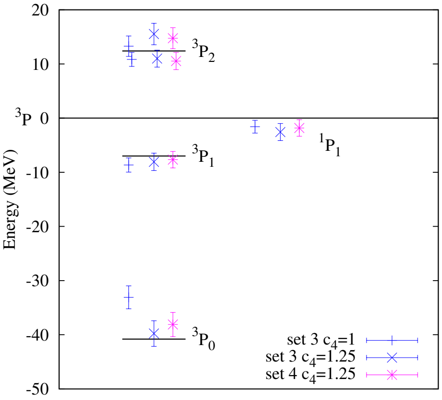


Figure: P-wave splittings on coarse ensembles. $^3P_2$ has E, T_2 irreps

S-wave hyperfine splitting

Hyperfine splitting $M_\Upsilon - M_{\eta_b}$ proportional to square of $c_4 \frac{g}{2aM_b}\,\sigma \cdot \tilde{\mathbf{B}}$ term
$\Longrightarrow$ Tree level calculation suffers from large $\mathcal{O}(2\alpha_s)$ systematic error

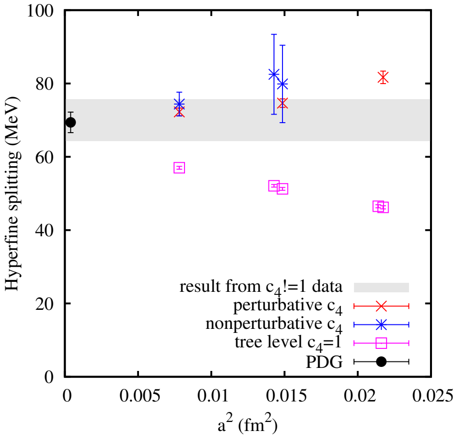
Both pert. and nonpert. values are fit to the same function as R_P, R_S including

- Adjustment for mass mistuning
- 4-quark operator corrections
- Correlated $\mathcal{O}(\alpha_s^2)$ systematic error on perturbative values
- Stat. + expmt error on non-pert tuned values
- Correlated NRQCD systematic error on non-pert tuned values
- 10% error from v^6 added to final answer

Consistent results are obtained from both

S-wave hyperfine splitting

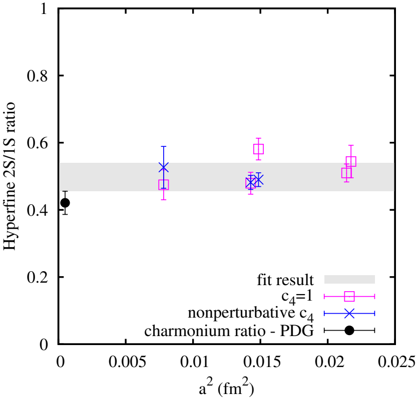
Combined fit to perturbative and nonperturbative values
($c_4 = 1$ data not included in fit)



- Only uncorrelated errors shown
- Previous HPQCD value [Gray et al 2004]:
  $M_\Upsilon - M_{\eta_b} = 61(14)$ MeV
- New result:
  $M_\Upsilon - M_{\eta_b} = 70(9)$ MeV
- 9 MeV systematic error dominated by missing $\mathcal{O}(v^6)$ terms
- Non-pert. tuned v^6 gives 60.3(5.5)(5.0)(2.1) MeV [Meinel 2010]
- Radiatively improved v^6 calculation underway

S-wave hyperfine ratio

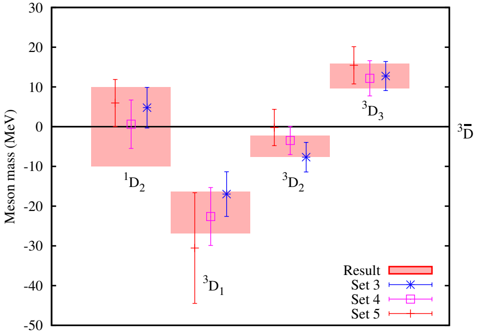
- Ratio $R_H = \frac{\Upsilon(2S) - \eta_b(2S)}{\Upsilon(1S) - \eta_b(1S)}$ should be independent of c_4
- Fit data the same way as $1S$ hyperfine but without c_4 errors



- Result:
  $M_{\Upsilon'} - M_{\eta_b'} = 35(3)$ MeV
- Prediction for $M_{\eta_b'} = 9988(3)$MeV
- Consistent with charmonium hyperfine ratio

D-wave states

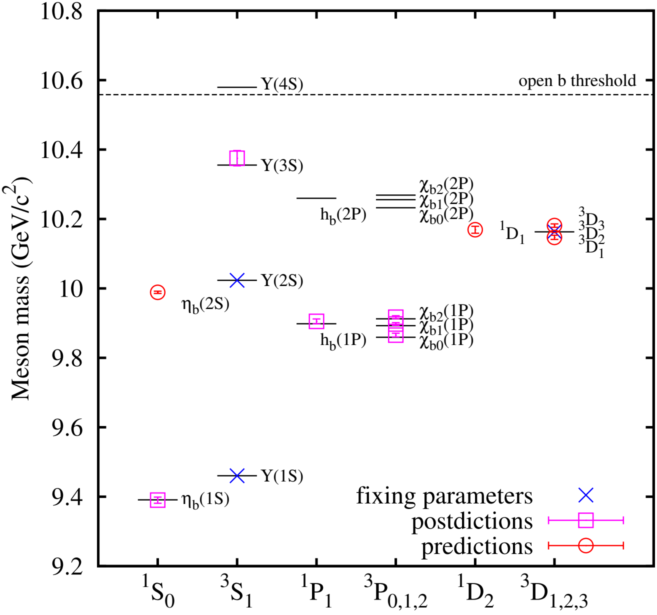
First prediction of bottomonium D-wave spin splittings from QCD



- 2 lattice spacings, 2 light quark masses
- All states fit simultaneously
- Mixing of $^3D_1$ with $^3S_1$ is non-zero but amplitudes of cross terms are small
- Correlated priors of 40(40)MeV relative to $^3D_2$
- Reconstruct states using ratios of splittings independent of c_3, c_4

$$\Delta_{\mathbf{L \cdot S}} = 14M_3 - 5M_2 - 9M_1 \propto c_3, \qquad \Delta_{S_{ij}} = -2M_3 + 5M_2 - 3M_1 \propto c_4^2$$

Bottomonium spectrum [PDG, CLEO]



B-meson Results

NRQCD-HISQ mesons

Heavy-light meson correlators constructed with NRQCD *b* and HISQ light quarks

- 16 time sources + forwards and backwards propagation
- Local and 2 exponentially smeared sources
- Random noise sources
- Fit ranges from $t_0 = 4, .., 8$ up to $L_t/2$
- Priors of ~500(250) MeV on energy splittings
- Unphysical energy shift removed with, e.g.

$$\Delta_{B_s} = aE_{B_s} - \frac{1}{2} aE_{b\bar{b}}$$

Adjusting for mistuning

Meson masses sensitive to the valence heavy and light quarks

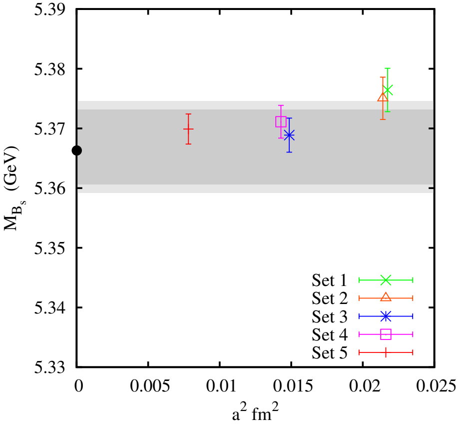
- Slope of Δ_{B_s} vs $M_{b\bar{b}}, M^2_{\eta_s}$ studied in [E.Gregory et al.]
- Shift found by interpolating to correct values
- Error on retuning taken as 50% of shift
- Mistuning is typically very small, less than a few MeV on all ensembles
- Similar shifts in the B_c case
- Sea quark mass mistuning accounted for in fits using

$$\delta x_q = \frac{\mathrm{m}_{\mathrm{q,sea}} - \mathrm{m}_{\mathrm{q,sea,phys}}}{m_{\mathrm{s,sea,phys}}}$$

Set	$\Delta_{M_{b\bar{b}}}$ (MeV)	$\Delta_{M^2_{\eta_s}}$ (MeV)	δx_l	δx_s
1	-1.9	0.0	0.17	0.01
2	-1.2	0.2	0.06	0.01
3	3.3	2.0	0.16	-0.04
4	0.0	2.4	0.06	-0.04
5	-2.8	-0.1	0.16	0.02

Table: Shifts applied to Δ_{B_s} due to mistuning of b and s quark

B_s



Systematics in light grey

- Obtain mass using $\Delta_{B_s} = aE_{B_s} - \frac{1}{2}aE_{b\bar{b}}$
- Scale error is 1/2 naive value when retuning is taken into account
- Error dominated by scale uncertainty and spin-ind NRQCD syst

Result

$$M_{B_s} = 5.367(8)_{\rm stat}(4)_{\rm sys}\,\mathrm{GeV}$$

Systematic errors

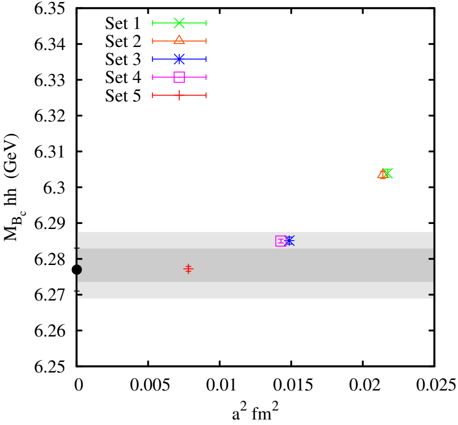
Various sources of systematic error and estimates:

- Spin indep. NRQCD syst:
  - Errors are $\mathcal{O}(\alpha_s^2 v^4)$ and v^6 in bottomonium
  - v^2 effects are $\sim$ 500 MeV
  - allow $0.3^2 \times 0.1 \times 500 = 4.5$ MeV and 5 MeV.
- Spin dep. NRQCD syst:
  - Only affects B_s due to spin average.
  - Dominant error from c_4 radiative corrections
  - Take 3/4 error in B_s hyperfine = 2.5 MeV
- EM: estimated at $\sim$ 0.1 MeV, include as error
- Finite volume effects: negligible
- $M_{b\bar{b},\mathrm{exp}}$:
  - Negligible effect on tuning.
  - $M_{b\bar{b},\mathrm{exp}} = 9.445(2)$ GeV adjusted for EM, annihilation.
  - 1 MeV error when reconstructing M_{B_s} from Δ_{B_s}
- M_{η_s}: 1.2 MeV error translates into 0.5 MeV in B_s

4 MeV total systematic error. Similar errors affect B_c

B_c heavy-heavy subtraction method

Two different methods of reconstructing the B_c mass



- Reconstruct mass using $\Delta_{B_c} = E_{B_c} - \frac{1}{2}(E_{b\bar{b}} + M_{\eta_c})$
- Discretisation errors set with scale $\Lambda = am_c$
- Error dominated by scale uncertainty and NRQCD syst

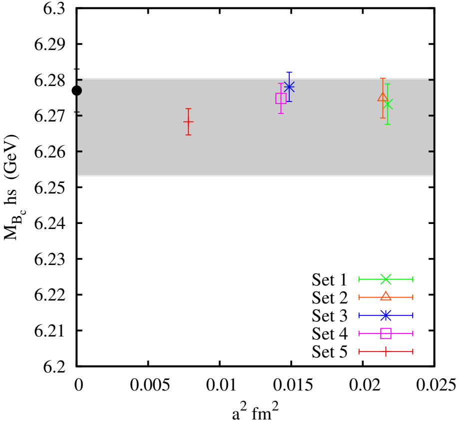
Result

$$M_{B_c} = 6.276(6)_{\rm stat}(8)_{\rm sys}\text{GeV}$$

Systematics in light grey

B_c heavy-strange subtraction method

Alternative splitting to compare systematics



- Reconstruct mass using $\Delta_{B_s} = E_{B_c} - (E_{B_s} + M_{D_s})$
- Scale error is 1/2 naive value when retuning is taken into account
- Discretisation errors set with scale $\Lambda = am_c$
- Error dominated by stats

Result

$$M_{B_c} = 6.267(14)_{\mathrm{stat}}\mathrm{GeV}$$

Systematics negligible

B meson

Heavy meson chiral perturbation theory used for fits [Jenkins 1992]

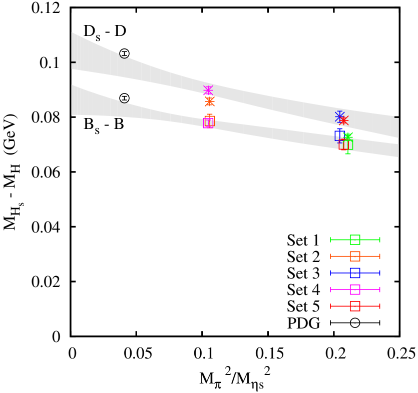
- NRQCD systematics cancel in $M_{B_s} - M_{B_d}$
- 1-loop formula up to M_π^3, including heavy meson spin symmetry breaking

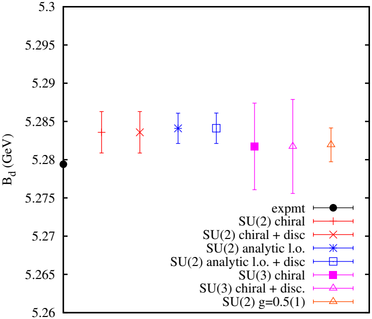
$$
\begin{aligned}
M_{B_s} - M_{B_d} \quad &= \quad -\frac{3}{4}(2a + 2\Delta^{(\sigma)})(m_s - m_l) - \frac{g^2\pi}{\Lambda^2}\left[\frac{3}{2}M_\pi^3 - 2M_K^3 - \frac{1}{2}M_\eta^3\right] \\
&\quad + \frac{3g^2\Delta}{4\Lambda^2}\left[-\frac{3}{2}l(M_\pi^2) + l(M_K^2) + \frac{1}{2}l(M_\eta^2)\right]
\end{aligned}
$$

- Chiral scale $\Lambda = 4\pi f_\pi$, leading hyperfine splitting Δ
- Prior of 0.5(5) on $BB^*\pi$ coupling g
- Chiral logarithms $l(M^2) = M^2\left(\ln\frac{M^2}{\Lambda^2} + \delta^{FV}(ML)\right)$
- Finite volume correction $\delta^{FV}(ML) = \frac{4}{ML}\sum_{\vec{n}\neq 0} K_1(|\vec{n}|ML)/|\vec{n}|$
- Very small partial quenching effect - m_s^{sea} well tuned
- Discretisation terms included $(1.0 + c_1(\Lambda a)^2 + c_2(\Lambda a)^4)$

$B_s - B$ #preliminary

Fit from $SU(2)$ with disc. terms





- Different fits give consistent results
- Error dominated by M_{B_s}
- Significant difference from HISQ $D_s - D$

Results:

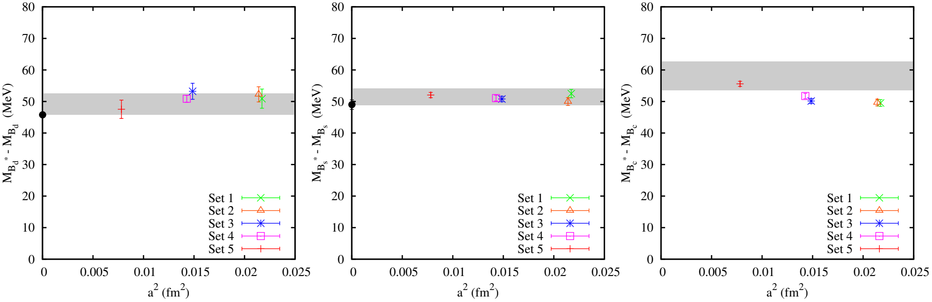
$$M_{B_s} - M_B = 83(3)\ \text{MeV}, \quad M_{D_s} - M_D = 98(4)\ \text{MeV}$$

$$M_B = 5.284(3)_{\text{stat/fit}}(8)_{B_s}\,\text{GeV}$$

B meson hyperfine splittings

Provides a good test of the radiative corrections

- smaller systematic error than bottomonium

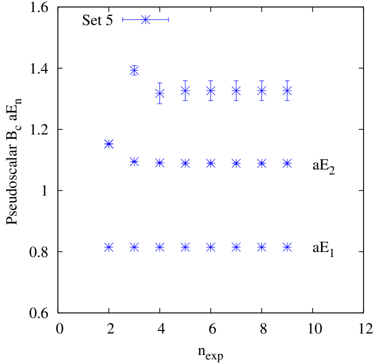


- Hyperfine splitting proportional to c_4, v^6 much smaller for heavy-light
- Correlated systematic for missing α_s^2 corrections (not shown)

$$
\begin{aligned}
\Delta^{\text{hyp}}_{B_d} &= 49(3)\text{MeV} \\
\Delta^{\text{hyp}}_{B_s} &= 51(3)\text{MeV} \\
\Delta^{\text{hyp}}_{B_c} &= 58(5)\text{MeV}.
\end{aligned}
$$

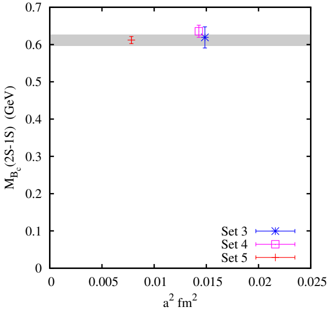
B_c radial excitations

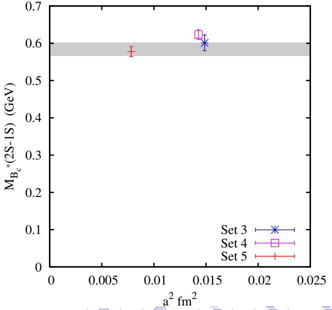
B_c correlators are accurate enough to extract first excited state on coarse and fine



These states are below *BD* threshold

$$M_{B_c'} - M_{B_c} = 612(15)\mathrm{MeV}$$
$$M_{B_c^{*\prime}} - M_{B_c^*} = 583(18)\mathrm{MeV}$$





Scalar and axial vector states

Staggered correlators have the form

$$C_{\text{meson}}(i,j,t) \quad = \quad \sum_{n=1}^{N_{\text{exp}}} b_{i,n} b^*_{j,n} e^{-E_n t} + \sum_{k=1}^{N_{\text{exp}}-1} d_{i,k} d^*_{j,k} (-1)^{t/a} e^{-E'_k t}$$

Energies of the oscillating states E'_k correspond to opposite parity states 0^+ and 1^+

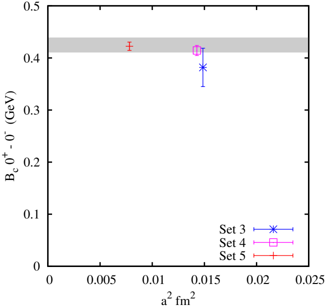
- Only clearly below threshold for the B_c - these are the B_{c0} and B_{c1}
- Identification for B_s, B_d is harder
- No light mass dependence and wrong energy for BK state so likely B_{s0} and B_{s1}

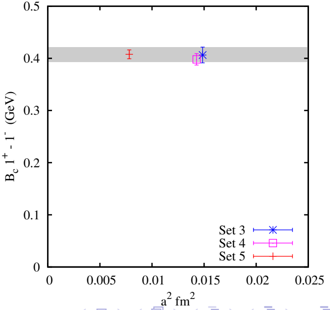
B_c scalar and axial-vector splittings

Signal on fine and coarse ensembles.
Results:

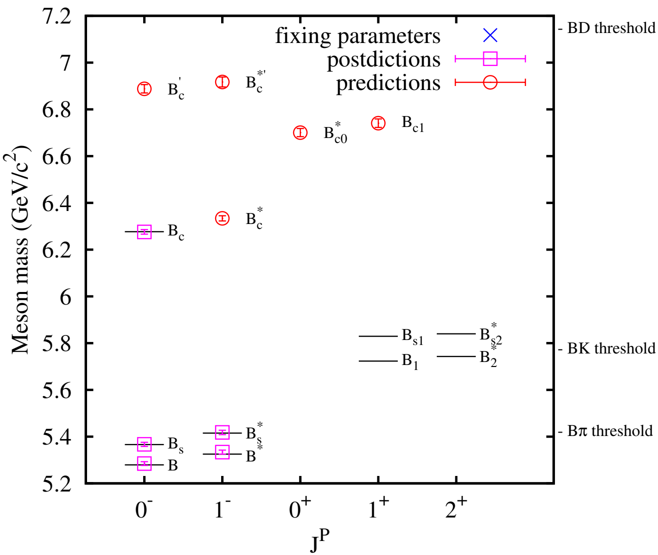
$$M_{B^*_{c0}} - M_{B_c} = 0.425(14)\ \mathrm{MeV}$$
$$M_{B_{c1}} - M_{B^*_c} = 0.407(14)\ \mathrm{MeV}$$

- $M_{B_{c1}}$ could be one of two 1^+ states
- Our result should project out lightest state
- Statistical error too large to distinguish





B-meson spectrum [PDG]



Summary

Meson masses from NRQCD in good agreement with experiment

- Radiative corrections to Wilson coefficients included for the first time
- Charm quarks included in the sea
- Ratios to $1.4, 2.4\%$ accuracy
- First prediction of D-wave splittings
- Accurate hyperfine splittings in HL and HH mesons

Future work:

- Leptonic width including current corrections
- f_B at the physical point
- Mixing amplitudes
- Spin dependent $\mathcal{O}(v^6)$ terms and 4-quark operators

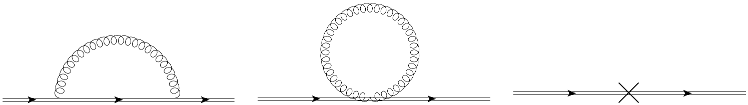
Appendix

Perturbative improvement - kinetic terms

- Matching coefficients in the action are a function of the cutoff am_b
- Short distance coefficients $\implies$ perturbative. They have the expansion:

$$c_i = 1 + \alpha_s c_i^{(1)} + \mathcal{O}(\alpha_s^2)$$

- $c_1^{(1)}, c_5^{(1)}, c_6^{(1)}$ are found by computing the NRQCD quark self-energy and ensuring the correct energy-momentum relation holds



- Diagrams generated automatically and evaluated numerically
- Gauge fields are tadpole improved to improve matching
- $\alpha_V(q)$ from heavy quark potential

$$V(q) = -\frac{C_f 4\pi\alpha_V(q)}{q^2}$$

- BLM scheme used to set the scale q^*

Perturbative improvement - c_4 and 4-quark terms

c_4 term controls size of the spin dependent splittings

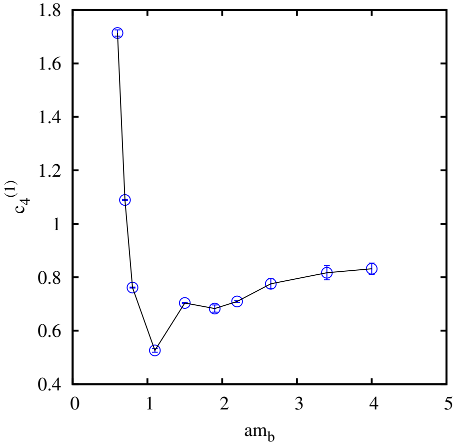
- Calculated by matching the effective action in NRQCD to continuum QCD using the background field method. [T. Hammant et al (2011)]

- Plot shows c_4 is well behaved in the region we are working
- Diverges for $am_b < 1$ as expected
- 4 quark terms:

$$S_{4q} = d_1 \frac{\alpha_s^2}{(am_b)^2} (\psi^\dagger \chi^*)(\chi^T \psi) + d_2 \frac{\alpha_s^2}{(am_b)^2} (\psi^\dagger \sigma \chi^*) \cdot (\chi^T \sigma \psi)$$

- Give a shift in the hyperfine of:

$$\Delta E_{\rm hyp} = \frac{6\alpha_s^2 (d_2 - d_1)}{m_b^2} |\psi(0)|^2$$



Gauge action

The action is a tadpole and one-loop improved Lüscher-Weisz action,

$$S_G = \beta\left[c_P \sum_P \left(1 - \frac{1}{3}\mathrm{ReTr(P)}\right) + c_R \sum_R \left(1 - \frac{1}{3}\mathrm{ReTr(R)}\right) + c_T \sum_T \left(1 - \frac{1}{3}\mathrm{ReTr(T)}\right)\right]$$

- Sums are over plaquettes P, rectangles R and twisted loops T
- Action is improved completely through order $\mathcal{O}(\alpha_s a^2)$
- Coefficients include gluonic loops and effect of N_f HISQ sea quarks
- Coefficients are

$$\begin{aligned} C_P &= 1.0 \\ C_R &= \frac{-1}{20u_{0P}^2}(1 - (0.6264 - 1.1746N_f)\log(u_{0P}^2)) \\ C_T &= \frac{1}{u_{0P}^2}(0.0433 - 0.0156N_f)\log(u_{0P}^2) \end{aligned} \tag{1}$$

Scale setting

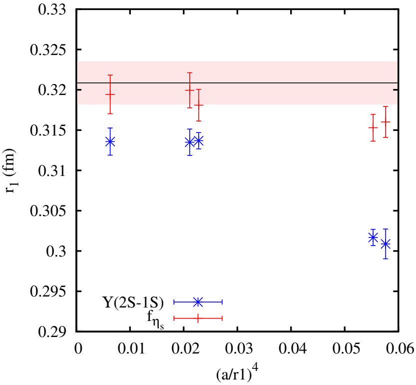
$\Upsilon(2S-1S)$ used to fix the lattice spacing

- Small systematic error
- Weak am_b dependence
  - insensitive to mistuning
- Accuracies of $\sim 1\%$ achieved

Set	a_Υ (fm)
1	0.1474(5)(14)(2)
2	0.1463(3)(14)(2)
3	0.1219(2)(9)(2)
4	0.1195(3)(9)(2)
5	0.0884(3)(5)(1)

Errors are: (stat)(syst)(expmt/EM)

- Also calculated with HISQ f_{η_s}
- Comparision of two methods by calculating r_1 using MILC r_1/a values

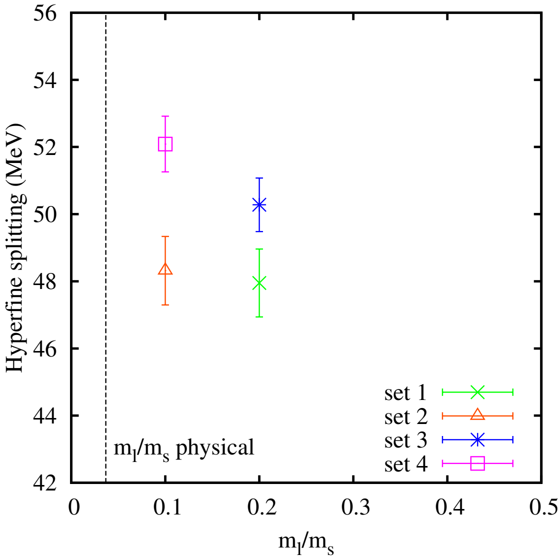


- Combined gives $N_f = 2+1+1$ value of

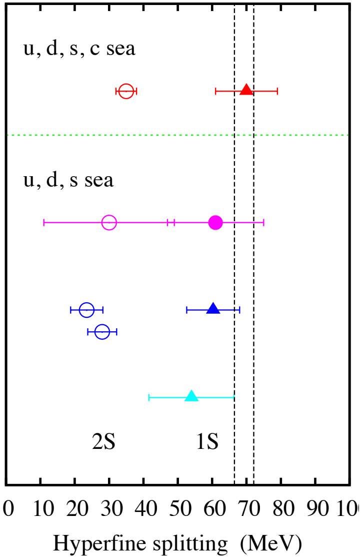
$$r_1 = 0.3209(26)\mathrm{fm}$$

S-wave hyperfine splitting

No significant evidence for light quark mass dependence in raw $c_4 = 1$ data



Comparison of hyperfine splittings



HPQCD(NRQCD) 2011

HPQCD(NRQCD) 2005

Meinel(NRQCD) 2010

Fermilab/MILC 2010

Tuned b quark masses

We can give tuned am_b values for each ensemble assuming $M_{Kin} = 2m_b + B$

Errors are:

- Stats from lattice spacing
- Syst from lattice spacing
- Stats from M_{Kin}
- Syst from M_{Kin}

Set	$am_b(a_\Upsilon)$
1	3.297(11)(35)(7)(16)
2	3.263(7)(35)(4)(16)
3	2.696(4)(22)(7)(13)
4	2.623(7)(22)(7)(13)
5	1.893(6)(12)(5)(9)

Quark masses are well tuned for sets 3,4,5

m_b/m_s

Quark mass ratio obtained by converting tuned masses to $\overline{MS}$ via the pole mass

$$\frac{m_b^{\overline{MS}}(\mu)}{m_s^{\overline{MS}}(\mu)} = \frac{am_b}{am_s}\left[1 + \alpha_s(A^{\mathrm{NRQCD}} - A^{\mathrm{HISQ}}) + \mathcal{O}(\alpha_s^2)\right]$$

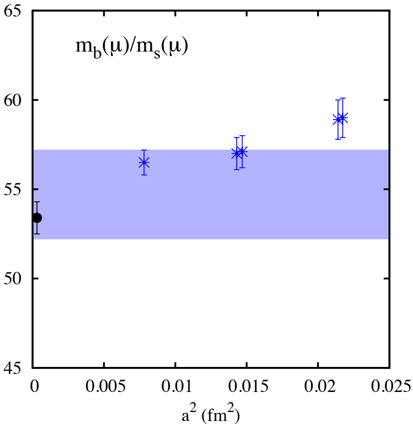
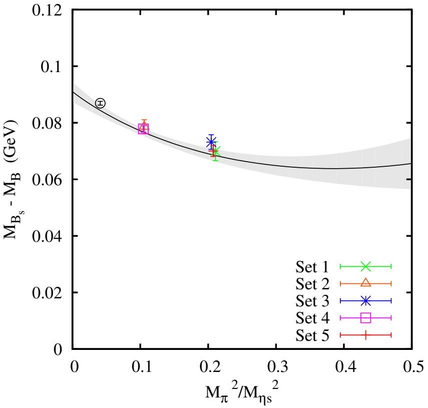


Figure: Plot of $m_b(\mu)/m_s(\mu)$ in $\overline{\mathrm{MS}}$

- Using tuned values for HISQ s quark
- Mass renormalisations calculated perturbatively to $\mathcal{O}(\alpha_s)$
- Result: $m_b/m_s = 54.7(2.5)$
- Error dominated by missing $\mathcal{O}(\alpha_s^2)$ corrections
- Compared to previous HISQ/HISQ ratio (black), independent results agree

$B_s - B$: different g prior

Changing the prior on $g_{BB^*\pi}$ to 0.5(1) does not alter the fit significantly.
Fit from $SU(2)$ with disc. terms:



Result:

$$M_B = 5.283(2)_{\rm stat/fit}(8)_{\rm B_s}\,\mathrm{GeV}$$