D and K Physics from Fermilab/MILC

Elvira Gámiz

for Lattice Fermilab/MILC Collaboration

Universidad de Granada

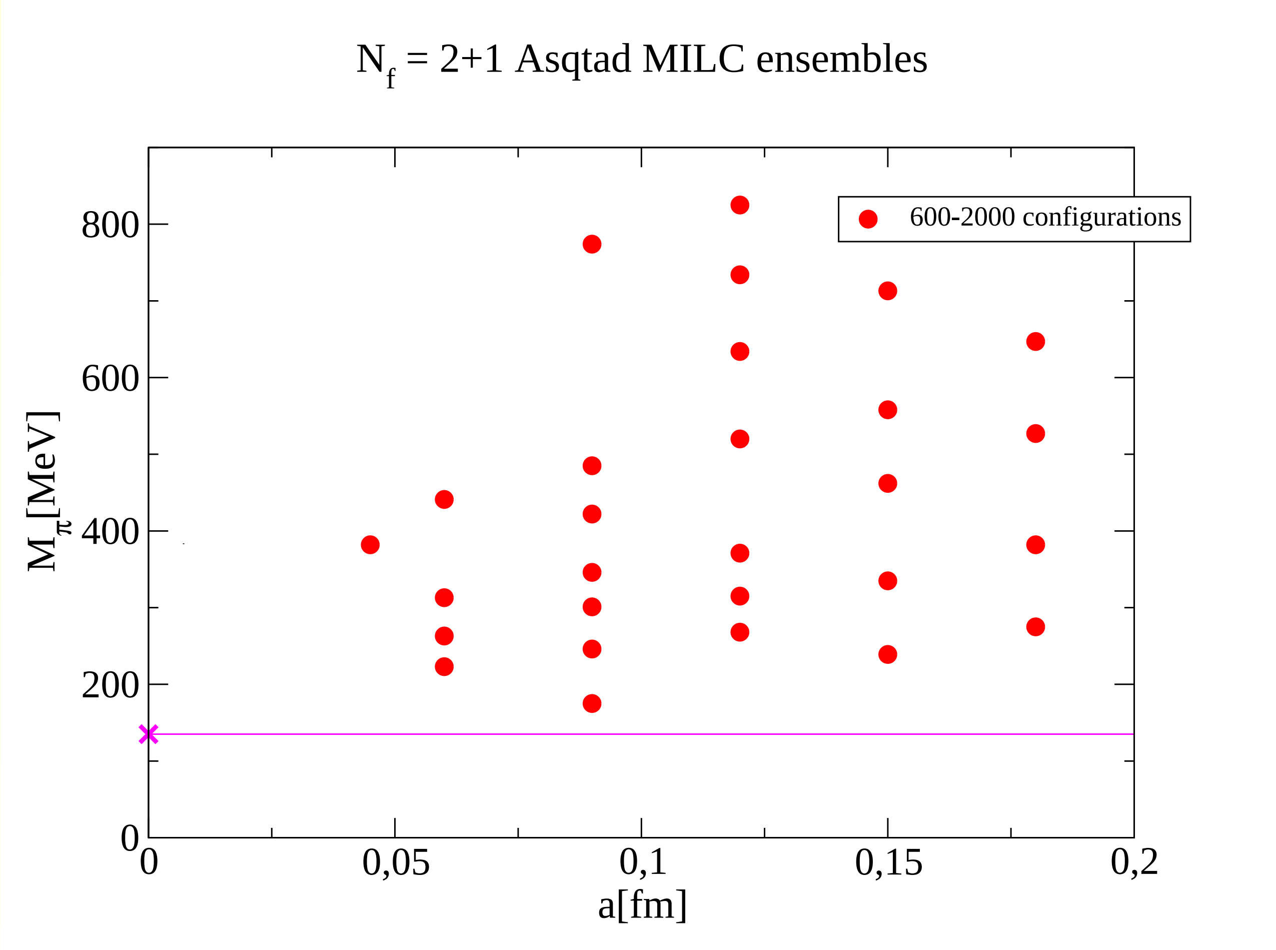
Beautiful Mesons and Baryons on the Lattice

· Trento (Italy) 2-6 March 2012 ·

Contents

1. Introduction

Last ten years: sucessful calculations of B-mesons and $D-$mesons weak matrix elements using Asqtad $N_f = 2+1$ MILC ensembles.



$D \to K(\pi) l\nu$, $B \to \pi l\nu$, $B \to D^{(*)} l\nu$, $B \to Dl\nu / B_s \to D_s l\nu$, decay constants f_B, f_{B_s}, f_D, f_{D_s}. And future plans: B and B_s mixing, $B \to Kll$, $B_s \to Kl\nu$ ... see **Andreas Kronfeld's** talk

1. Introduction: Lattice actions

Sea quarks: $N_f = 2 + 1$ **MILC** configurations with improved staggered Asqtad u, d and s sea quarks, and improved glue

A. Bazavov *et al*, arXiv:0903.3598, and refs. therein

* Asqtad: Tree-level order a^2 effects removed
  $\rightarrow$ leading errors are $\mathcal{O}(\alpha_s a)$, $\mathcal{O}(a^4)$

* One-loop Symanzik-Weisz improved gauge action (with gluonic $\mathcal{O}(\alpha_s)$ corrections but not $\mathcal{O}(n_f \alpha_s)$ corrections)

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Valence c and b: Fermilab action **El-Khadra et al**, PRD 1997

Sheikholeslami-Wohlert action with Fermilab interpretation via HQET

** Tune hopping parameter and clover coefficient to eliminate discretization effects through `NLO`

** Heavy quark errors: $\mathcal{O}(\alpha_s \Lambda_{QCD}/M), \mathcal{O}(\Lambda_{QCD}/M)^2$

** It can be efficiently used for both b and c quarks.

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Valence light quarks: Asqtad action.

1. Introduction: Moving to Hisq $N_f = 2+1+1$

Higher precision requires reduction of discretization errors

Hisq (Highly improved staggered action):

E. Follana et al, HPQCD coll., Phys.Rev.D75:054502 (2007)

Reduce $\mathcal{O}(a^2\alpha_s)$ errors compared to Asqtad

$\rightarrow$ more continuum-like behavior

Reduce $\mathcal{O}((am_Q)^4)$ errors $\rightarrow$ especially designed to treat `charm` quarks in a relativistic way.

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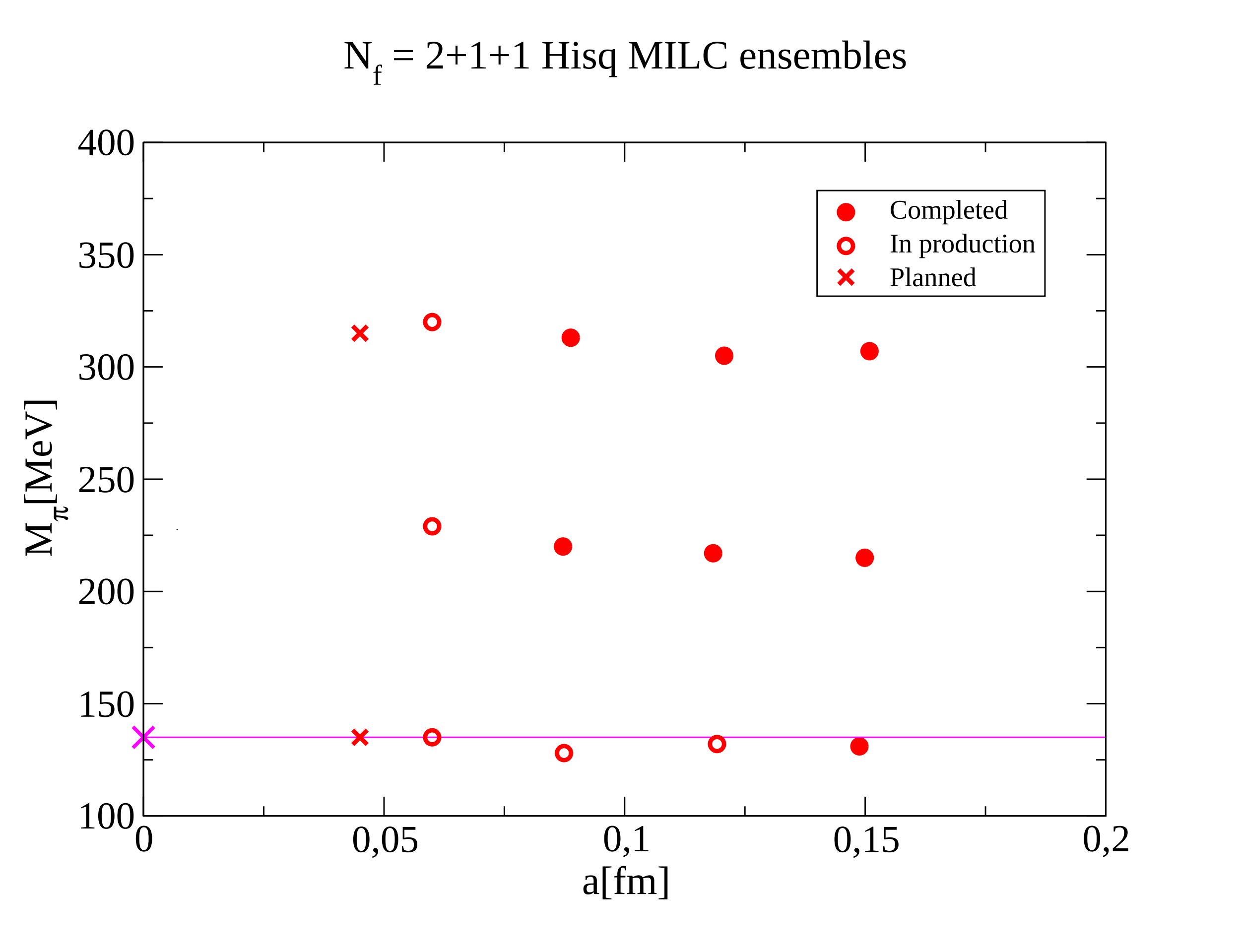
Can achieve the same precision in $D-$`meson` quantities as in K quantities

* Using the same action for light and charm quarks: simplifies renormalization of heavy-light quantities, allows to cancel systematics in ratios . . . .

1. Introduction: New library of gauge configurations.

Hisq sea quarks and same improved gauge action ($\mathcal{O}(n_f \alpha_s)$ correc.)



$\sim$ 1000 configurations per ensemble.

* Including the effects of sea charm quarks.

* **Physical light quark masses** (plus $am_l = 0.2am_s,\ 0.1am_s$)

2. Decay constants

In parallel to configuration generation.

* Tuning of the am_l, am_s, and am_c quark masses.

** Sea strange and charm quark masses closer than physical values than for Asqtad MILC $N_f = 2+1$ configurations.

Sea strange quark mass tuned at the 7-1% level of accuracy

* Generating decay constant data: Pseudoscalar code run on all ensembles generated/being benerated.

$$f_\pi,\ f_K,\ f_D,\ f_{D_s}$$

2.1. The decay constants ratio: f_K/f_π

One of the most thoroughly-studied quantities on the lattice.

* Simple matrix element: $\langle 0|\bar{a}\gamma_\mu\gamma_5 b|P_{ab}(p)\rangle = i f_{P_{ab}} p_\mu$ → precise calculations

* Ratio even more precise (cancellation of statistical fluctuations and systematic uncertainties).

Marciano 2004:

$$\underbrace{\frac{\Gamma\left(K\to l\bar{\nu}_l(\gamma)\right)}{\Gamma\left(\pi\to l\bar{\nu}_l(\gamma)\right)}}_{\textbf{experiment}} \propto \frac{|V_{us}|^2}{|V_{ud}|^2}\underbrace{\left(\frac{f_K}{f_\pi}\right)^2}_{\textbf{lattice}}$$

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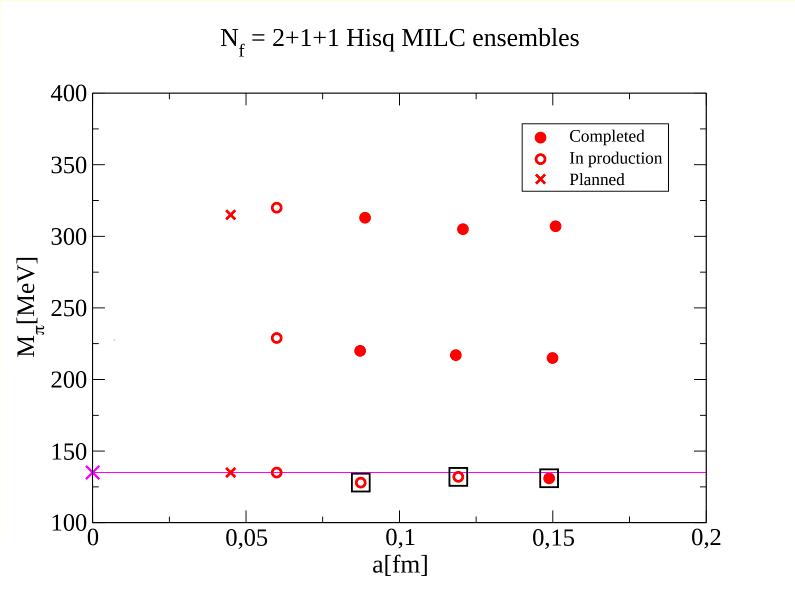

At each ensemble

* Calculate m_{xy} and f_{xy} with 10 values of m_x m_y from m_l^{sea} to m_s^{sea}.

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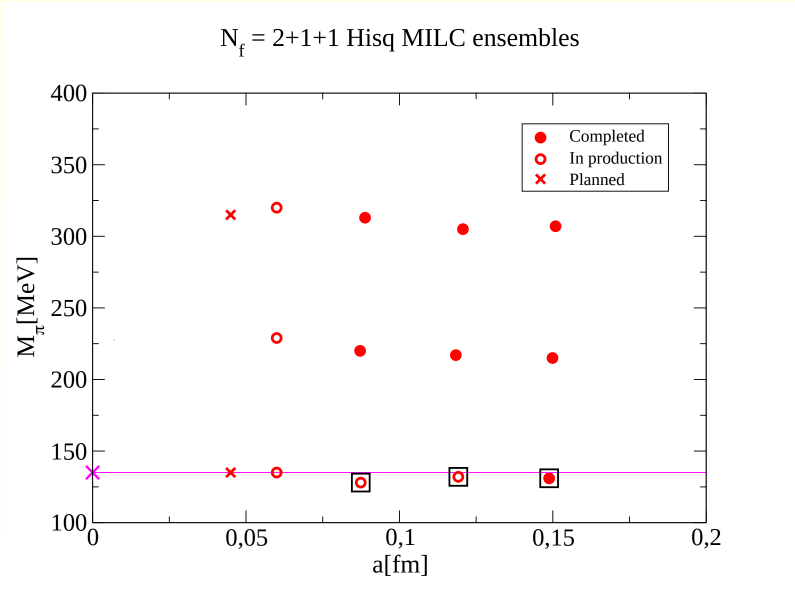

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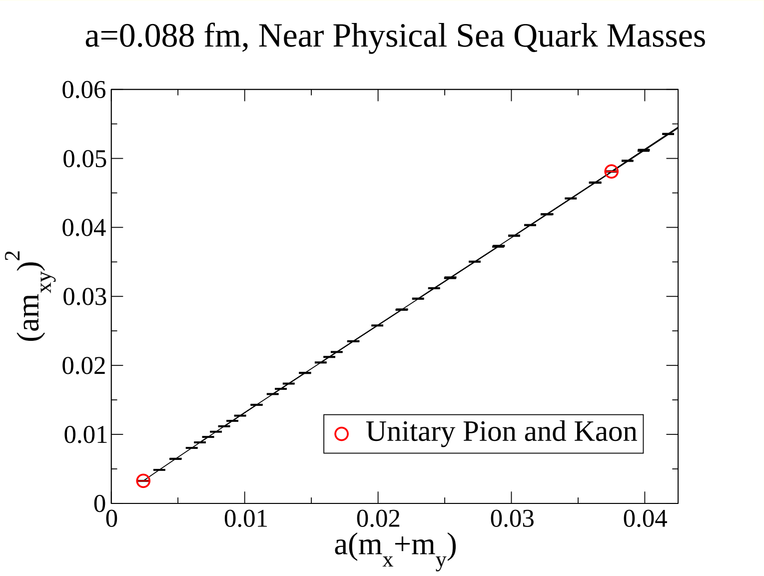

Data trend suggests linear interpolation between $(m_x, m_y) = (m_l, m_l)$ and its nearest neighbour

$$(am_{xy})^2 = A_1 + B_1 a(m_x + m_y)$$

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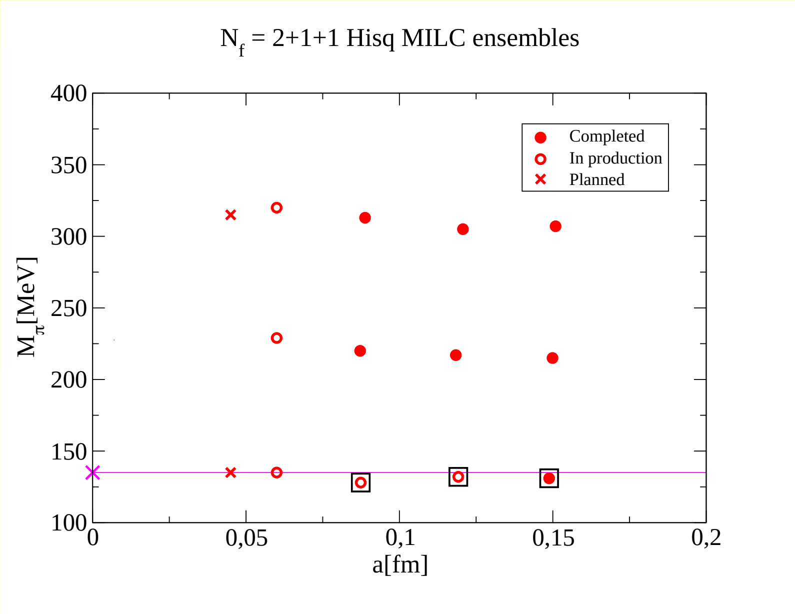

At each ensemble

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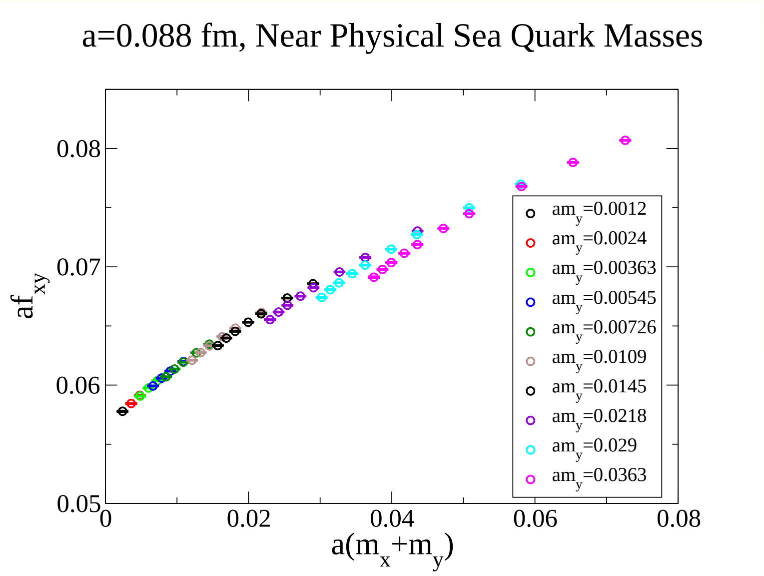

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$$af_{xy} = \begin{array}{ll} A_2 + B_2 a(m_x + m_y) & (m_x, m_y) \sim (m_l, m_l) \\ A_3 + B_3 a m_x + C_3 a m_y & (m_x, m_y) \sim (m_l, m_s) \end{array}$$

2.1. The decay constants ratio: f_K/f_π

On each ensemble

* Require $(f_{xx}/m_{xx})^2 = (f_\pi/m_\pi)^2_{phys}$, solve for m_x to determine m_l^{phys}.

(Checked quadratic form makes little difference)

* Require $(m_{ly}/m_{ll})^2 = (m_K/m_\pi)^2_{phys}$, solve for m_y to determine m_s^{phys}.

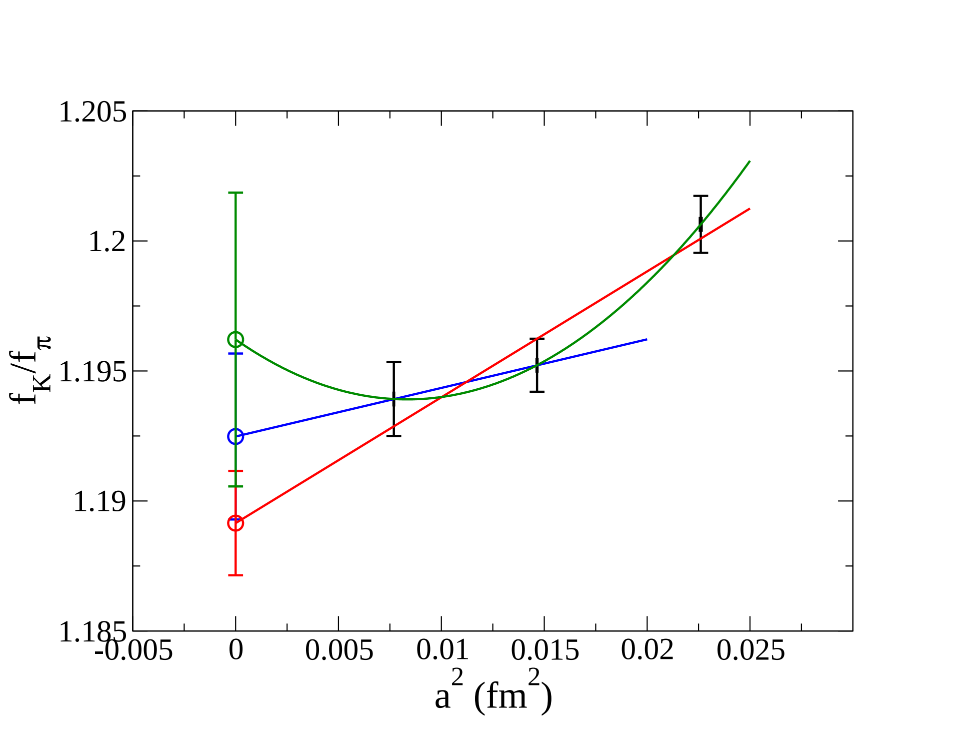
(Checked that using $2m_{xy}^2 - m_{xx}^2$ makes little difference)

* Linearly interpolate f_{xx} and f_{xy} to $m_x = m_l^{phys}$ and $m_y = m_s^{phys}$ to obtain a value of $(f_K/f_\pi)_{phys}$.

We avoid using a intermediate scale like r_1 $\rightarrow$ reduced scale setting error

2.1. The decay constants ratio: f_K/f_π

Preliminary: continuum extrap.



$f_K/f_\pi = 1.1962(56)(37)_{cont}(32)_{FV}$

* FV error from NNLO ChPT + data at 3 volumes.

* Systematic to investigate: fit form ($SU(2)$ sChPT), partial quenching, . . .

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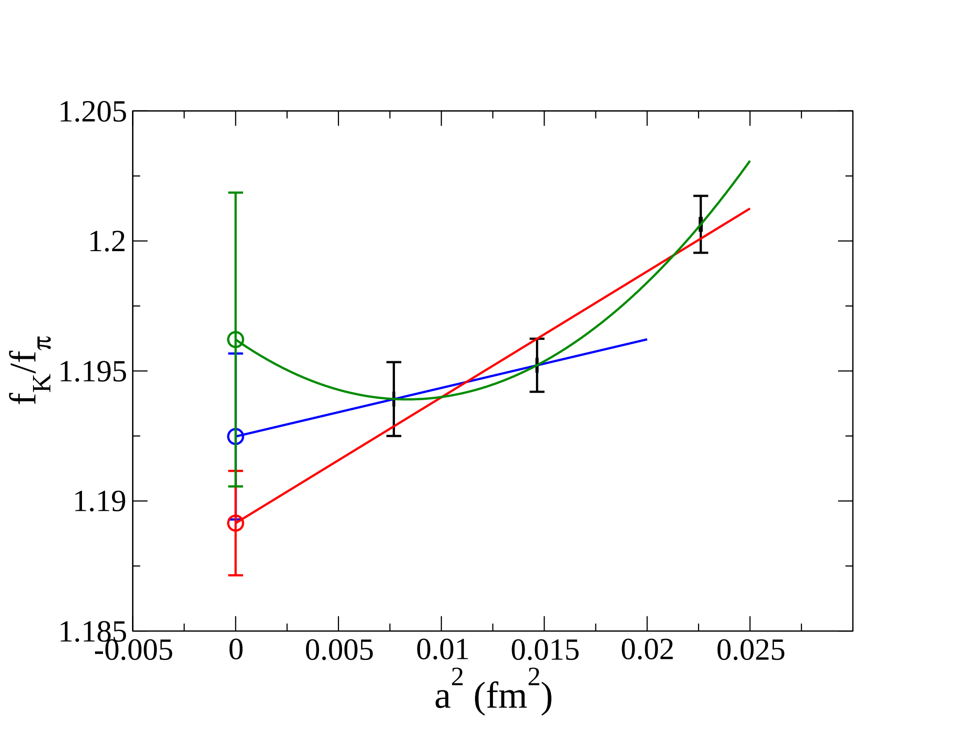

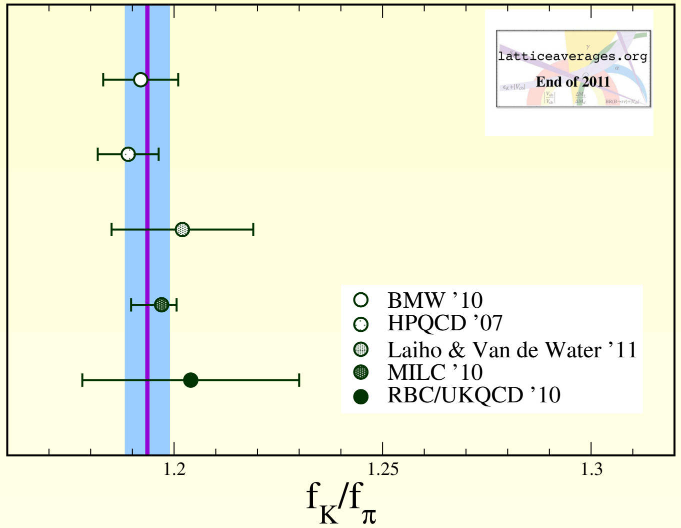

Average: 1.1936 ± 0.0053

Laiho, Lunghi, Van de Water latticeaverages.org

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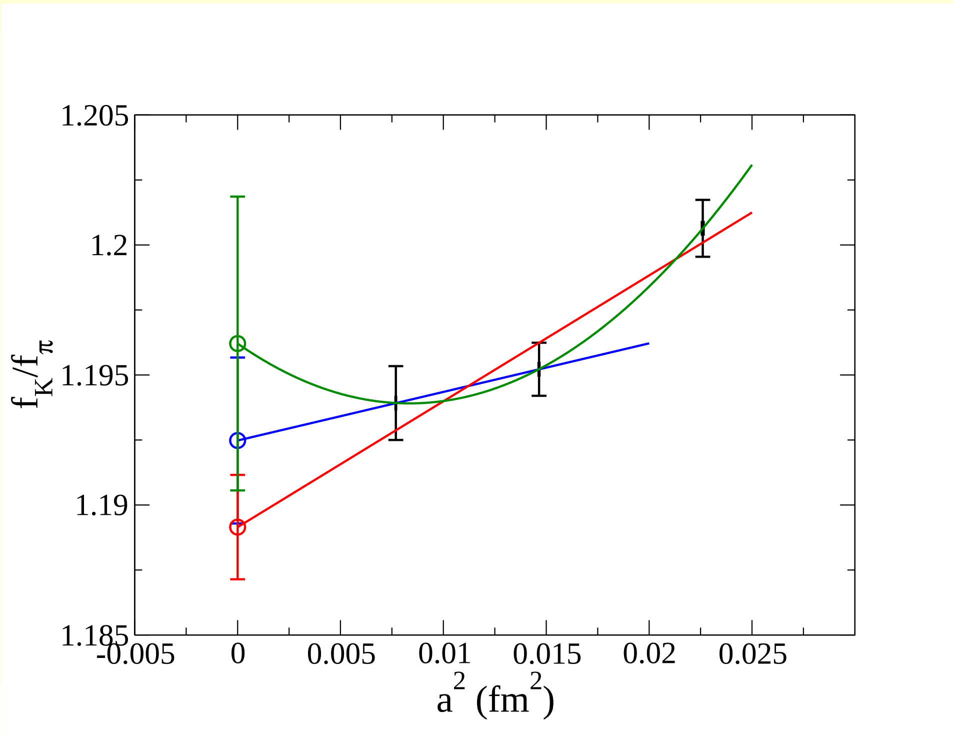

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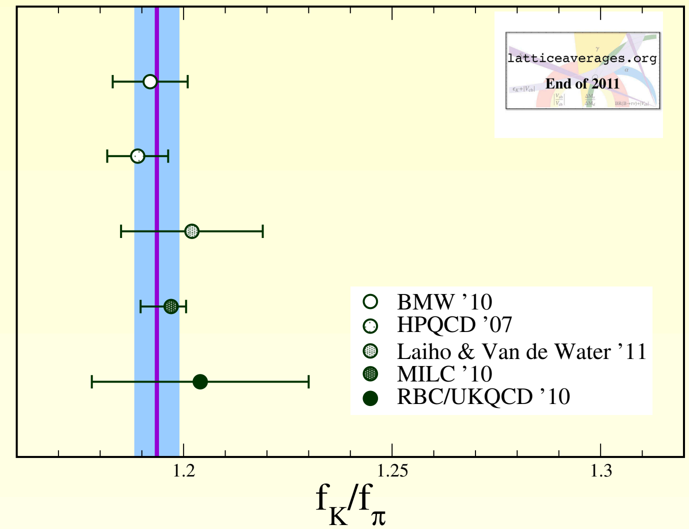

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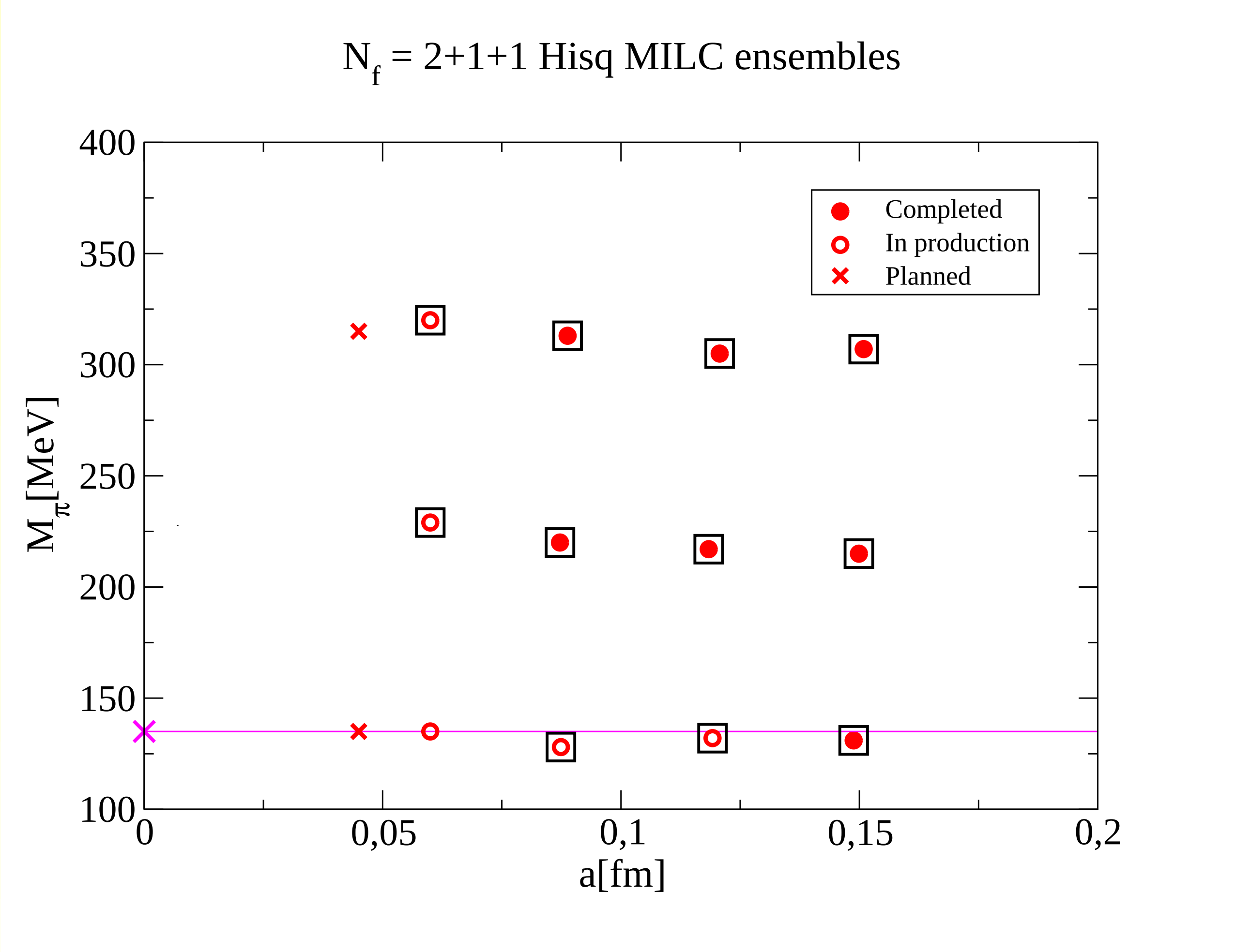
Experimental average:

Antonelli et al., EPJC69(2010)

$$\left|\frac{V_{us}}{V_{ud}}\right| \times \frac{f_K}{f_\pi} = 0.2758(5).$$

$\rightarrow |V_{us}| = 0.2251 \pm 0.0010_{lat} \pm 0.004_{exp}$

2.1. Decay constants: f_D, f_{D_s}



Generate heavy-light decay constants with

$$am_h^{valence} = 0.9am_c^{sea},\ am_c^{sea}$$

and 8 values of $am_l^{valence} = am_l^{sea} - am_s^{sea}$

2.1. Decay constants: f_D, f_{D_s}

Procedure: on each ensemble

* $(M_\pi/F_\pi)^2 \rightarrow am_{light}^{phys.}$ and a.

(Cubic interpolation through three light valence masses)

* $(2M_K^2 - M_\pi^2) \rightarrow am_s^{phys.}$ (linear interp./extrap. through two valence am_s)

* $M_{D_s} \rightarrow am_c$ (linear interp./extrap. through two valence am_c)

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* $f_D = f$ at tuned light and charm physical masses.

$f_{D_s} = f$ at tuned strange and charm physical masses.

(linear interpolation/extrapolation in light/strange and charm quark masses)

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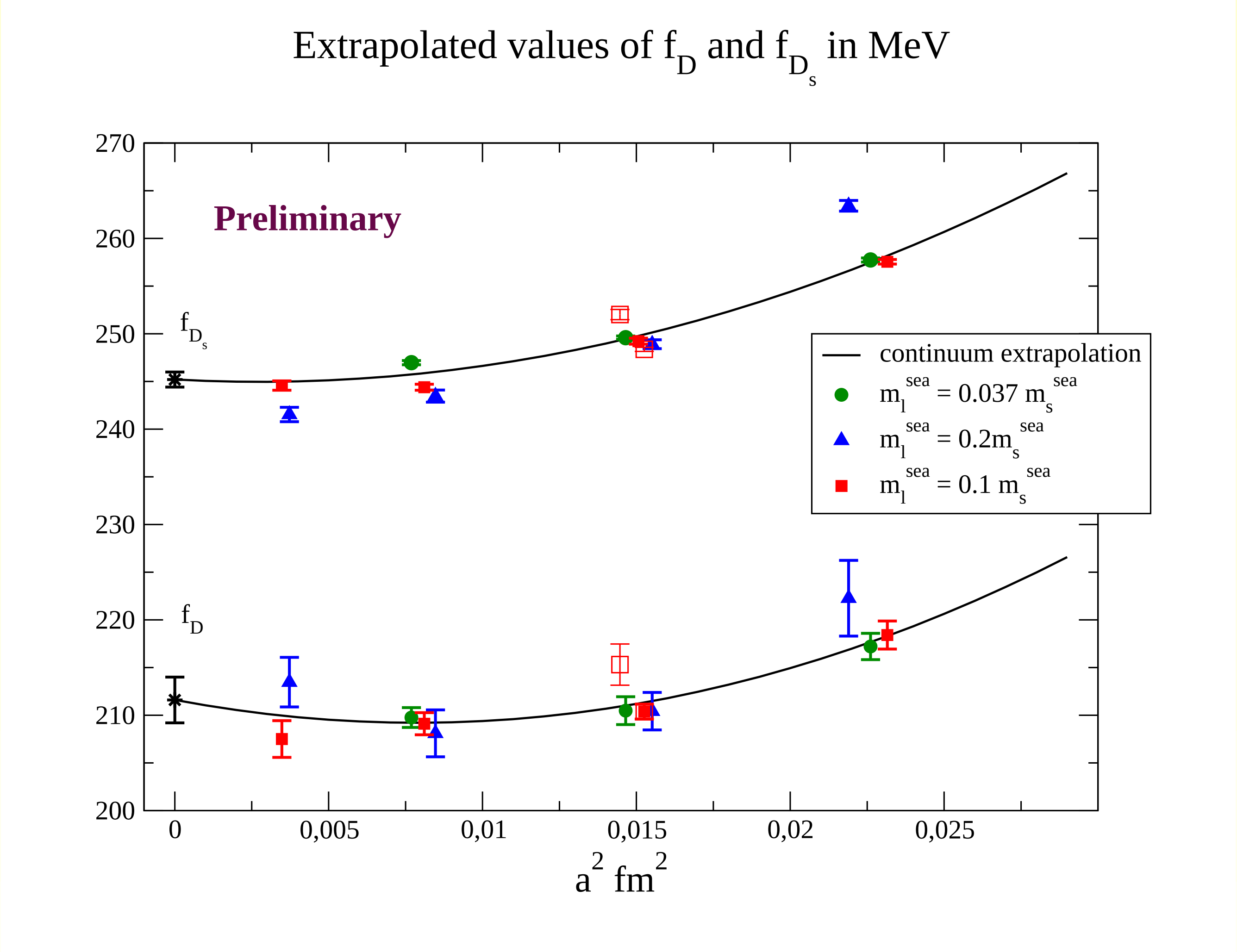
(linear interpolation/extrapolation in light/strange and charm quark masses)

Essentially: computing f_D/f_π and f_{D_s}/f_π and using f_π to set the scale

* Do not use intermediate quantities like r_1

$\to$ scale setting error becomes part of statistical error

2.1. Decay constants: f_D, f_{D_s}



Valence masses fixed at physical values

Fits include ensembles with $am_l^{sea} = 0.1 m_s^{sea}$ and $m_l^{sea} = am_l^{phys}$

$$f_D(a) = f_D + c_1 a^2 + c_2 a^4 + c_3 (m_l^{\rm sea} - m_l^{\rm physical})$$

$$f_{D_s}(a) = f_{D_s} + c_1 a^2 + c_2 a^4 + c_3 (m_l^{\rm sea} - m_l^{\rm physical}) + c_4 a^2 (m_l^{\rm sea} - m_l^{\rm physical})$$

2.1. Decay constants: f_D, f_{D_s}

Preliminary results ($N_f = 2+1+1$)

$f_D = 211.6 \pm 2.4 \pm \text{systematic}$ $\qquad$ $f_{D_s} = 245.2 \pm 0.8 \pm \text{systematic}$

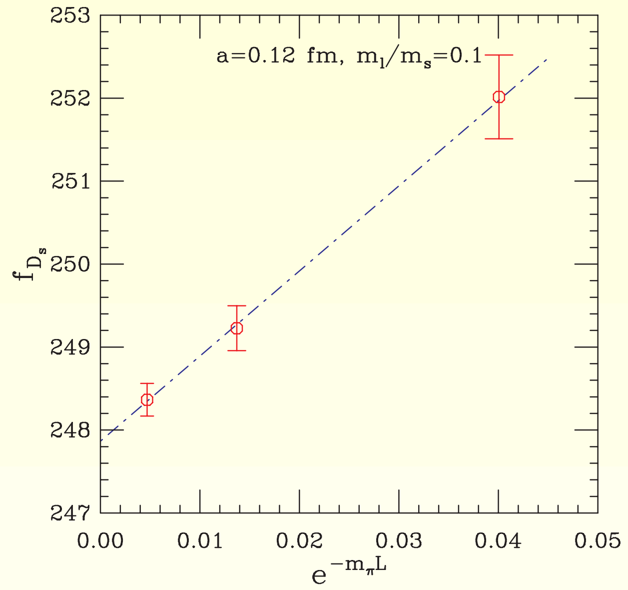
Collaboration	N_f	$f_D(\text{MeV})$	$f_{D_s}(\text{MeV})$
FNAL/MILC 2011	2+1	$218.9 \pm 2.3 \pm 11.0$	$260.1 \pm 2.3 \pm 10.5$
HPQCD 2010	2+1	$213.0 \pm 1.4 \pm 3.4$	$248.0 \pm 1.4 \pm 2.0$
PACS-CS 2011	2+1	$226 \pm 6 \pm 5$	$257 \pm 2 \pm 5$
ETMC 2011	2	212 ± 8	248 ± 6
Exper. av. 2012 Rosner & Stone		206.7 ± 8.9	260.0 ± 5.4

To do: Systematic errors

* **Choice of continuum fit function, charm/strange mass uncertainty**, finite volume, electromagnetic and isosping breaking effects, . . .

2.1. Decay constants: f_D, f_{D_s}

Finite volume corrections



3. Semileptonic decays

Extraction of CKM matrix elements

$$\frac{d}{dq^2}\Gamma(P_1 \to P_2 l\nu) \propto |V_{ab}|^2 |f_+^{P_1 \to P_2}(q^2)|^2 \qquad q = p_{P_2} - p_{P_1}$$

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CLEO-c, Besson et al PRD80 (2009)

$$|V_{cs}|f_+(0)^{D\to K} = 0.719(\pm 0.8\% \pm 0.7\%)$$
$$|V_{cd}|f_+(0)^{D\to \pi} = 0.150(\pm 3\% \pm 0.7\%)$$

Na et al. (HPQCD)
PRD82(2010) ,PRD84(2011)

$f_+(0)^{D\to K,latt}$: 2.5% error
$f_+(0)^{D\to \pi,latt}$: 5% error

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$|V_{cs}|f_+(0)^{D\to K} = 0.717(\pm 0.8\% \pm 0.7 \pm 0.7\%)$ (last error from $B(D^0 \to K^-\pi^+)$)

* For D decays error in $|V_{cj}|$ dominated by lattice errors
* Testing lattice QCD: shape of the form factors

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$\Longrightarrow$ $B-$ semileptonic decays

3. Semileptonic decays

Experimental average, Antonelli et al. (Flavianet), EPJC69(2010)

$$|V_{us}|f_+(0)^{K\to\pi} = 0.2163(\pm 0.23\%)$$

$$f_+(0)^{K\to\pi} : \begin{matrix} +.5\% \\ -0.6\% \end{matrix}$$

RBC/UKQCD, EPJC69(2010)

* Check unitarity in the first row of CKM matrix.

$$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0001(6)$$ **M. Antonelli et al**

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Look for new physics effects in the comparison of $|V_{us}|$ from helicity suppressed $K_{\mu 2}$ versus helicity allowed K_{l3}

$$R_{\mu 23} = \left(\frac{f_K/f_\pi}{f_+^{K\pi}(0)}\right) \times \mathrm{experim.\,data\,on}\, K_{\mu 2}\pi_{\mu 2}\,\mathrm{and}\, K_{l3}$$

* In the SM $R_{\mu 23} = 1$. Not true for some BSM theories (for example, charged Higgs)

* Current value $R_{\mu 23} = 0.999(7)$, limited by lattice inputs.

3.1. Semileptonic decays at $q^2 = 0$

First step in the project: Form factors at zero momentum transfer.

Extraction of CKM matrix elements

$$
\begin{aligned}
K \to \pi l \nu \quad &\to \quad |V_{us}| \\
D \to \pi l \nu \quad &\to \quad |V_{cd}| \\
D \to K l \nu \quad &\to \quad |V_{cs}|
\end{aligned}
$$

3.1.1. Semileptonic decays at $q^2 = 0$: Methodology

For the extraction of $|V_{f_1 f_2}|$ we need $f_+^{P1 \to P_2}(0)$ for mesons P_1 and P_2.

$$\langle P_2 | V^\mu | P_1 \rangle = f_+^{P_1 P_2}(q^2) \left[p_{P_1}^\mu + p_{P_2}^\mu - \frac{m_{P_1}^2 - m_{P_2}^2}{q^2} q^\mu \right] + f_0^{P_1 P_2}(q^2) \frac{m_{P_1}^2 - m_{P_2}^2}{q^2} q^\mu$$

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We use HPQCD method for D semileptonic decays

* In the continuum, the Ward identity ($S = \bar{a}b$)

$$q^\mu \langle P_2|V^{cont.}_\mu|P_1\rangle = (m_b - m_a)\langle P_2|S^{cont}|P_1\rangle$$

relates matrix elements of vector and scalar currents. In the lattice

$$q^\mu \langle P_2|V^{lat.}_\mu|P_1\rangle Z = (m_b - m_a)\langle P_2|S^{lat.}|P_1\rangle$$

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$$q^\mu \langle P_2|V_\mu^{lat.}|P_1\rangle Z = (m_b - m_a)\langle P_2|S^{lat.}|P_1\rangle$$

→ substitute the 3-point function with a V_μ insertion by a 3-point function with a S insertion

$$f_0^{P_1 P_2}(q^2) = \frac{m_b - m_a}{m^2_{P_1} - m^2_{P_2}}\langle P_2|S|P_1\rangle_{q^2}$$

3.1.1. Semileptonic decays at $q^2 = 0$: Methodology

For the extraction of $|V_{f_1 f_2}|$ we need $f_+^{P1\to P_2}(0)$ for mesons P_1 and P_2.

$$\langle P_2|V^\mu|P_1\rangle = f_+^{P_1 P_2}(q^2)\left[p^\mu_{P_1} + p^\mu_{P_2} - \frac{m^2_{P_1} - m^2_{P_2}}{q^2} q^\mu\right] + f_0^{P_1 P_2}(q^2)\frac{m^2_{P_1} - m^2_{P_2}}{q^2} q^\mu$$

We use **HPQCD** method for D semileptonic decays

* In the continuum, the Ward identity ($S = \bar{a}b$)

$$q^\mu \langle P_2|V_\mu^{cont.}|P_1\rangle = (m_b - m_a)\langle P_2|S^{cont}|P_1\rangle$$

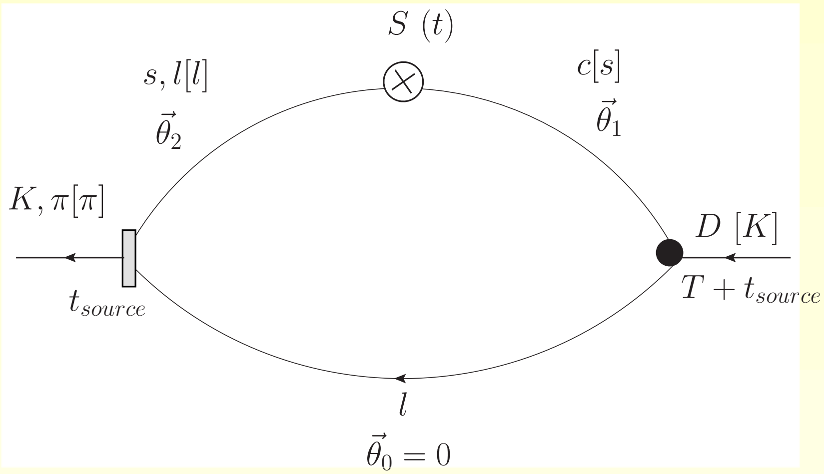
relates matrix elements of vector and scalar currents. In the lattice

$$q^\mu \langle P_2|V_\mu^{lat.}|P_1\rangle Z = (m_b - m_a)\langle P_2|S^{lat.}|P_1\rangle$$

→ substitute the 3-point function with a V_μ insertion by a 3-point function with a S insertion

$$f_0^{P_1 P_2}(q^2) = \frac{m_b - m_a}{m^2_{P_1} - m^2_{P_2}}\langle P_2|S|P_1\rangle_{q^2} \Longrightarrow \boxed{f_+^{P_1 P_2}(0) = f_0^{P_1 P_2}(0) = \frac{m_b - m_a}{m^2_{P_1} - m^2_{P_2}}\langle S\rangle_{q^2=0}}$$

3.1.2. Semileptonic decays at $q^2 = 0$: Simulations setup



* Color random wall sources $\rightarrow$

Reduction of statistical errors by a factor of 2-3

Quantities inside [ ] correspond to $K \rightarrow \pi l \nu$

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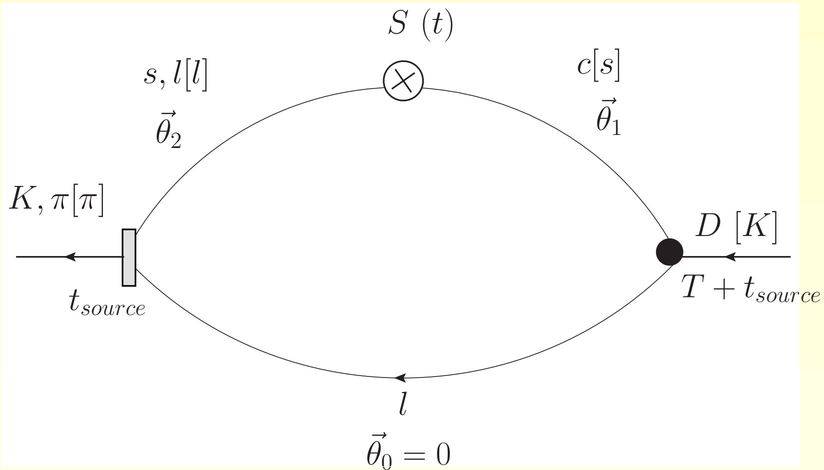


* **Color random wall sources** $\rightarrow$

Reduction of statistical errors by a factor of 2-3

Quantities inside [ ] correspond to $K \rightarrow \pi l \nu$

* **Twisted boundary conditions** $\rightarrow$ allow generating correlation functions with non-zero external momentum such that $q^2 \simeq 0$

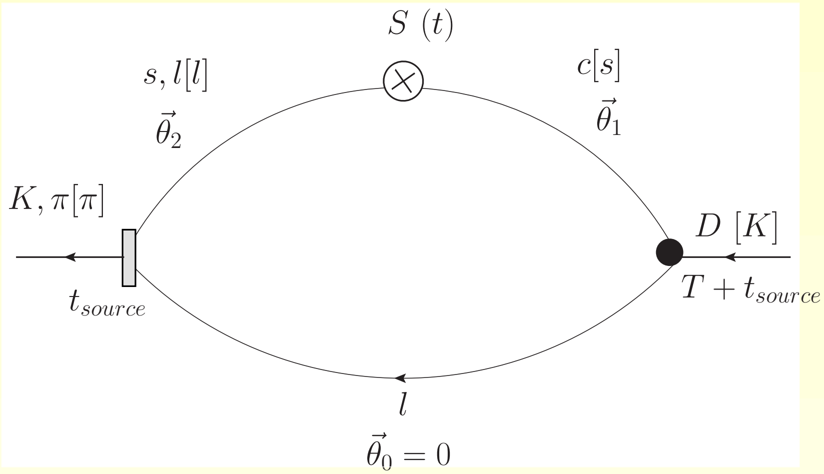
$$q(x + L\hat{k}) = e^{i\theta_k} q(x)$$

** $K \rightarrow \pi l \nu$: momentum injected on the K ($\vec{\theta}_1 \neq 0$) or π ($\vec{\theta}_2 \neq 0$)

$$\vec{\theta}_1(q^2 = 0) = \sqrt{\left(\frac{m_K^2 + m_\pi^2}{2m_\pi}\right)^2 - m_K^2}\ \frac{L}{\pi} \quad \Longrightarrow \quad \vec{p}_K = \vec{\theta}_1 \frac{\pi}{L}$$

$$\vec{\theta}_2(q^2 = 0) = \sqrt{\left(\frac{m_K^2 + m_\pi^2}{2m_K}\right)^2 - m_\pi^2}\ \frac{L}{\pi} \quad \Longrightarrow \quad \vec{p}_\pi = \vec{\theta}_2 \frac{\pi}{L}$$

3.1.2. Semileptonic decays at $q^2 = 0$: Simulations setup



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Reduction of statistical errors by a factor of 2-3

Quantities inside [ ] correspond to $K \rightarrow \pi l \nu$

* **Twisted boundary conditions** $\rightarrow$ allow generating correlation functions with non-zero external momentum such that $q^2 \simeq 0$

** $D \rightarrow K(\pi) l\nu$: D-meson always at rest. Momentum injected on the $K(\pi)$ ($\vec{\theta}_0 = \vec{\theta}_1 = 0$, $\vec{\theta}_2 \neq 0$)

$$\vec{\theta}_2(q^2 = 0) = \sqrt{\left(\frac{m_D^2 + m_{K(\pi)}^2}{2m_D}\right)^2 - m_{K(\pi)}^2}\,\frac{L}{\pi} \quad \Longrightarrow \quad \vec{p}_{K(\pi)} = \vec{\theta}_2 \frac{\pi}{L}$$

3.1.3. Testing the method: Hisq valence on $N_f = 2+1$ Asqtad configurations

$\approx a$ (fm)	am_l/am_s	Volume	N_{conf}	$N_{sources}$	N_T
0.12	0.4	$20^3 \times 64$	2052	4	5
	0.2	$20^3 \times 64$	2243	4	8
	0.14	$20^3 \times 64$	2109	4	5
	0.1	$24^3 \times 64$	2098	8	5
0.09	0.4	$28^3 \times 96$	1996	4	5
	0.2	$28^3 \times 96$	1946	4	5

with N_T is the number of source-sink separations (need even and odd values of T to eliminate contamination with wrong-spin states (lattice artifacts)).

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* Strange valence quark masses are tuned to their physical values

C.T.H. Davies et al, PRD81(2010)

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* Strange valence quark masses are tuned to their physical values

C.T.H. Davies et al, PRD81(2010)

* Light valence quark masses: $\frac{m_l^{val}(Hisq)}{m_s^{phys}(Hisq)} = \frac{m_l^{sea}(Asqtad)}{m_s^{phys}(Asqtad)}$

3.1.3. Testing the method: Hisq valence on $N_f = 2+1$ Asqtad conf.

Fitting and statistical errors

We want to extract the value of the form factor $f_0(q^2)$ from the relation

$$f_0(q^2) = \frac{m_s - m_q}{m_K^2 - m_\pi^2} \langle S \rangle_{q^2=0} = \frac{1}{2} A_{00}(q^2) \sqrt{2E_\pi^0 2E_K^0} \frac{m_s - m_q}{m_K^2 - m_\pi^2}$$

3.1.3. Testing the method: Hisq valence on $N_f = 2+1$ Asqtad conf.

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Strategy: Combined fits of two-point functions with and without external momentum (4) + three-point functions with $q^2 = 0$ (2):

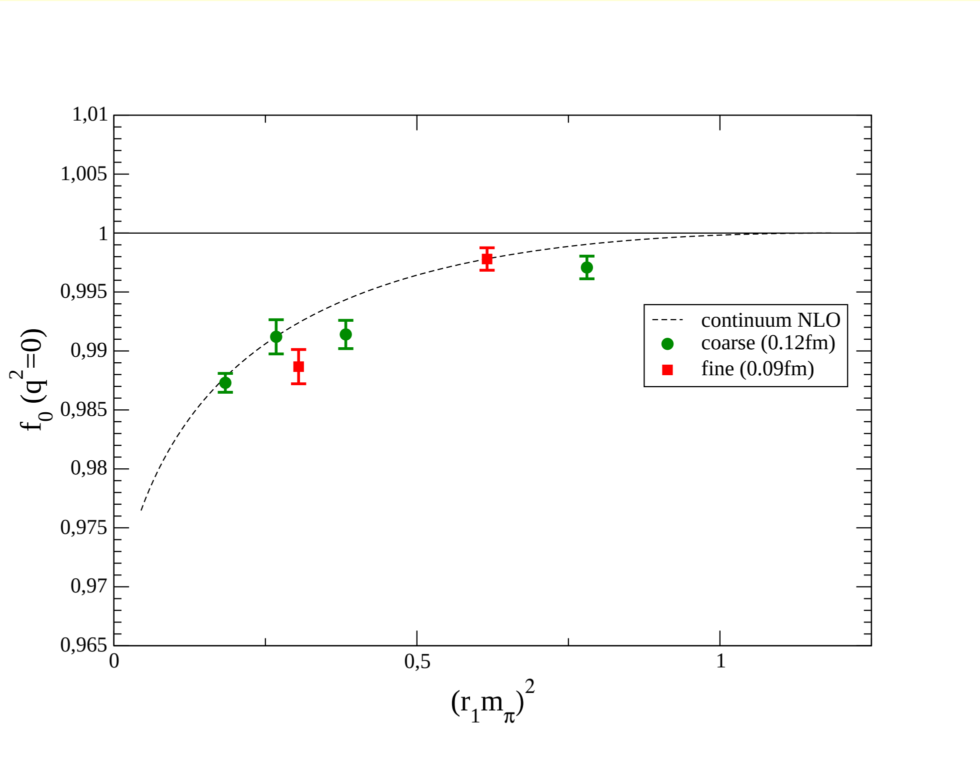
$$C_{3pt}^{K\to\pi}(t,T;\vec{p}_\pi,\vec{p}_K) = \sum_{m,n=0}^{N_{exp}^{3pt}} (-1)^{mt}(-1)^{n(T-t)} A_{mn}(q^2) \sqrt{Z_m^{\pi,\vec{p}_\pi} Z_n^{K,\vec{p}_K}} \times \left(e^{-E_\pi^m t - E_\pi^m (L_t - t)}\right) \left(e^{-E_K^n (T-t) - E_K^n (T - L_t + t)}\right) ;$$

$$C_{2pt}^{P}(t;\vec{p}_P) = \sum_{m}^{N_{exp}^{2pt}} (-1)^{mt} \sqrt{Z_m^{P,\vec{p}_P}} e^{-E_P^m t - E_P^m (L_t - t)} \qquad P = \pi, K$$

* Use several (3 or 4) values of T (even and odd) to fit out oscillatory terms.

3.1.3. Testing the method: Hisq valence on $N_f = 2 + 1$ Asqtad conf.

Statistical errors $0.1 - 0.15\%$.



Find it very difficult to make changes in the fitting procedure that change the fit results outside the one statistical sigma range

* Choice of source-sink separation $T's$, number of exponentials, time ranges, fitting function.

3.1.3. Testing the method: Hisq valence on $N_f = 2+1$ Asqtad conf.

Chiral-continuum extrapolation for $f_+^{K\pi}(0)$

The form factor $f_+(0)$ can be written in χPT as

$$f_+(0) = 1 + f_2 + f_4 + f_6 + ... = 1 + f_2 + \Delta f$$

- # $f_+(0)$ goes to 1 in the $SU(3)$ limit due to vector current conservation
- # Ademollo-Gatto theorem $\rightarrow$ SU(3) breaking effects are second order in $(m_K^2 - m_\pi^2)$ and f_2 is completely fixed in terms of experimental quantities.

3.1.3. Testing the method: Hisq valence on $N_f = 2+1$ Asqtad conf.

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Ademollo-Gatto theorem $\rightarrow$ SU(3) breaking effects are second order in $(m_K^2 - m_\pi^2)$ and f_2 is completely fixed in terms of experimental quantities.

* At finite lattice spacing systematic errors can enter due to violations of the dispersion relation needed to derive

$$f_+(0) = f_0(0) = \frac{m_s - m_q}{m_K^2 - m_\pi^2} \langle S \rangle_{q^2=0}$$

Dispersion relation violations in our data are $\leq 0.15\%$.

3.1.3. Testing the method: Hisq valence on $N_f = 2+1$ Asqtad conf.

Chiral-continuum extrapolation for $f_+^{K\pi}(0)$

Fitting strategy I:

One-loop (NLO) partially quenched Staggered χPT +

two-loop (NNLO) continuum χPT by **Bijnens & Talavera**, arXiv:0303103.

$$f_+^{K\pi}(0) = 1 + f_2^{PQ,stag.}(a) + C_a\left(\frac{a}{r_1}\right)^2 + f_4^{cont.}(\text{logs}) + f_4^{cont.}(L_i's)$$
$$+ r_1^4(m_\pi^2 - m_K^2)^2\left[C_6'^{(1)} + C_6^a\left(\frac{a}{r_1}\right)^2\right]$$

where $C_6'^{(1)} \propto C_{12} + C_{34} - L_5^2$.

L_5 is an $\mathcal{O}(p^4)$ LEC and $C_{12,34}$ are $\mathcal{O}(p^6)$ LECs

* Staggered χPT: logs are known non-analytical functions of $m_{K,\pi}$ containing dominant taste-breaking a^2 effects
→ remove the dominant light discretization errors

3.1.3. Testing the method: Hisq valence on $N_f = 2+1$ Asqtad conf.

Fitting strategy II:

One-loop (NLO) partially quenched Staggered χPT +

analytical parametrization of NNLO terms.

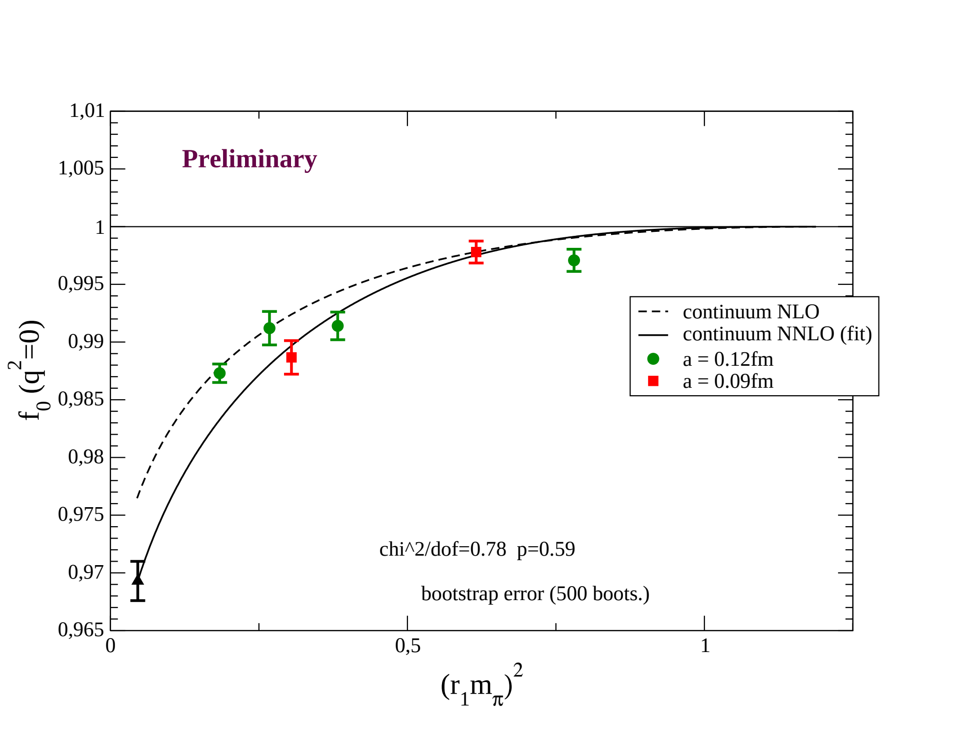
$$f_+^{K\pi}(0) = 1 + f_2^{PQ\,,stagg.}(a) + C_a \left(\frac{a}{r_1}\right)^2 + r_1^4\,(m_\pi^2 - m_K^2)^2$$

$$\times \left[C_6^{(1)} (r_1 m_\pi)^2 + C_6^{(2)} (r_1 m_K)^2 + C_6^a \left(\frac{a}{r_1}\right)^2 \right]$$

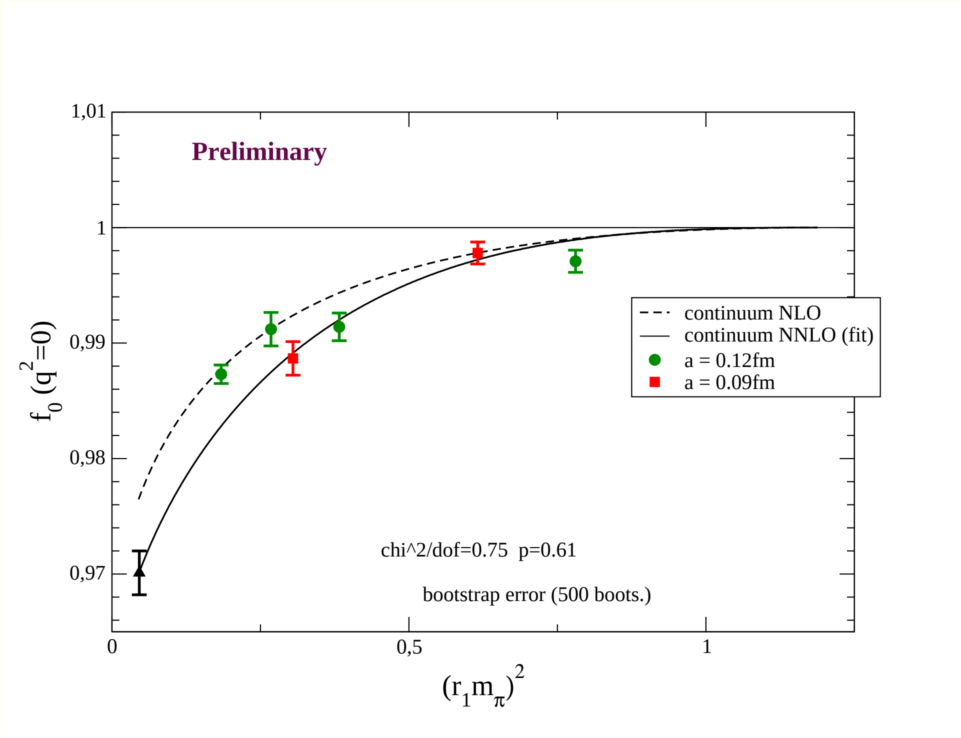
* We also add terms of order $(r_1 m_\pi)^4$, $(r_1 m_\pi)^2 log((r_1 m_\pi)^2)$.

3.1.3. Testing the method: Hisq valence on $N_f = 2 + 1$ Asqtad configurations

Results: some examples

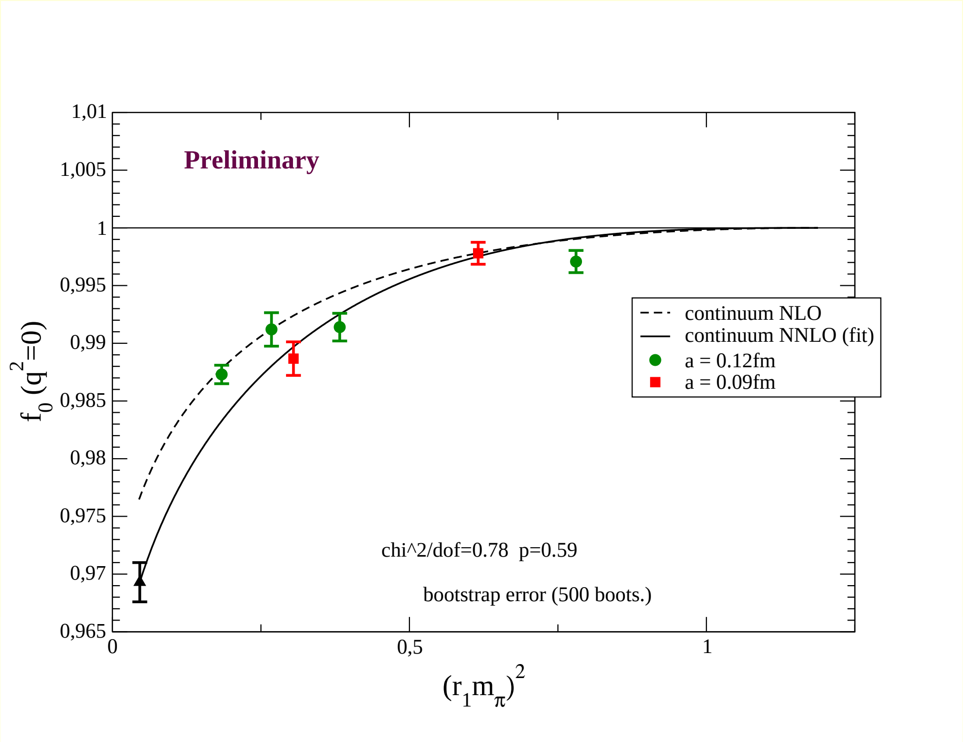


NLO sχPT +
NNLO continuum χPT



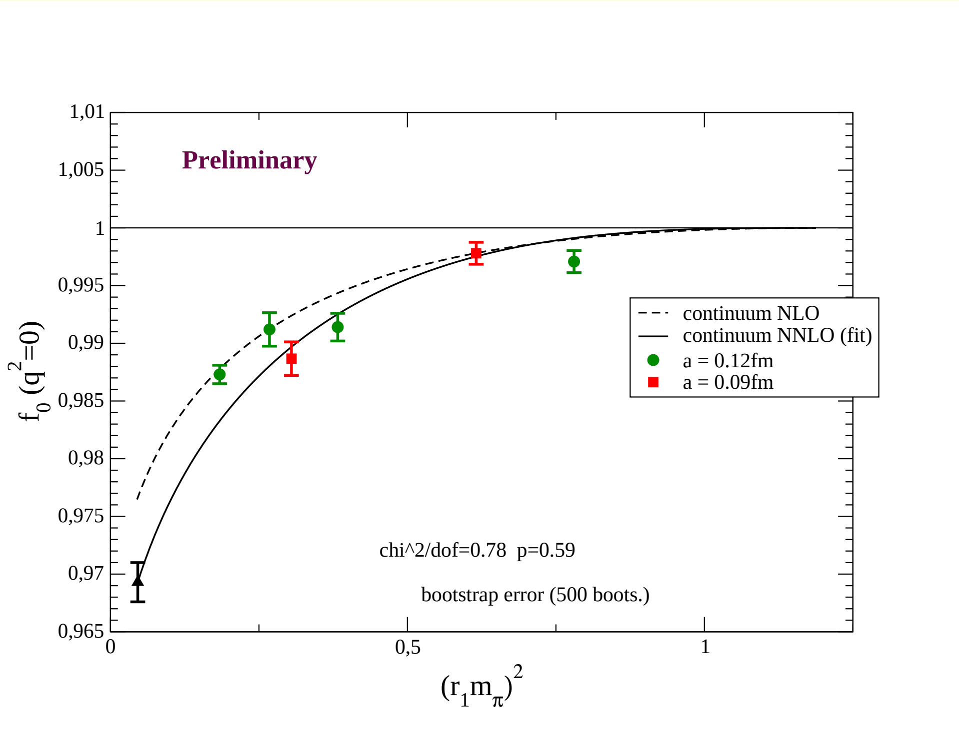
NLO sχPT +
NNLO analytic terms

3.1.3. Testing the method: Hisq valence on $N_f = 2+1$ Asqtad configurations



* Statistical (bootstrap) errors around $0.2 - 0.3\%$.
* Different results (fitting functions, fitting strategies, ...) agree within 1 statistical σ.
* Violations of AG theorem are $\sim 0.32 - 0.15\%$ for $a \approx 0.12$ fm and $\sim 0.15 - 0.1\%$ for $a \approx 0.09$ fm.

3.1.3. Testing the method: Hisq valence on $N_f = 2 + 1$ Asqtad configurations



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* Violations of AG theorem are $\sim 0.32 - 0.15\%$ for $a \approx 0.12$ fm and $\sim 0.15 - 0.1\%$ for $a \approx 0.09$ fm.

Need to check sχPT and clarify the origin of a^2 effects.

3.1.3. Testing the method: Hisq valence on $N_f = 2 + 1$ Asqtad configurations

Expected error budget

* Statistical+extrapolation : 0.2-0.3%

* Chiral extrapolation/fitting function: 0.1%

* Discretization errors: 0.15%

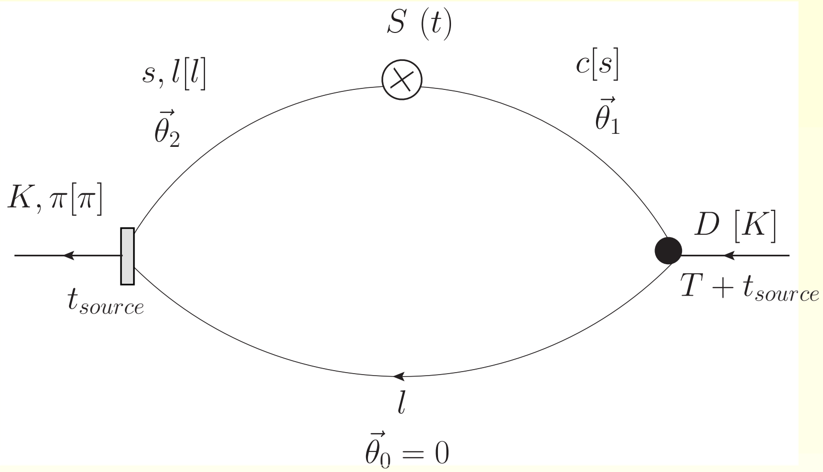
* Mistuning of m_s on the sea: 0.2%

TOTAL: 0.35-0.5%

Already competitive with the state-of-the-art calculation RBC/UKQCD, EPJC69(2010)

3.1.4 Semileptonic $q^2 = 0$ form factors with $N_f = 2+1+1$

$K \to \pi l\nu$ and $D \to K(\pi) l\nu$ at $q^2 = 0$ on the MILC Hisq $N_f = 2+1+1$ ensembles.



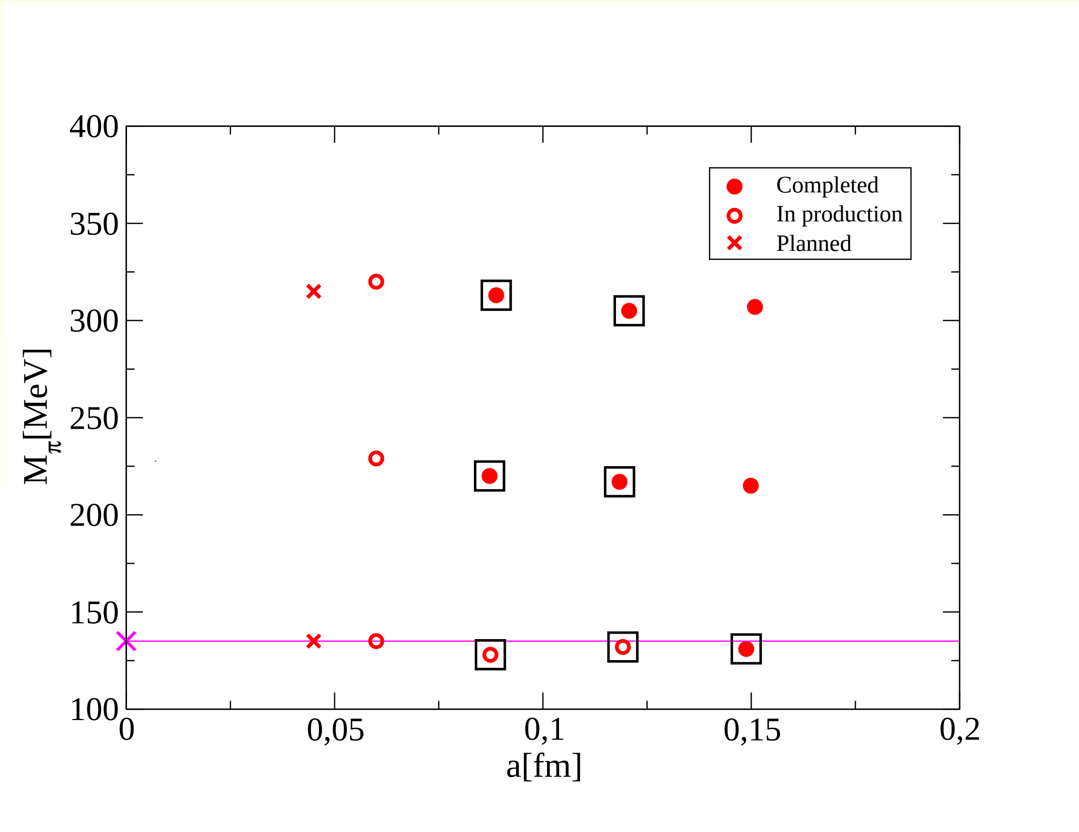
* Reduction of discretization errors from the sea.

* Physical quark masses.

* Incorporates effects of m_c^{sea}.

* Better tuning of sea quark masses ($am_s^{phys} - am_s^{sea} \leq 7\%$, much smaller for $0.09\ fm$ ensembles).

3.1.4 Semileptonic $q^2 = 0$ form factors with $N_f = 2+1+1$



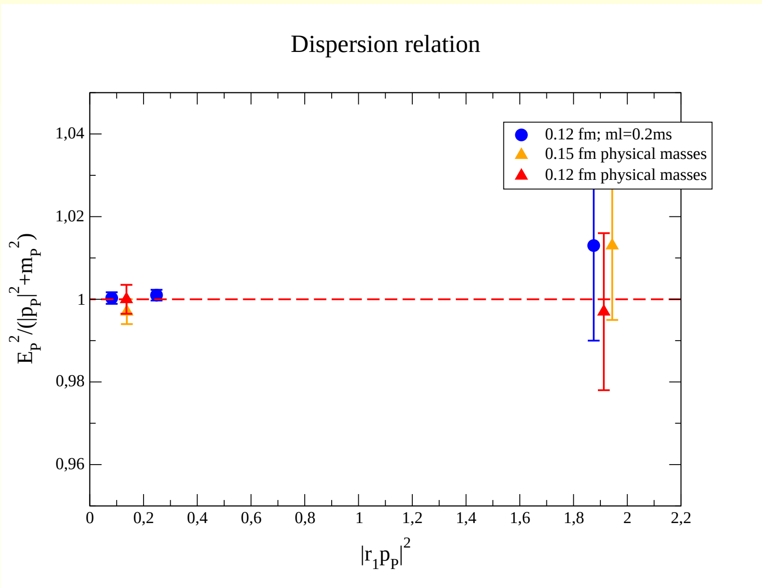
$$am_l^{valence} = am_l^{sea},\ am_s^{valence} = am_s^{physical},$$ and $$am_c^{valence} = am_c^{sea}, \approx am_c^{phys}.$$

Not all final am_c^{phys} are available $\rightarrow$ simulate at two values of the charm quark mass.

3.1.4 Semileptonic $q^2 = 0$ form factors with $N_f = 2+1+1$

First preliminary results

* Good fulfillment of dispersion relation.



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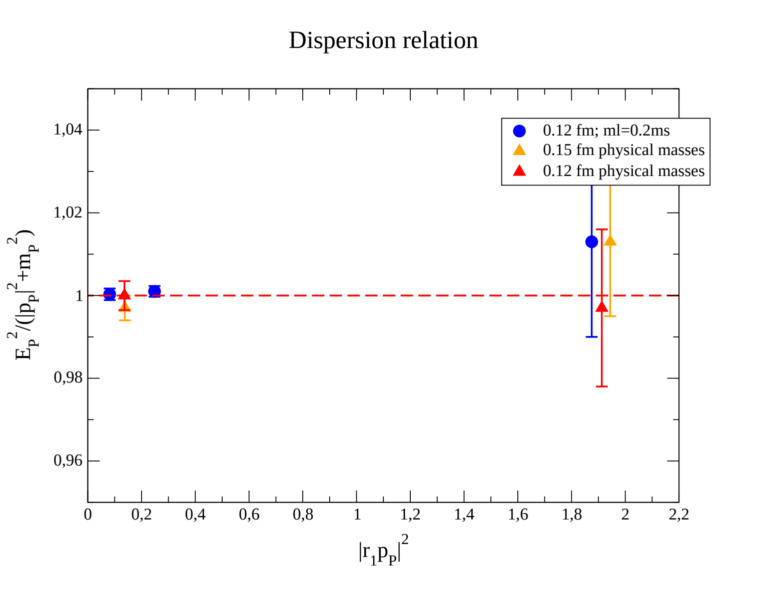
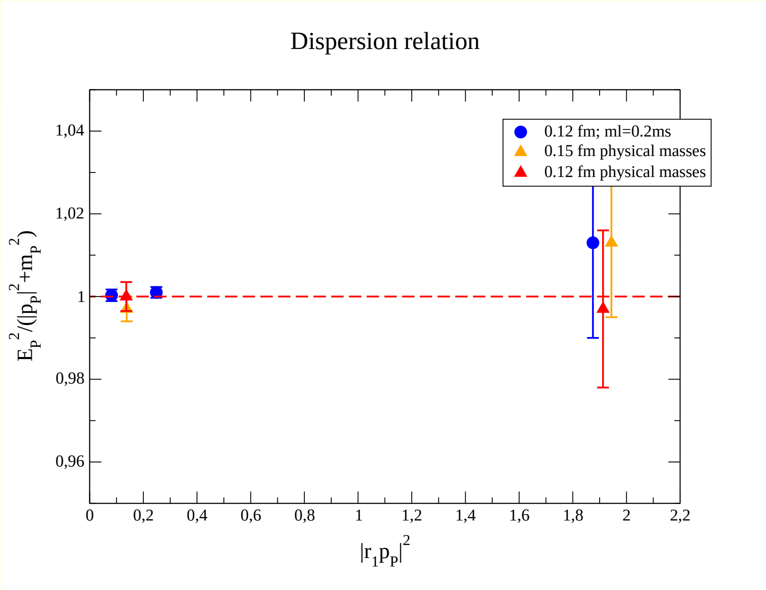


* Statistical errors do not change much with the sea content (for similar a and valence masses).
  ** Ensemble with $a \approx 0.12$ fm, $am_l = 0.2am_s$: statistical errors $\leq 0.05\%$, around ~ 1.3 larger than for the corresponding Asqtad ensemble (with twice as many configurations).

3.1.4 Semileptonic $q^2 = 0$ form factors with $N_f = 2 + 1 + 1$

First preliminary results

* Good fulfillment of dispersion relation.



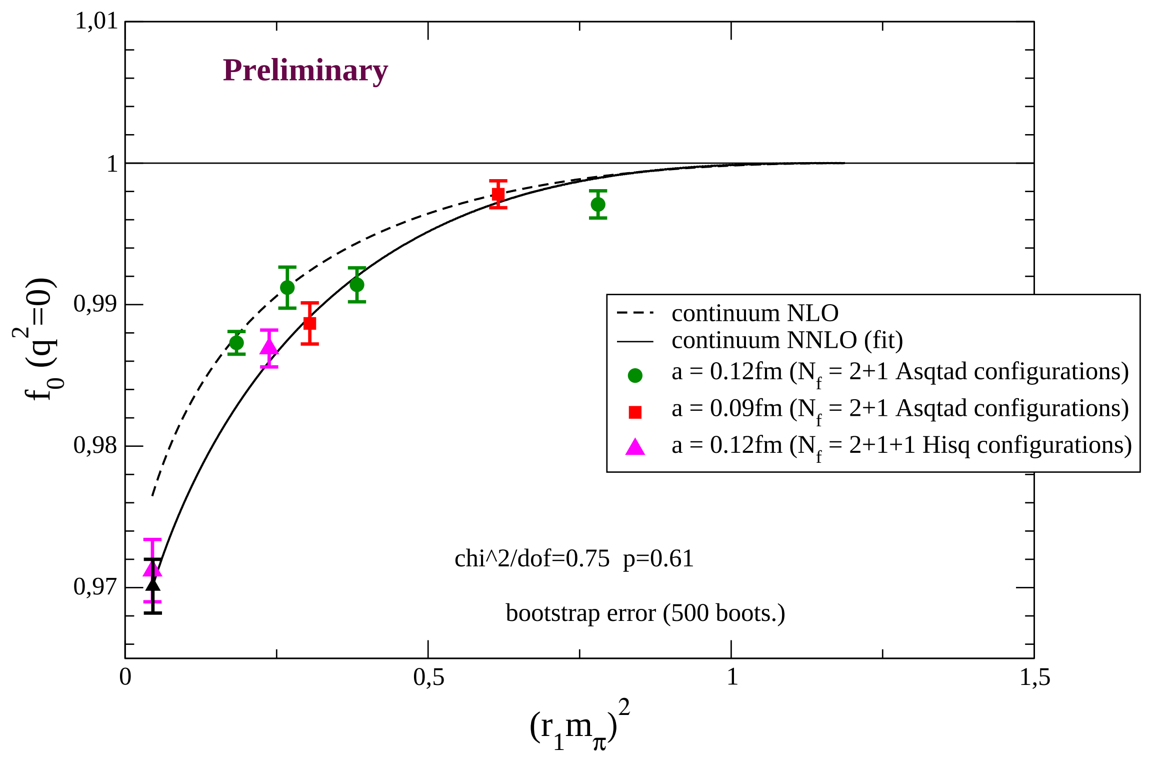
* Statistical errors do not change much with the sea content (for similar a and valence masses).
  ** Ensemble with $a \approx 0.12$ fm, $am_l = 0.2am_s$: statistical errors $\leq 0.05\%$, around ~ 1.3 larger than for the corresponding Asqtad ensemble (with twice as many configurations).

* Statistical errors are larger for smaller quark masses and the external momentum ap needed for $q^2 = 0$ is larger.

$\rightarrow$ Need data at unphysical light masses to reduce statistic. and discretization errors

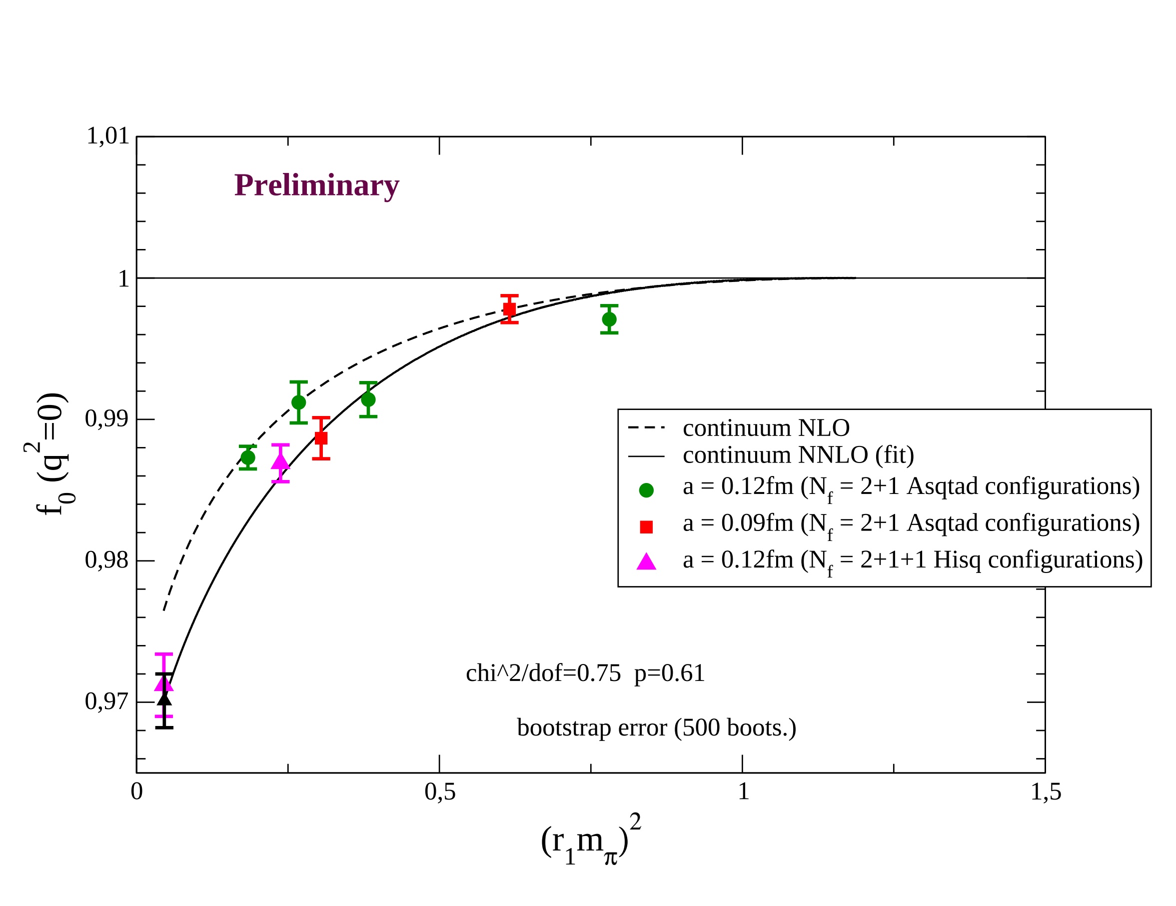
3.1.4 Semileptonic $q^2 = 0$ form factors with $N_f = 2 + 1 + 1$

First very preliminary results



3.1.4 Semileptonic $q^2 = 0$ form factors with $N_f = 2+1+1$

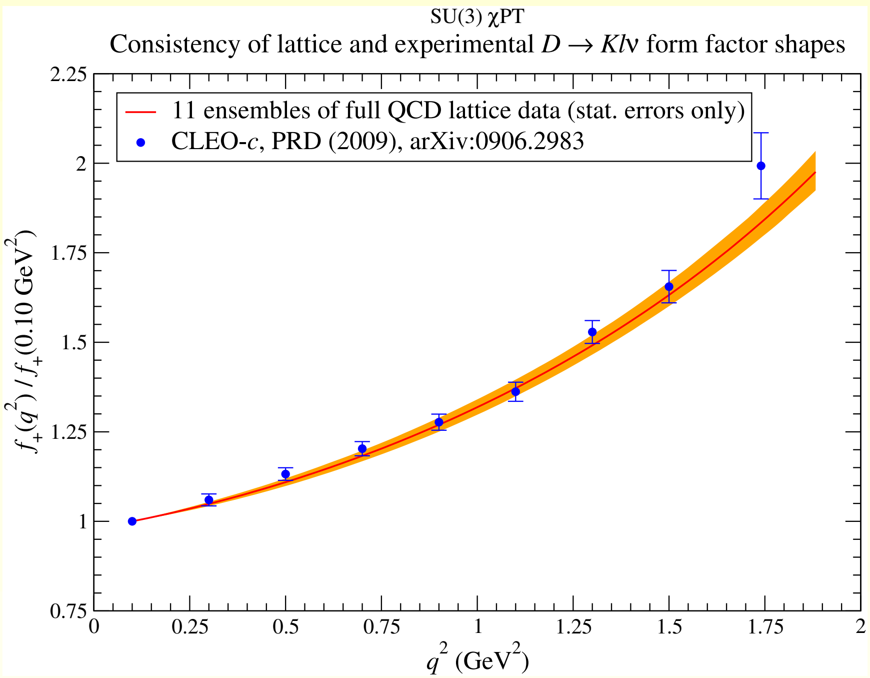
First very preliminary results

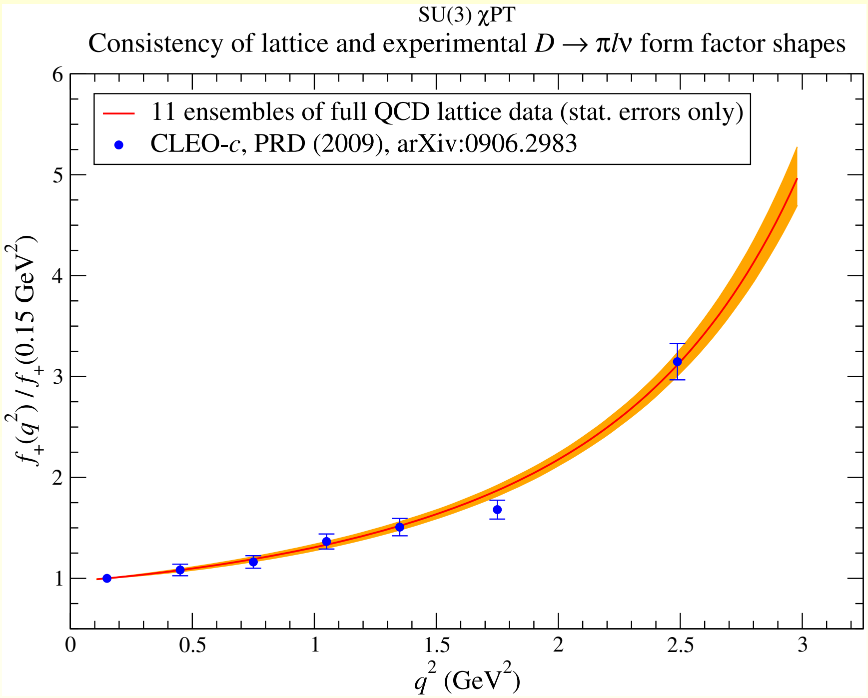


$D \to K(\pi) l\nu$ Still working on the (more challenging) fits

3.2. D semileptonic decays at $q^2 \neq 0$

Comparing shape of the form factor with experiment:





... to test lattice QCD → use same methodology for another semileptonic decays: $B \to \pi l \nu$, $B \to K l \bar{l}$, $B_s \to K l \nu$ ...

* Global fit in SM + experiment → $|V_{cs(cd)}|$ and $f_+^{D\to K(\pi)}(q^2)$

3.2. D semileptonic decays at $q^2 \neq 0$

Plan for the (near) future: extend the $q^2 = 0$ calculation with Hisq fermions to $q^2 \neq 0$.

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... meanwhile, we expect to finish the analysis of the shape of the form factors parametrizing $D \to K(\pi) l\nu$ on the $N_f = 2+1$ Asqtad ensembles with Fermilab charm quarks.

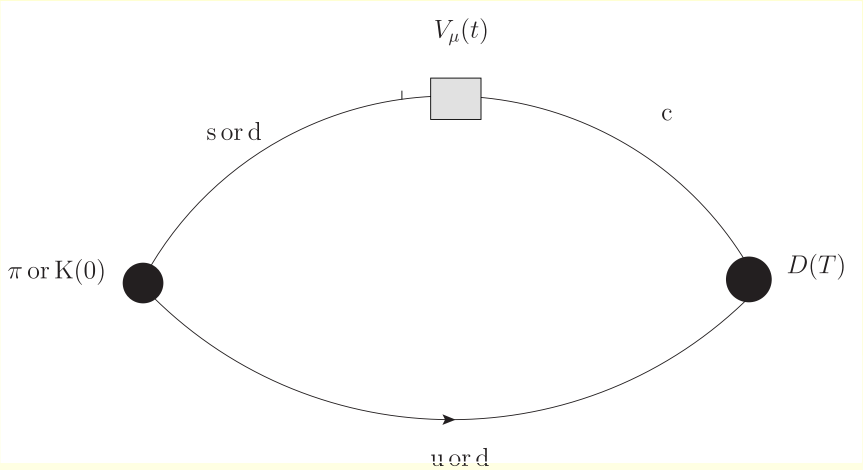
* Same strategy as for FNAL/MILC $B-$physics programme described by **Andreas Kronfeld's talk**

** MILC staggered Asqtad $N_f = 2+1$ configurations.

** Staggered Asqtad formulation for the light ($u = d$ and s) quarks.

** Fermilab charm quarks.

3.2. $D \to K(\pi) l\nu$ at $q^2 \neq 0$: Fermilab charm

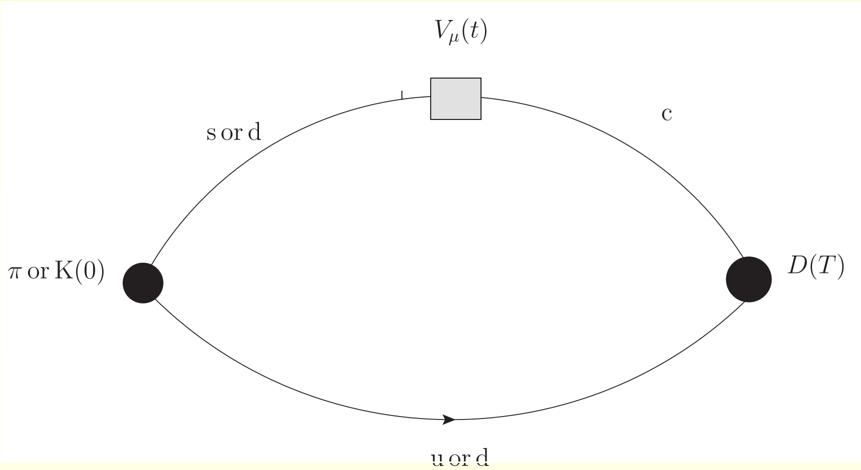


$$\langle P|V^\mu|D\rangle = f_+(q^2)\left[p_D^\mu + p_P^\mu - \frac{m_D^2 - m_P^2}{q^2} q^\mu\right]$$

$$+ f_0(q^2)\frac{m_D^2 - m_P^2}{q^2} q^\mu$$

* Work on D rest frame

3.2. $D \to K(\pi) l\nu$ at $q^2 \neq 0$: Fermilab charm



$$\langle P|V^\mu|D\rangle = f_+(q^2)\left[p_D^\mu + p_P^\mu - \frac{m_D^2 - m_P^2}{q^2} q^\mu\right]$$

$$+ f_0(q^2)\frac{m_D^2 - m_P^2}{q^2} q^\mu$$

* Work on D rest frame

A more convenient choice of parameters is

$$\langle P|V_\mu|D\rangle = \sqrt{2m_D}\left[v_\mu f_\parallel^{D\to P}(q^2) + p_{\perp\mu} f_\perp^{D\to P}(q^2)\right]$$

with $f_\parallel^{D\to P}(q^2) = \frac{\langle\pi|\mathcal{V}^0|D\rangle}{\sqrt{2m_D}}$ and $f_\perp^{D\to P}(q^2) = \frac{\langle\pi|\mathcal{V}^i|D\rangle}{\sqrt{2m_D}}\frac{1}{p_\pi^i}$ (D rest frame)

3.2. $D \to K(\pi) l\nu$ at $q^2 \neq 0$: Fermilab charm

Improved vector currents.

* Rotate heavy quark field to remove $\mathcal{O}(1/m_c)$ errors:

$$\psi_c \to \Psi_c = \left(1 + a d_1 \vec{\gamma} \cdot \vec{D}_{lat}\right) \psi_c$$

** d_1 its fixed to its tadpole-improved tree-level value.

3.2. $D \to K(\pi) l\nu$ at $q^2 \neq 0$: Fermilab charm

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** d_1 its fixed to its tadpole-improved tree-level value.

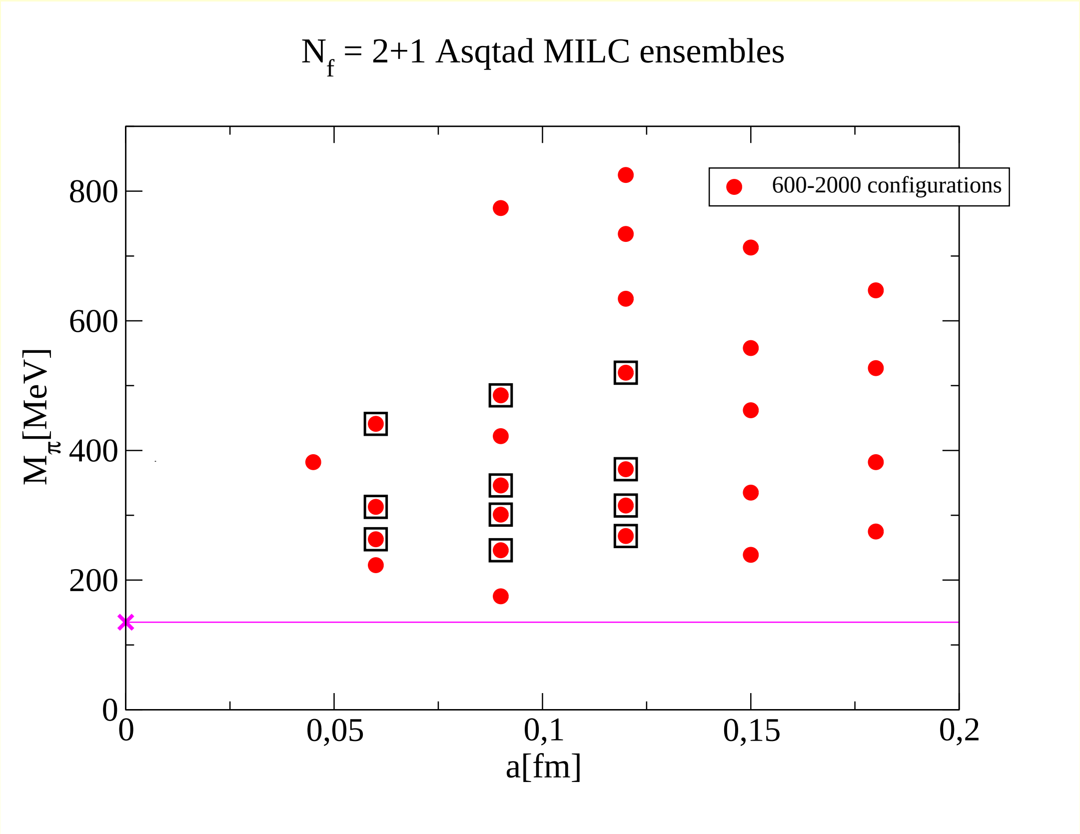
Renormalization: Partially non-perturbative.

$$Z^{hl}_{V_\mu} = \rho^{hl}_{V_\mu} \sqrt{Z^{hh}_V Z^{ll}_V}$$

* Z^{hh}_V and Z^{ll}_V determined non-perturbatively

* $\rho^{hl}_{V_\mu}$ perturbative correction (one-loop). Small correction.

3.2. $D \to K(\pi)l\nu$ at $q^2 \neq 0$: Fermilab charm



* Physical charm masses and $am_{l(s)}^{valence} = am_{l(s)}^{sea}$.
* Four source-sink separations (two physical ones and two for constructing ratio).
* 5 values of q^2 (= values outgoing momenta).

3.2. $D \to K(\pi) l\nu$ at $q^2 \neq 0$: Fermilab charm

Generate 3-point and 2-point correlation functions

$$C_{3,\mu}^{D\to K(\pi)}(t,T;\vec{p}_{K(\pi)}) = \sum_{\vec{x},\vec{y}} e^{i\vec{p}_{K(\pi)}\cdot\vec{y}} \langle \mathcal{O}_{K(\pi)}(t_{source},\vec{0}) V_\mu(t,\vec{y}) \mathcal{O}_D^\dagger(T,\vec{x})\rangle$$

$$C_2^{K(\pi)}(t;\vec{p}_{K(\pi)}) = \sum_{\vec{x}} e^{i\vec{p}_{K(\pi)}\cdot\vec{x}} \langle \mathcal{O}_{K(\pi)}(t_{source},\vec{0}) \mathcal{O}_{K(\pi)}^\dagger(t,\vec{x})\rangle$$

$$C_2^{D}(t) = \sum_{\vec{x}} \langle \mathcal{O}_{D}(t_{source},\vec{0}) \mathcal{O}_{D}^\dagger(t,\vec{x})\rangle$$

* D rest frame: $\vec{p}_\pi = (0,0,0),(1,0,0),(1,1,0),(1,1,1),(2,0,0)$.
* Randomizing spatial location of the sources $\to$ decreases autocorrelations.
* Smearing: D-`meson` interpolating operators are smeared with a $1S$ charmoniun wavefunction.

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* Randomizing spatial location of the sources $\to$ decreases autocorrelations.

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Build combinations of $C_3^{D\to\pi}(t,T)$ and $C_3^{D\to\pi}(t,T+1)$ to suppress contributions from opposite parity (staggered artifacts) and excited states

$\to$ fit to a plateau or plateau + dominant oscillating contamination

3.2. $D \to K(\pi) l\nu$ **at** $q^2 \neq 0$: **Fermilab charm**

SU(3) SχPT

Partially-quenched heavy-meson staggered ChPT **Aubin & Bernard**,PRD76(2007)

* Use complete SU(3) NLO + analytic NNLO expressions.

$$f_{\parallel} = \frac{c_0}{f_\pi}\left[1 + \mathrm{logs} + c_1 m_l + c_2 E_\pi + c_3 (E_\pi)^2 + c_4 a^2 + \mathrm{NNLO\,analy.\,terms}\right]$$

$$f_{\perp} = \frac{c'_0 g_\pi}{f_\pi}\left\{\frac{1}{E_\pi + \Delta_l^* + \mathrm{logs}} + \frac{1}{E_\pi + \Delta_l^*}\left[\mathrm{logs} + c'_1 m_l + c'_2 E_\pi + c'_3 (E_\pi)^2 + c'_4 a^2 + \mathrm{NNLO\,analy.\,terms}\right]\right.$$

** No sea quark mass dependence is included (m_s^{sea} similar in all ensembles and $m_l^{val.} = m_l^{sea}$)

** Tried including $a^4 \to$ not much impact.

3.2. $D \to K(\pi) l\nu$ at $q^2 \neq 0$: Fermilab charm

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$$\left.\left. + c_3' (E_\pi)^2 + c_4' a^2 + \mathrm{NNLO\,analy.\,terms}\right]\right.$$

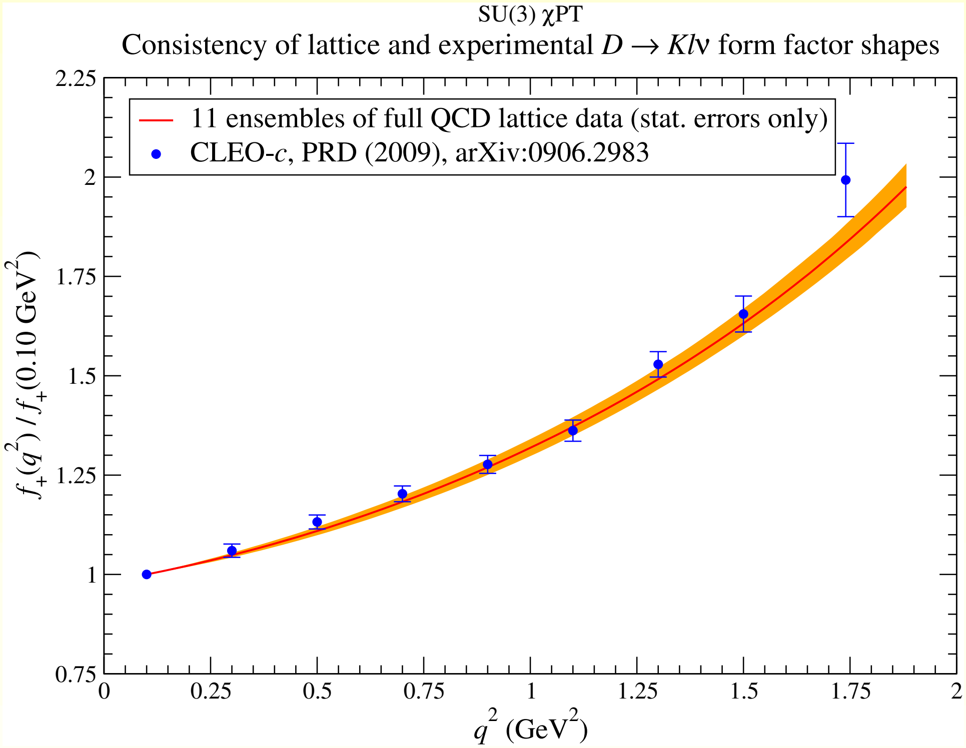
** No sea quark mass dependence is included (m_s^{sea} similar in all ensembles and $m_l^{val.} = m_l^{sea}$)

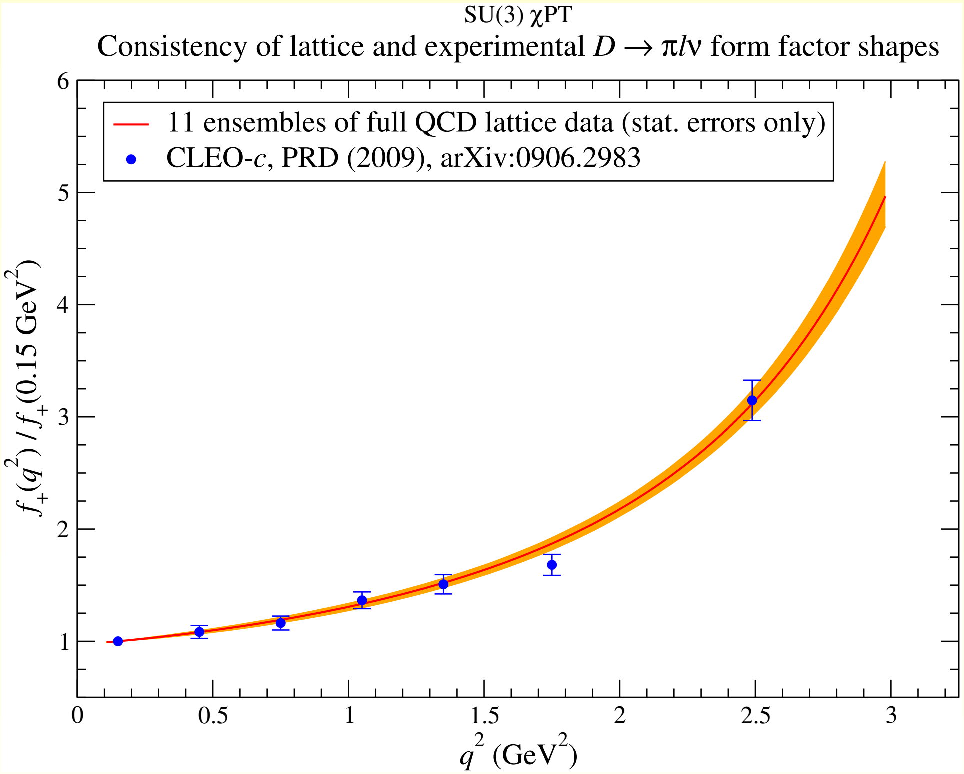
** Tried including $a^4 \to$ not much impact.

* ChPT not reliable for $\chi_\pi = \frac{\sqrt{2}E_\pi}{4\pi f_\pi} > 1 \to \vec{p}_\pi = (1,1,1), (2,0,0)$ not used in the fits.

3.2. $D \to K(\pi)l\nu$ at $q^2 \neq 0$: Fermilab charm

SU(3) SχPT



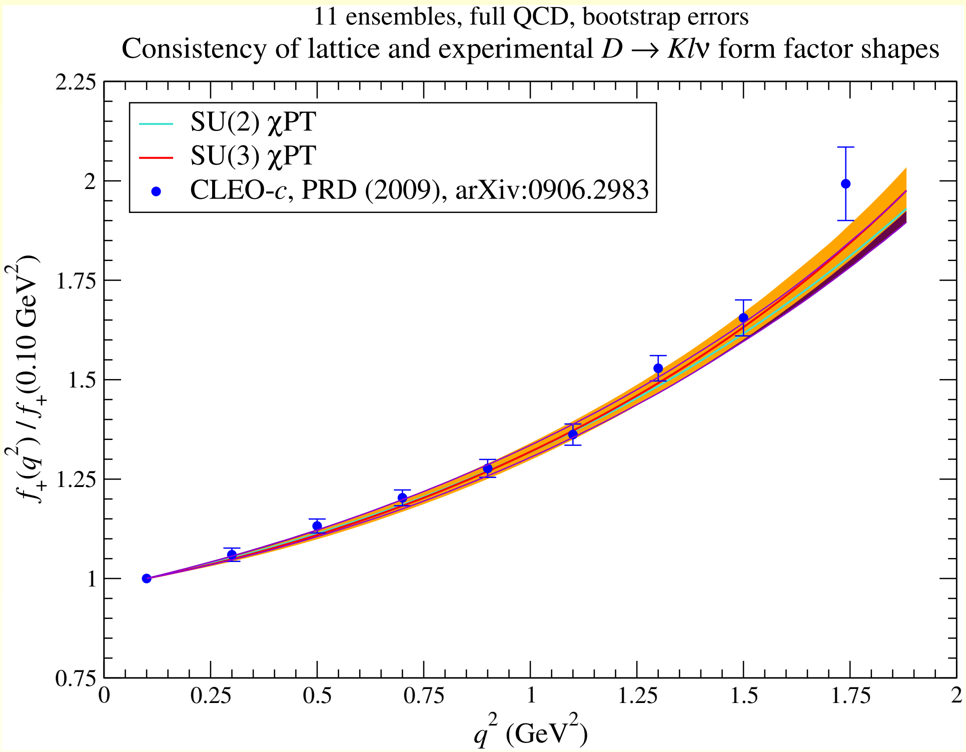


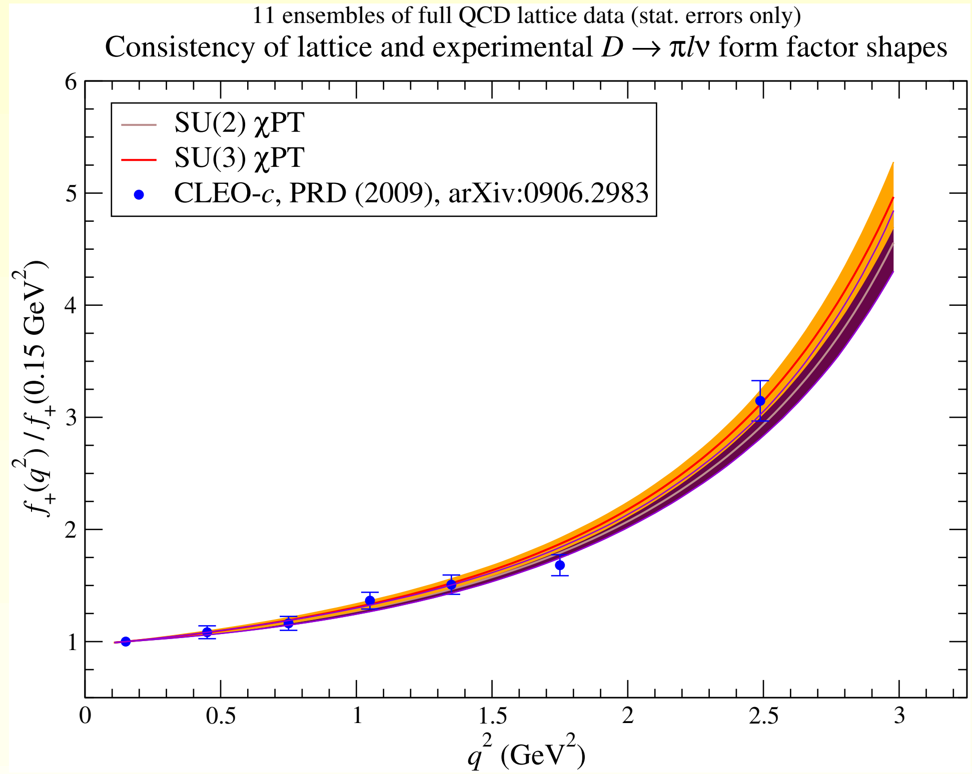
$f_+(q^2)$ rescaled by its value at $q^2 = 0.10\ \mathrm{GeV}^2$

\# Good description of the experimentally measured shape of the form factors.

3.2. $D \to K(\pi) l\nu$ at $q^2 \neq 0$: Fermilab charm

SU(3) vs SU(2) SχPT: Cross-check of chiral+continuum extrapolation.





* $SU(3)$ or $SU(2)$? Still under investigation. Very consistent results.

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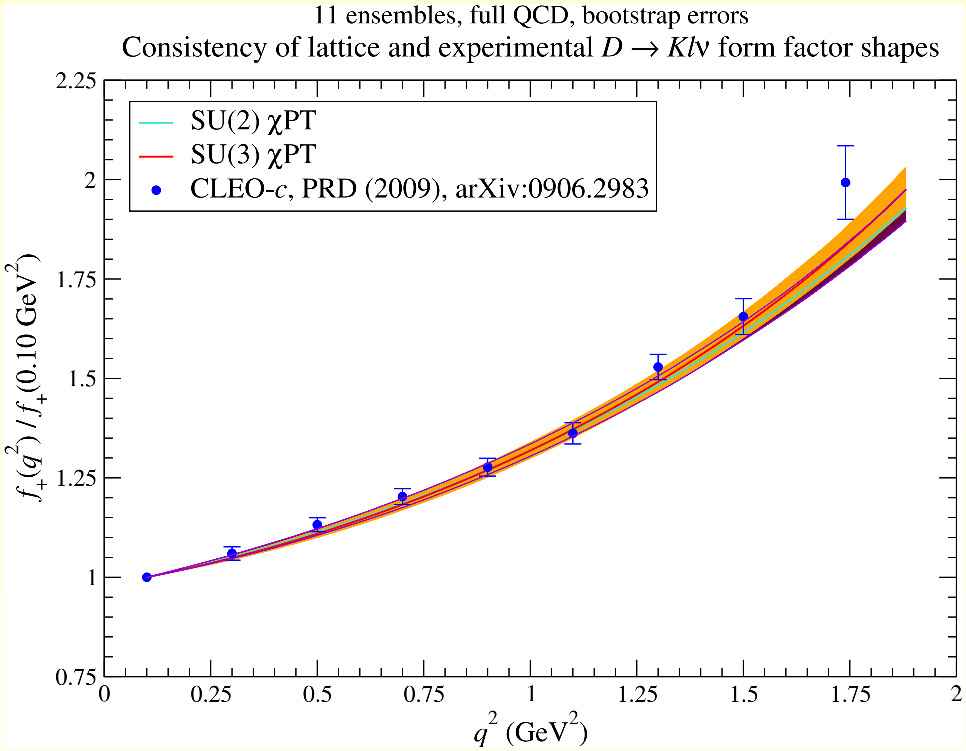
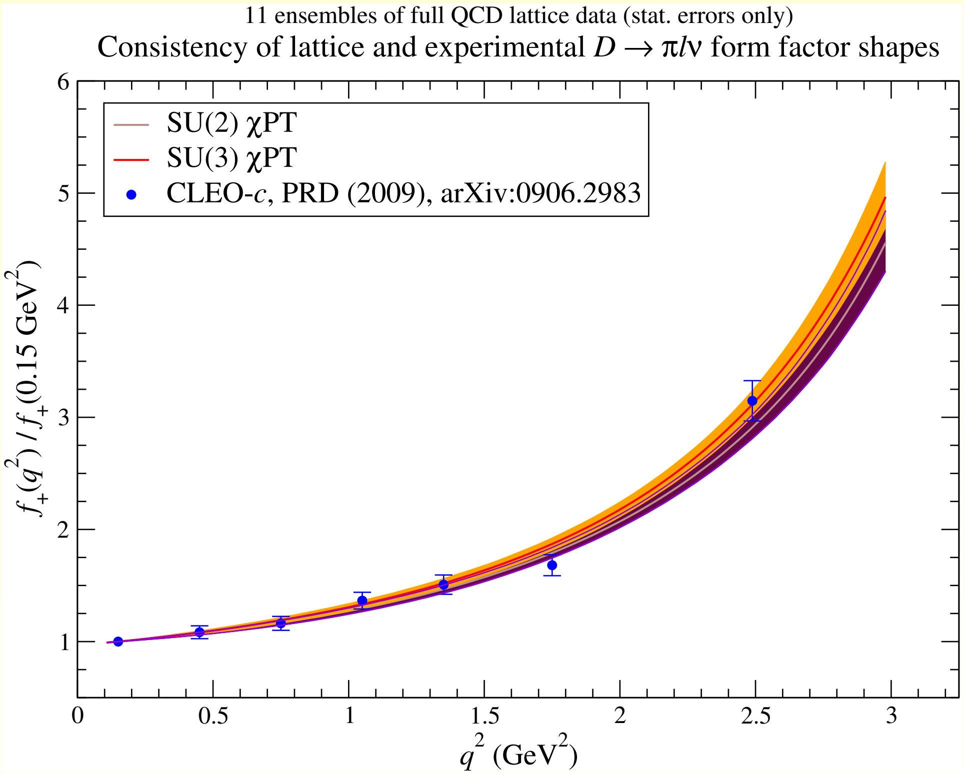




* $SU(3)$ or $SU(2)$? Still under investigation. Very consistent results.

Plan: Use z-expansion to extract $|V_{cd(cs)}|$ in a model-independent way from a simultaneous fit of lattice and experimental data over q^2.

$z-$expansion based on unitarity, analyticity, crossing symmetry, and heavy-quark symmetry

3.2. $D \to K(\pi) l\nu$ at $q^2 \neq 0$: Fermilab charm

Projected error budget for $f_+^{D\to K(\pi)}$

stat. + χPT (%)	2.0
g_π	2.9
r_1	1.4
$\hat{m}$	0.3
m_s	1.3
m_c	0.2
heavy quark discr.	2.5
renormalization	0.8
Finite Volume	0.5
total system. (%)	4.4
Total (%)	**4.8**

4. Conclusions

Starting a new program with Hisq $N_f = 2+1+1$ ensembles.

* BES 3 charm and CERN K programs makes D and K physics timely.

* Reduction of discretization errors on the sea, physical quark masses, sea charm quarks included.

First preliminary results for $f_K/f\pi$ and f_D, f_{D_S} very encouraging.

For $f_+^{K\pi}(0)$ we expect errors $0.35 - 0.5\%$, smaller when using Hisq ensembles.

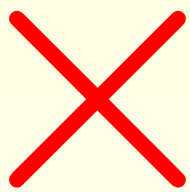
* $f_+^{DK}(0)$ and $f_+^{D\pi}(0)$ on the $N_f = 2+1+1$ Hisq ensembles.

* Also $f_+^{DK}(q^2)$ and $f_+^{D\pi}(q^2)$.

D semileptonic form factors with Fermilab c on the Asqtad $N_f = 2+1$ close to be completed (errors of the order of 5%).

Very good check against pure relatavistic calculation

B physics: Fermilab or Oktay-Kronfeld action versus relativistic Hisq b quarks?



MILC $N_f = 2+1+1$ ensembles

$\approx a(fm)$	am_l	am_s	am_c	Volume	$N_{conf.}$	am_s^{phys}
0.15	0.013	0.065	0.838	$16^3 \times 48$	1021?	0.06860(23)
	0.0064	0.064	0.828	$24^3 \times 48$	1000?	0.06860(13)
	0.00235	0.0647	0.831	$32^3 \times 48$	1020	0.06905(8)
0.12	0.0102	0.0509	0.635	$24^3 \times 64$	1053	0.05348(15)
	0.00507	0.0507	0.628	$32^3 \times 64$	1020	0.05295(13)
	0.00184	0.0507	0.628	$48^3 \times 64$	460	0.05325(5)
0.09	0.0074	0.037	0.440	$32^3 \times 96$	1011	0.03705(13)
	0.00363	0.0363	0.430	$48^3 \times 96$	1000	0.03668(8)
	0.0012	0.0363	0.432	$64^3 \times 96$	497	0.03655(5)
0.06	0.0048	0.024	0.286	$48^3 \times 144$	844	0.02295(8)
	0.0024	0.024	0.286	$64^3 \times 144$	555	0.02240(5)
	0.00084	0.0231	0.274	$96^3 \times 192$	1	??
0.045	0.0035	0.0175	0.213	$64^3 \times 192$	-	
	0.0007	0.018	0.21	$128^3 \times 256$	-	