

Exclusive semileptonic B and Λ_b decays at large hadronic recoil

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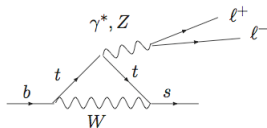
Workshop: Beautiful Mesons and Baryons on the Lattice,

ECT* Trento, 2-6 April 2012

Part 1: $B \rightarrow K\ell^+\ell^-$ at large hadronic recoil

$b \rightarrow s$ flavour-changing neutral currents (FCNC)

- $b \rightarrow sl^+l^-$, via virtual $t, Z, W \oplus$ New Physics?



\Rightarrow effective local operators
 $C(m_b) \otimes \{\bar{s} b \bar{l} l\}$

- experiment (LHCb) prefers exclusive channels,
 $B \rightarrow K l^+ l^-$, $B \rightarrow K^* l^+ l^-$, $B_s \rightarrow \phi l^+ l^-$,
- the accuracy improving :
recently, LHCb observed the zero in the FB asymmetry in $B \rightarrow K^* l^+ l^-$
- can the theory provide a competitive accuracy?
(in what follows: mainly $B \rightarrow K l^+ l^-$ mode)

The anatomy of the effective Hamiltonian

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \Big|_{\mu \sim m_b}$$

- local $b \rightarrow sll$ operators:

$$O_{9(10)} = \frac{\alpha_{em}}{4\pi} [\bar{s}_L \gamma_\mu b_L] \ell \gamma^\mu (\gamma_5) \ell, \quad C_{9(m_b)} \simeq 4.4$$

$$C_{10(m_b)} \simeq -4.7,$$

- operators combined (nonlocally) with e.m. lepton-pair emission:

$$O_7 = -\frac{em_b}{8\pi^2} [\bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b] F^{\mu\nu}, \quad C_7(m_b) \simeq -0.3$$

$$O_1^{(c)} = [\bar{s}_L \gamma_\rho c_L] [\bar{c}_L \gamma^\rho b_L], \quad C_1(m_b) \simeq 1.1$$

$$O_2^{(c)} = [\bar{c}_L \gamma_\rho c_L] [\bar{s}_L \gamma^\rho b_L], \quad C_2(m_b) \simeq -0.25$$

$$O_{8g} = -\frac{m_b}{8\pi^2} \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b G^{\mu\nu}, \quad C_8(m_b) \simeq 0.2$$

$$O_{3-6} \text{ - quark-penguin operators, } \quad C_{3,4,5,6} < 0.03$$

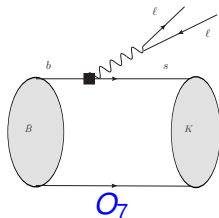
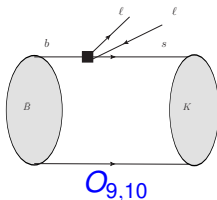
- neglected the $\sim V_{ub} V_{us}^*$ part

$B \rightarrow K \ell^+ \ell^-$ decay in Standard Model

- the decay amplitude:

$$\begin{aligned}
 A(B \rightarrow K \ell^+ \ell^-) &= \langle K \ell^+ \ell^- | H_{\text{eff}} | B \rangle \\
 &= -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i \langle K \ell^+ \ell^- | O_i | B \rangle
 \end{aligned}$$

- dominant contributions factorize: $\Rightarrow B \rightarrow K$ form factors

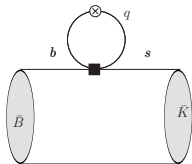


$$\langle K(p) | \bar{s} \gamma_\mu b | B(p+q) \rangle \Rightarrow f_{BK}^+(q^2), \quad \langle K(p) | \bar{s} \sigma_{\mu\nu} b | B(p+q) \rangle \Rightarrow f_{BK}^T(q^2)$$

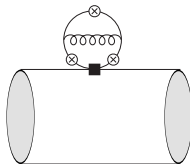
- additional **nonlocal** contributions: combining O_{1-2}^c , O_{3-6} , O_{8g} with the emission of $\gamma^* \rightarrow \ell^+ \ell^-$

The nonlocal effects in $B \rightarrow K\ell^+\ell^-$

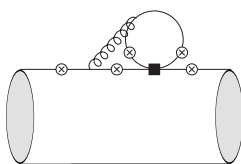
- 4-quark operators: $O_{1,2}^c$, O_{3-6}^q ($q = u, d, s, c, b$)



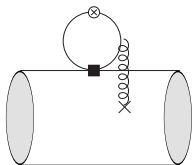
LO loop



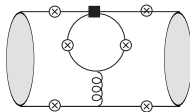
NLO loops



\Rightarrow factorizable



soft gluon emission

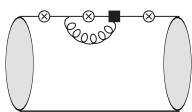


spectator scatt.

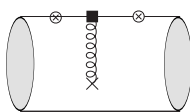
\Rightarrow nonfactoriz.

The nonlocal effects in $B \rightarrow K\ell^+\ell^-$

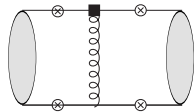
- gluon-penguin operator O_{8g}



factoriz. NLO

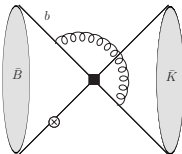
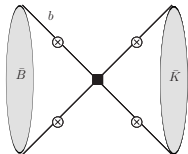


nonfact.: soft gluon



spectator scatt.

- weak annihilation: O_{3-6}^q $q = u, d$



Including nonlocal effects in the decay amplitude

- schematically:

$$A(B \rightarrow K \ell^+ \ell^-) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[\sum_{i=7,9,10} C_i \otimes (\bar{\ell} \ell) \otimes f_{BK}^i + \sum_{i=1,2,3..6,8} C_i \otimes (\bar{\ell} \ell) \otimes \mathcal{H}_{BK}^i \right]$$

- hadronic matrix elements of nonlocal contributions:

$$\mathcal{H}_{BK}^i(p, q) = \langle K(p) | \int d^4x e^{iq \cdot x} T \{ j^{em}(x), O^i(0) \} | B(p+q) \rangle$$

- in LO reducible to form factors \otimes loop factor (OPE)
- first estimates of NLO contributions to $B \rightarrow K^{(*)} \ell^+ \ell^-$ in the QCD factorization scheme ($m_b \rightarrow \infty$)
at small q^2 (large hadronic recoil)

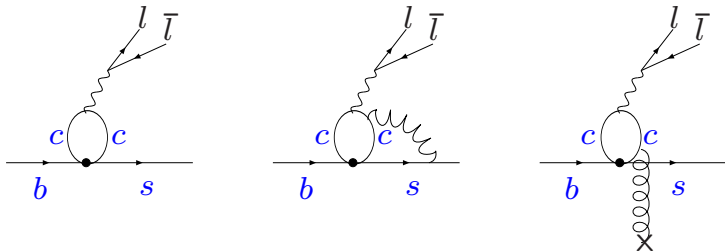
[Beneke, Feldmann, Seidel (2001)], ...

QCD calculation of $B \rightarrow K \ell^+ \ell^-$

- $B \rightarrow K$ form factors:
 - lattice QCD,
 - light-cone QCD sum rules (LCSR)
(the same accuracy as LCSR for $B \rightarrow \pi$ FF's including $m_s \neq 0$ effects)
- \mathcal{H}_{BK}^i , the size of nonfactorizable **nonperturbative** contributions? ("soft" gluons, vacuum quark-antiquark pairs)
- the dominant part : $\mathcal{H}_{BK}^{1,2}$ due to $O_{1,2}^c$ (**charm loops**)

Charm-loops in $B \rightarrow Kl^+l^-$

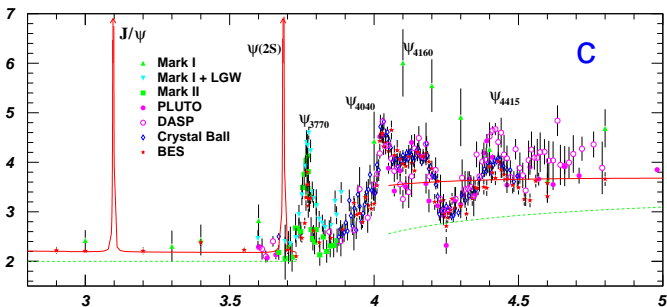
- Charm-loop effect: a combination of the $(\bar{s}c)(\bar{c}b)$ weak interaction ($O_{1,2}$) and e.m. interaction $(\bar{c}c)(\bar{l}l)$



- similar u, d, s, b -quark loops: suppressed by Wilson coeffs. (quark-penguin operators O_{3-6}) or by CKM (u -loops from $O_{1,2}^u$)

Charm loop turns charmonium

- at $q^2 \rightarrow m_{J/\psi}^2$, $\bar{c}c$ loop becomes a **hadronic state**:
e.g., $B \rightarrow K \ell^+ \ell^- = \{ B \rightarrow J/\psi K \otimes J/\psi \rightarrow \ell^+ \ell^- \}$
- heavier ψ -levels (charmonia with $J^P = 1^-$) at $q^2 = m_\psi^2$,
 $\bar{c}c$ states with the masses up to $m_B - m_K^{(*)} \simeq 4.8\text{GeV} (\simeq 4.4\text{GeV})$
- spectrum of ψ states as seen in $e^+ e^- \rightarrow \text{hadrons}$



[PDG, V.V. Ezhela et al. hep-ph/0312114]

The $B \rightarrow \psi K^{(*)}$ amplitudes

- naive factorization fails in $B \rightarrow \psi K^{(*)}$, ($\psi = J/\psi, \psi(2S)$),

$$A(B \rightarrow \psi K) \sim \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (c_1(\mu) + c_2(\mu)/3) f_\psi f_{BK}^+(q^2 = m_\psi^2)$$

yields $\Gamma(B \rightarrow \psi K) \ll \text{exp}$.

- indicating large nonfactorizable contributions
(which should effectively remove the μ -dependence)
- QCD factorization should not (and does not) work in these decays, due to heavy-mass final states

How to account for the charm-loop effect?

- $\bar{c}c$ loops \oplus ψ resonances - a double counting !!
- the experimentalists (BABAR,Belle,CDF,LHCb):
subtract the “bins” of J/ψ and $\psi(2S)$ from the q^2 -distribution data in $B \rightarrow K^{(*)} \ell^+ \ell^-$
- the effect of intermediate/virtual $\bar{c}c$ states remains at $q^2 \ll m_{J/\psi}^2$ (nonperturbative at $q^2 \sim 4m_c^2$)
- Can we use the { loop \oplus corrections } ansatz in the small $q^2 \ll 4m_c^2$ region (large hadron recoil) ?
- soft-gluon emission from c -quark loop from OPE on the light-cone

[A.K., Th. Mannel, A. Pivovarov and Yu-M. Wang, 1006.4945 [hep-ph]]

Charm-loop in $B \rightarrow Kl^+l^-$ at $q^2 \ll 4m_c^2$

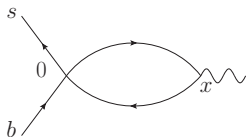
- the hadronic matrix element:

$$\mathcal{H}_{BK}^\mu(p, q) = i \langle K(p) | \int d^4x e^{iq \cdot x} T \left\{ \bar{c}(x) \gamma^\mu c(x), \right. \\ \left. [C_1 [\bar{s}_L(0) \gamma_\rho c_L(0) \bar{c}_L(0) \gamma^\rho b_L(0)] + C_2 \dots] \right\} | B(p+q) \rangle$$

- at $q^2 \ll 4m_c^2$, contracting c -quark fields in propagators: $\langle c(0) \bar{c}(x) \rangle$ and $\langle c(x) \bar{c}(x) \rangle$, including gluon emission:

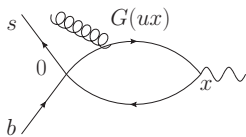
Expansion near the light-cone

- at $q^2 \ll 4m_c^2$, the dominant region: $\langle x^2 \rangle \sim 1/(2m_c - \sqrt{q^2})^2$
- T - product of $\bar{c}c$ -operators can be expanded near the light-cone $x^2 \sim 0$, diagrammatically:



the simple loop

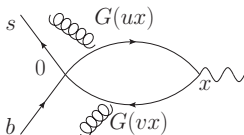
(unit operator LO \oplus NLO)



one-gluon emission

(non-local, $x \neq 0$)

($\bar{s}(0)G(x)b(0)$)



two-gluon emission

.....

The resulting effective operators

- LO reduced to simple $\bar{c}c$ -loop,
no difference between local and LC,

$$\mathcal{O}_\mu(q) = (q_\mu q_\rho - q^2 g_{\mu\rho}) \frac{9}{32\pi^2} g(m_c^2, q^2) \bar{s}_L \gamma^\rho b_L.$$

- gluon emission: use c -quark propagator near the light-cone in the external gluon field [I. Balitsky, V. Braun (1999)]
- define LC kinematics (n_\pm) in the rest-frame of B ,
 $q \simeq (m_b/2)n_+$
- one-gluon emission yields a new **nonlocal** operator:

$$\tilde{\mathcal{O}}_\mu(q) = \int d\omega I_{\mu\rho\alpha\beta}(q, m_c, \omega) \bar{s}_L \gamma^\rho \delta[\omega - \frac{(in_+ \mathcal{D})}{2}] \tilde{G}_{\alpha\beta} b_L,$$

Hadronic matrix elements for the charm-loop effect

- the LO: factorized $\bar{c}c$ loop

$$\left[\mathcal{H}_{BK}^\mu(p, q) \right]_{\text{fact}} = \left(\frac{C_1}{3} + C_2 \right) \langle K(p) | \mathcal{O}^\mu(q) | B(p+q) \rangle,$$

- reduced to $f_{BK}^+(q^2)$ form factors, (\otimes loop function)
- The gluon emission yields:

$$\left[\mathcal{H}_{BK}^\mu(p, q) \right]_{\text{nonfact}} = 2C_1 \langle K(p) | \tilde{\mathcal{O}}^\mu(q) | B(p+q) \rangle.$$

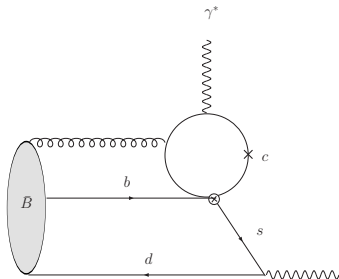
- new hadronic matrix element (\otimes coeff.function)

$$\langle K(p) | \bar{s}_L \gamma^\rho \delta[\omega - \frac{(in+\mathcal{D})}{2}] \tilde{G}_{\alpha\beta} b_L | B(p+q) \rangle,$$

LCSR for the soft-gluon hadronic matrix element

- the correlation function:

$$\mathcal{F}_{\nu\mu}^{(B\rightarrow K)}(p, q) = i \int d^4y e^{ip\cdot y} \langle 0 | T \{ j_\nu^K(y) \tilde{\mathcal{O}}_\mu(q) \} | B(p+q) \rangle,$$



- hadronic dispersion relation in the kaon channel

$$\mathcal{F}_{\nu\mu}^{(B\rightarrow K)}(p, q) = \frac{if_K p_\nu}{m_K^2 - p^2} \langle K(p) | \tilde{\mathcal{O}}^\mu(q) | B(p+q) \rangle + \int_{s_h}^{\infty} ds \frac{\tilde{\rho}_{\mu\nu}(s, q^2)}{s - p^2}$$

- B meson DA's used as nonperturbative input

Nonlocal effects in $B \rightarrow K\ell^+\ell^-$ in terms of ΔC_9

- the Wilson coefficient $C_9(\mu = m_b) \simeq 4.4$,
a process-dependent correction to be added:

$$\Delta C_9^{BK}(q^2) \sim \frac{\sum_i \mathcal{H}_{BK}^i(q^2)}{f_{BK}^+(q^2)}$$

- previously we investigated only the role of c -loop soft nonfact. effect vs LO and found

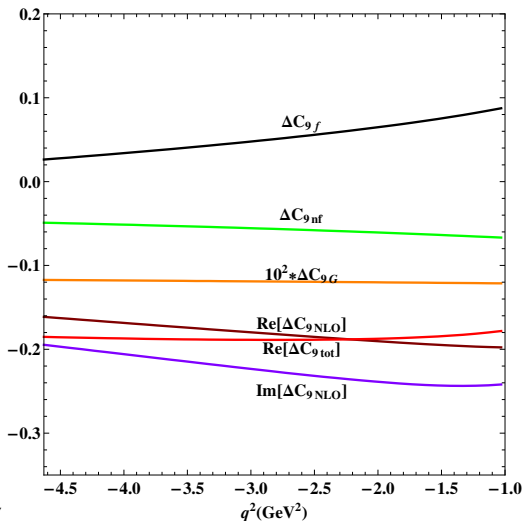
$$\Delta C_9^{BK(\bar{c}c, LO \oplus nf)}(q^2 = 0) \sim 0.2$$

Nonlocal effects in $B \rightarrow K\ell^+\ell^-$ in terms of ΔC_9

[A.K., Th.Mannel, Y.M. Wang, work in progress]

- preliminary results including nonlocal effects due to
 - $O_{1,2}^C$ LO and soft nonfact.
 - O_{3-6} LO and soft nonfact.
 - O_8 soft nonfact. (new LCSR estimate)
 - $O_{1,2}^C$ NLO two-loop factorizable contributions using diagrams from [Asatrian, Greub, Walker(2002)] ,
 - $O_{1,2}^C$ NLO spectator contributions [Beneke, Feldmann, Seidel (2001)], ...
- we use OPE results at negative q^2 having in mind the presence of light-quark loops

Nonlocal effects for $B \rightarrow K\ell^+\ell^-$ in spacelike region



preliminary

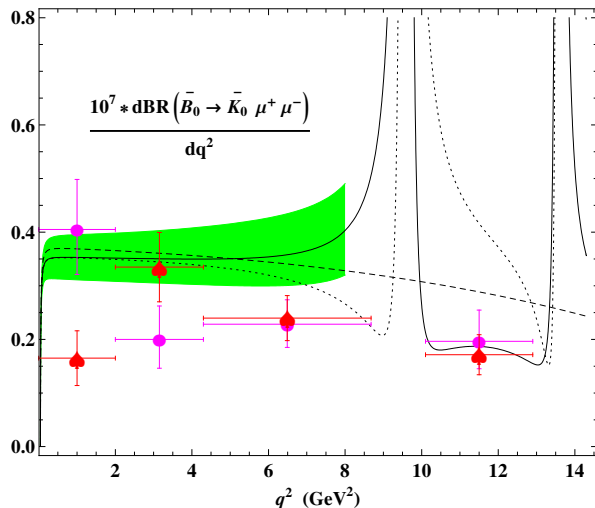
Accessing the timelike q^2 region ?

- analyticity of the hadronic matrix element in q^2 ,
⊕ unitarity \Rightarrow **hadronic dispersion relation**:

$$\mathcal{H}^{(B \rightarrow K)}(q^2) = \mathcal{H}^{(B \rightarrow K)}(0) + q^2 \left[\sum_{\psi=J/\psi, \psi(2S), \dots} \frac{f_\psi A_{B\psi K}}{m_\psi^2 (m_\psi^2 - q^2 - im_\psi \Gamma_\psi^{tot})} + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s - q^2 - i\epsilon)} \right]$$

- the residues $|A_{B\psi K}|$ and $|f_\psi|$ determined by $BR(B \rightarrow \psi K)$, $BR(\psi \rightarrow \ell^+ \ell^-)$
- FSI phase attributed to $A(B \rightarrow \psi K)$, (Im part in $(p + q)^2$)
- a certain ansatz (z-parameterization) used for the integral
- dispersion relation fitted to the calculated $\mathcal{H}^{(B \rightarrow K)}$ at $q^2 < 0$

The width of $B \rightarrow K\ell\ell$ [preliminary]

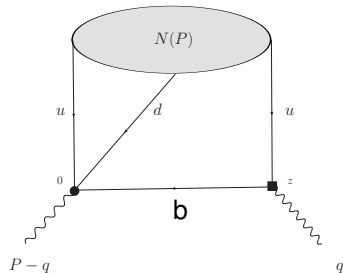


- exp. data on $B \rightarrow K\ell\ell$, [Belle '09 (violet), CDF'11(red)]
- solid (green shaded-theor. uncertainties) and dotted - two different ansaetze for dispersion relation; dashed - without nonlocal effects

Part 2: $\Lambda_b \rightarrow p$ form factors from LCSR

[A.K., Ch.Klein, Th.Mannel, Y.-M. Wang arXiv:1108.2971]

The LCSR method



- vacuum-to-nucleon correlation function:

$$\Pi_{\mu(5)}(P, q) = i \int d^4z e^{iq \cdot z} \langle 0 | T \{ \eta_{\Lambda_b}(0), \bar{b}(z) \gamma_\mu (\gamma_5) u(z) \} | N(P) \rangle .$$

- $q^2 \ll m_b^2$, $(P - q)^2 \ll m_b^2$, $P^2 = m_N^2$,
- Λ_b interpolating 3-quark current, we use

$$\eta_{\Lambda_b}^{(P)} = (u C \gamma_5 d) b, \quad \eta_{\Lambda_b}^{(A)} = (u C \gamma_5 \gamma_\lambda d) \gamma^\lambda b .$$

Nucleon Distribution Amplitudes (DA's)

[V.Braun, A.Lenz et al (2000-2009)],

- definition, schematically ($z^2 \sim 0$):

$$\langle 0 | \epsilon^{ijk} u_{\alpha}^i(0) u_{\beta}^j(z) d_{\gamma}^k(0) | N(P) \rangle = \sum_t S_{\alpha\beta\gamma}^t \times \int dx_1 dx_2 dx_3 \delta(1 - \sum_{i=1}^3 x_i) e^{-ix_2 P \cdot z} F_t(x_i, \mu),$$

- twist expansion: 27 DA's of twist 3,4,5,6
- coefficients and normalization parameters determined from 2-point sum rules
- proton e.m. form factors were calculated from LCSR

Accessing the $\Lambda_b \rightarrow p$ form factors

- hadronic dispersion relation, schematically

$$\begin{aligned} \Pi_{\mu(5)}(P, q) &= \frac{\langle 0 | \eta_{\Lambda_b} | \Lambda_b \rangle \langle \Lambda_b | \bar{b} \gamma_\mu (\gamma_5) u | N \rangle}{m_{\Lambda_b}^2 - (P - q)^2} \\ &+ \frac{\langle 0 | \eta_{\Lambda_b} | \Lambda_b^* \rangle \langle \Lambda_b^* | \bar{b} \gamma_\mu (\gamma_5) u | N \rangle}{m_{\Lambda_b^*}^2 - (P - q)^2} + \int_{s_0^h}^{\infty} ds \frac{\rho_{\mu(5)}(s, q^2)}{s - (P - q)^2} \end{aligned}$$

- 6 form factors, standard definitions (cf nucleon β decay):

$$\langle \Lambda_b(P - q) | \bar{b} \gamma_\mu u | N(P) \rangle = \bar{u}_{\Lambda_b}(P - q) \left\{ f_1(q^2) \gamma_\mu + i \frac{f_2(q^2)}{m_{\Lambda_b}} \sigma_{\mu\nu} q^\nu + \frac{f_3(q^2)}{m_{\Lambda_b}} q_\mu \right\} u_N(P),$$

$$0 \leq q^2 \leq (m_{\Lambda_b}^2 - m_N^2), \quad \gamma_\mu \rightarrow \gamma_\mu \gamma_5, \quad f_i(q^2) \rightarrow g_i(q^2)$$

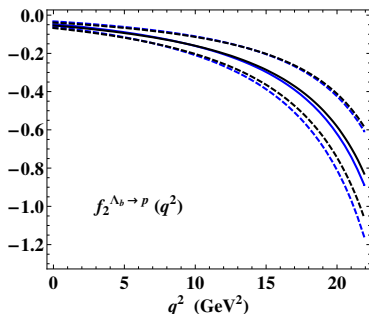
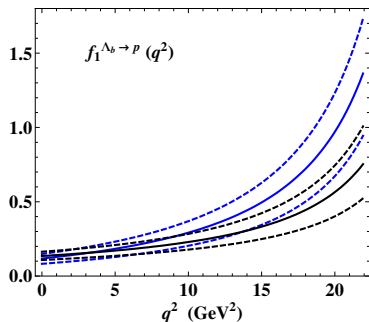
- decay constant of Λ_b from two-point sum rules

LCSR in detail

- specific problems for baryon QCD sum rules
 - the contributions of Λ_b^* , ($J^P = 1/2^-$ state, $m_{\Lambda_b^*} - m_{\Lambda_b} \sim 200 - 300$ MeV)
we used linear combinations of kinematical structures in the correlation function to eliminate Λ_b^*
 - baryon interpolating current: multiple choice
we used pseudoscalar and axial currents
- replace b by c in LCSR $\Rightarrow \Lambda_c \rightarrow N$ form factors
(used to calculate strong couplings)
- inputs:
finite m_b , a few universal parameters of nucleon DA's ,
two-point sum rules for η_{Λ_b} currents:

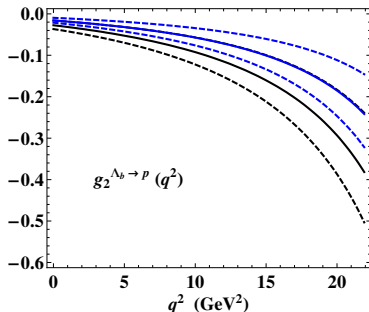
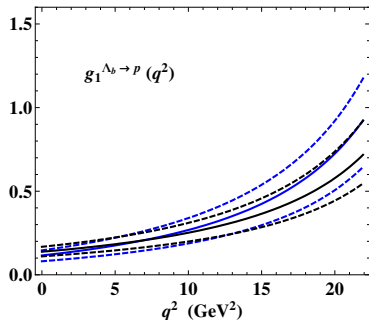
$$\lambda_{\Lambda_b}^{(\mathcal{A})} = 1.27_{-0.34}^{+0.35} \times 10^{-2} \text{ GeV}^2, \quad \lambda_{\Lambda_b}^{(\mathcal{P})} = 1.09_{-0.30}^{+0.31} \times 10^{-2} \text{ GeV}^2,$$

Numerical results for the $\Lambda_b \rightarrow p$ vector form factors



- $q^2 \leq 11 \text{ GeV}^2$ direct calculation from LCSR, at larger q^2 z-parameterization and extrapolation
- reasonable agreement between sum rules with different baryon currents

Numerical results for the axial-vector $\Lambda_b \rightarrow p$ form factors

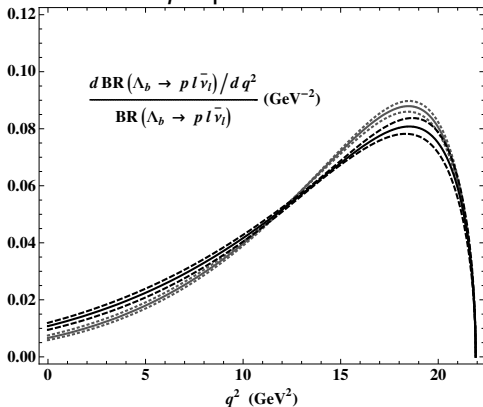


The width of $\Lambda_b \rightarrow p l \nu_l$ decay

- can be used to extract $|V_{ub}|$

$$\frac{d\Gamma}{dq^2}(\Lambda_b \rightarrow p l \nu_l) = \frac{G_F^2 m_{\Lambda_b}^3}{192\pi^3} |V_{ub}|^2 \left\{ k_1(q^2, m_{\Lambda_b}, m_N) |f_1(q^2)|^2 + \dots \right\}$$

- normalized q^2 -spectrum



The width of $\Lambda_b \rightarrow p\ell\nu_\ell$ decay

- partially integrated width: pure prediction of LCSR

$$\begin{aligned}\Delta\zeta(0, q_{max}^2) &= \frac{1}{|V_{ub}|^2} \int_0^{q_{max}^2} dq^2 \frac{d\Gamma}{dq^2}(\Lambda_b \rightarrow p\ell\nu_\ell) \\ &= 5.5_{-2.0}^{+2.5} \text{ ps}^{-1} \left(= 5.6_{-2.9}^{+3.2} \text{ ps}^{-1} \right)\end{aligned}$$

for axial-vector (pseudoscalar) interpolating current of Λ_b

- improvements in the future possible: nucleon DA parameters, α_S corrections

Conclusions

- with LC OPE and LCSR the effects of four-quark and penguin operators in $b \rightarrow sl^+l^-$ exclusive decays can be systematically accounted including soft-gluon effects in the region $q^2 < \text{few GeV}^2$,
- $B \rightarrow Kl^+l^-$ allows for more accurate treatment than $B \rightarrow K^*l^+l^-, K^*\gamma$ both in FF's and nonlocal effects
- are (or will be) the \mathcal{H}_{BK}^i amplitudes accessible on the lattice ?
- Λ_b form factors can be calculated from LCSR

*both topics will be discussed further
in the talk by Thorsten Feldmann*

BACKUP SLIDES

The hierarchy of contributions in LC OPE

after integrating over x and taking hadronic matrix element

- each extra gluon brings one power of $\sim \frac{\Lambda_{QCD}^2}{4m_c^2 - q^2}$ suppression
- perturbative gluon corrections are α_S suppressed
- reexpanding the one-gluon nonlocal operator near $x = 0$ in derivatives of $G_{\mu\nu}(0)$:

$$\text{term with } k\text{-th derivative} \Rightarrow \sum_{k=0}^{\infty} \frac{(q\Lambda_{QCD})^k}{(4m_c^2 - q^2)^{k+1}}$$

$$q \sim m_b/2 \text{ and } m_b\Lambda_{QCD} \sim m_c^2.$$

the OPE near the light-cone works, but not the local OPE

The local OPE limit

- $\omega \rightarrow 0$ in the nonlocal operator, no derivatives of $G_{\mu\nu}$

$$\tilde{O}_\mu^{(0)}(q) = I_{\mu\rho\alpha\beta}(q) \bar{s}_L \gamma^\rho \tilde{G}_{\alpha\beta} b_L ,$$

$$I_{\mu\rho\alpha\beta}(q, m_c) = (q_\mu q_\alpha g_{\rho\beta} + q_\rho q_\alpha g_{\mu\beta} - q^2 g_{\mu\alpha} g_{\rho\beta}) \\ \times \frac{1}{16\pi^2} \int_0^1 dt \frac{t(1-t)}{m_c^2 - q^2 t(1-t)}$$

At $q^2 = 0$, the quark-gluon operator obtained

in $B \rightarrow X_S \gamma$ in [M.Voloshin (1997)]

in $B \rightarrow K^* \gamma$ [A.K., G. Stoll, R. Rueckl, D. Wyler (1997)]

- the necessity of resummation was discussed before
[Z. Ligeti, L. Randall and M.B. Wise, (1997);
A.K. Grant, A.G. Morgan, S. Nussinov and R.D. Peccei (1997);
J. W. Chen, G. Rupak and M. J. Savage, (1997);
G. Buchalla, G. Isidori and S.J. Rey (1997)]

Charm-loop effect for $B \rightarrow K^* \ell^+ \ell^-$

- factorizable part determined by the three $B \rightarrow K^*$ form factors $V^{BK^*}(q^2)$, $A_1^{BK^*}(q^2)$, $A_2^{BK^*}(q^2)$,
- three kinematical structures for the nonfactorizable part:

$$\Delta C_9^{(\bar{c}c, B \rightarrow K^*, V)}(q^2) = (C_1 + 3C_2) g(m_c^2, q^2) - 2C_1 \frac{32\pi^2}{3} \frac{(m_B + m_{K^*}) \tilde{A}_V(q^2)}{q^2 V^{BK^*}(q^2)},$$

- nonfactorizable part enhances the effect, $1/q^2$ factor

$$\Delta C_9^{(\bar{c}c, B \rightarrow K^*, V)}(1.0 \text{ GeV}^2) = 0.7^{+0.6}_{-0.4}$$

$$\Delta C_9^{(\bar{c}c, B \rightarrow K^*, A_1)}(1.0 \text{ GeV}^2) = 0.8^{+0.6}_{-0.4}$$

$$\Delta C_9^{(\bar{c}c, B \rightarrow K^*, A_2)}(1.0 \text{ GeV}^2) = 1.1^{+1.1}_{-0.7}$$

