Bottomonium masses and radiative transitions

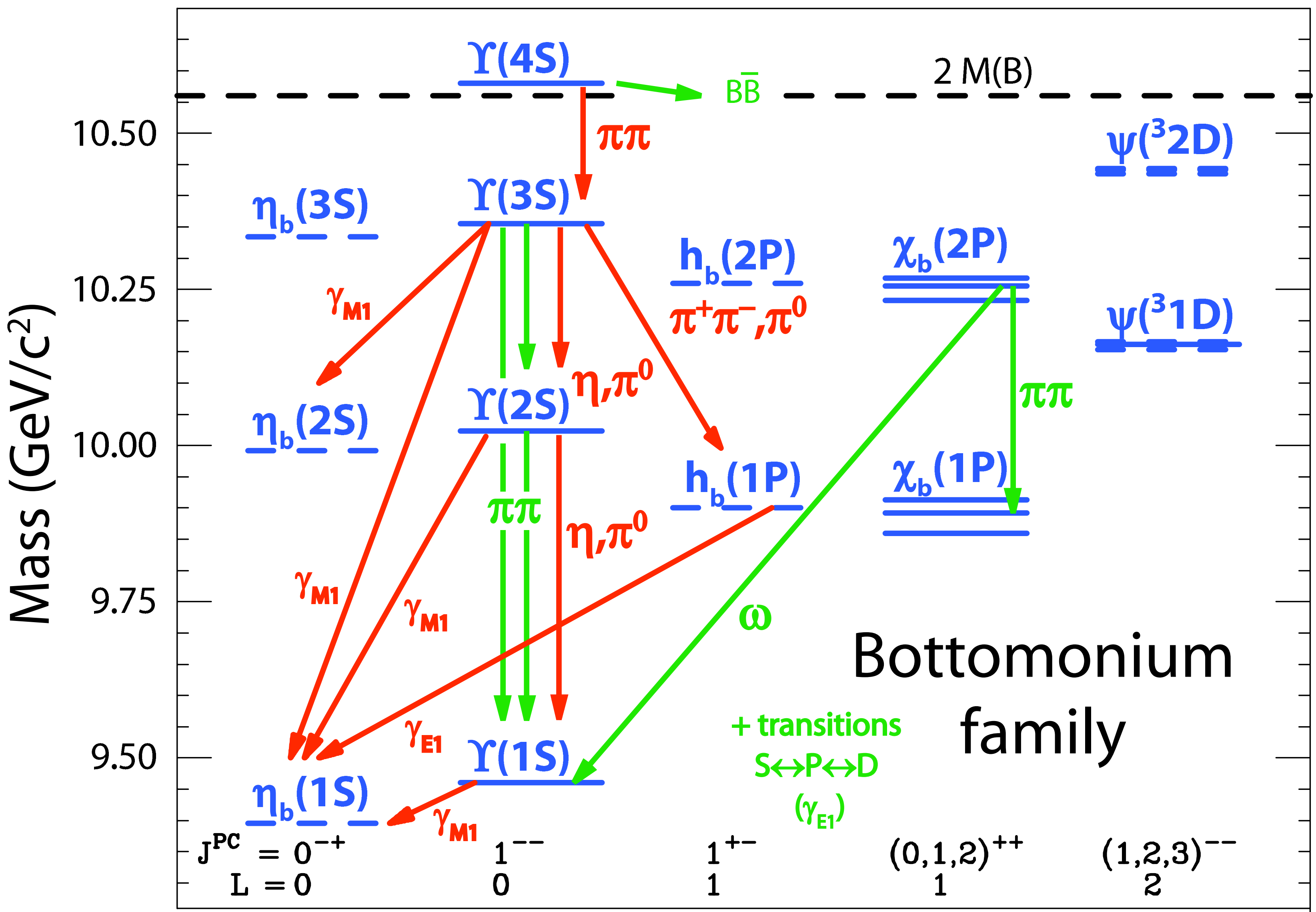
Lattice NRQCD results for

- M1 transitions among S waves (a refinement of [1])
- masses of S, P, D and F waves (and a glimpse beyond)

This work is being done in collaboration with R. M. Woloshyn.

[1] R. Lewis and R. M. Woloshyn, Phys. Rev. D85, 014504 (2012).

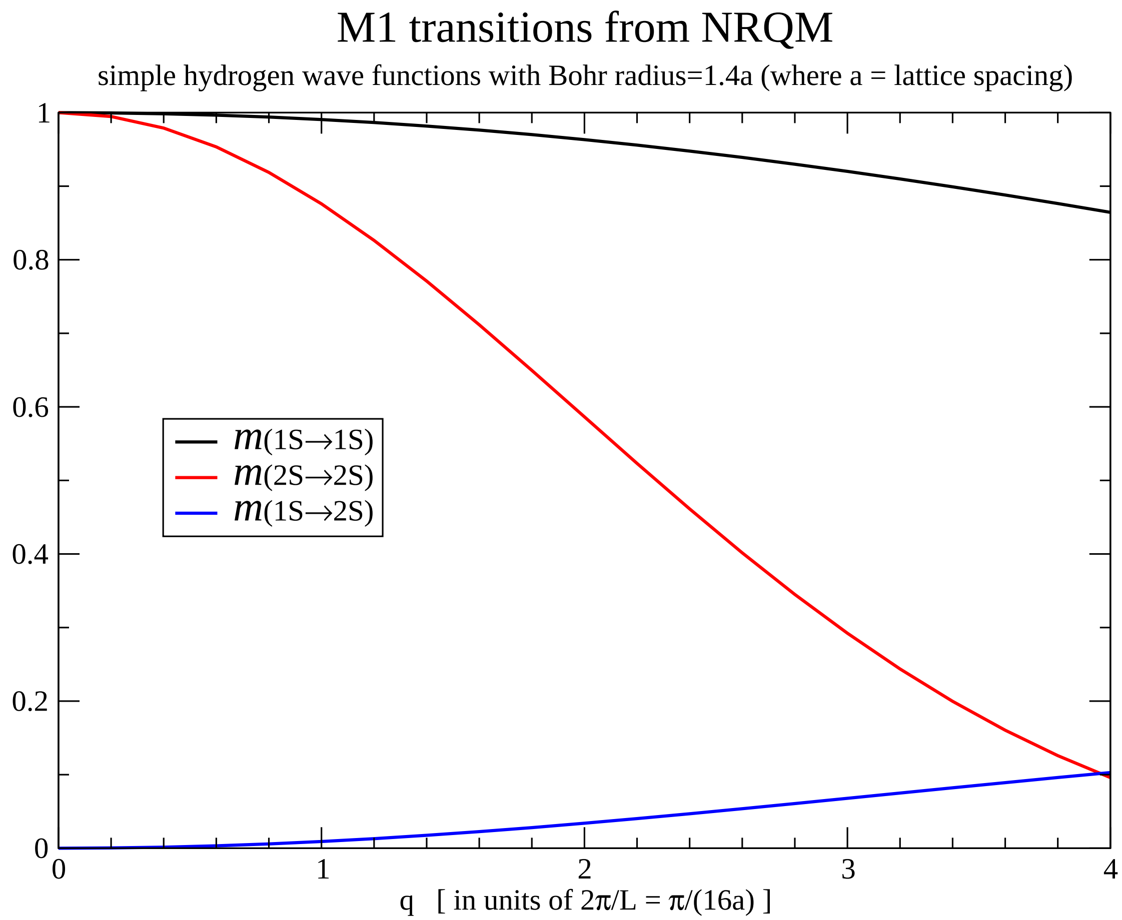
Radiative transitions in bottomonium



Eichten, Godfrey, Mahlke, Rosner, RMP80, 1161 (2008)

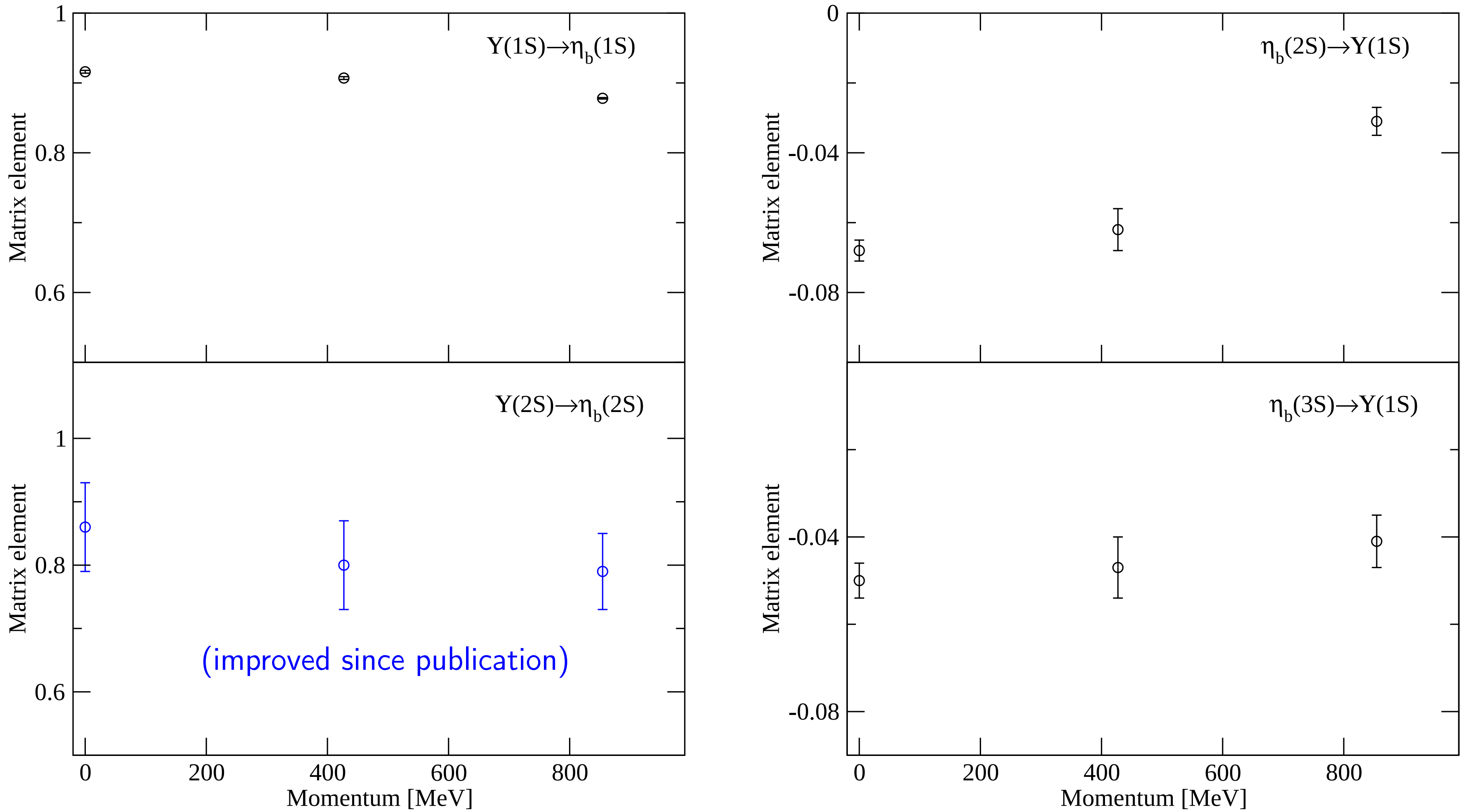
Pseudoscalar/vector M1 transitions in the nonrelativistic quark model require

$$\mathcal{M}(nS \to n'S) = \int_0^\infty R_{n'}(r) R_n(r) j_0(qr/2) r^2 dr$$



Therefore hindered transitions are subtle: recoil, spin, relativistic, . . .

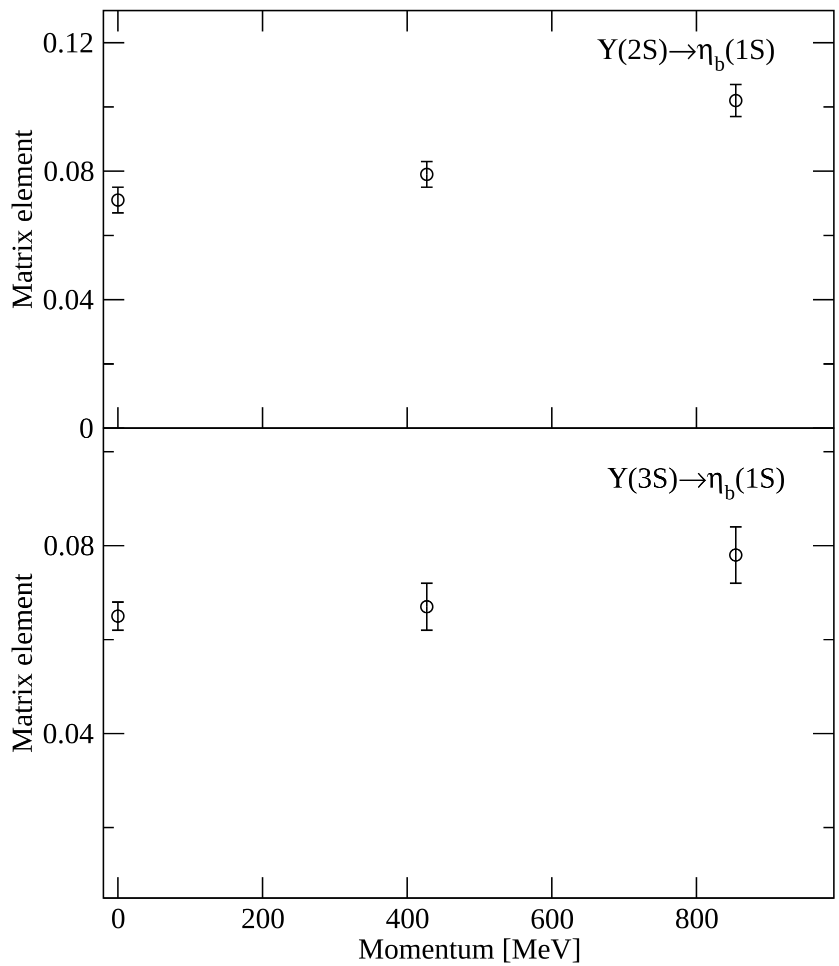
Qualitative success



Near unity; modest momentum dependence.

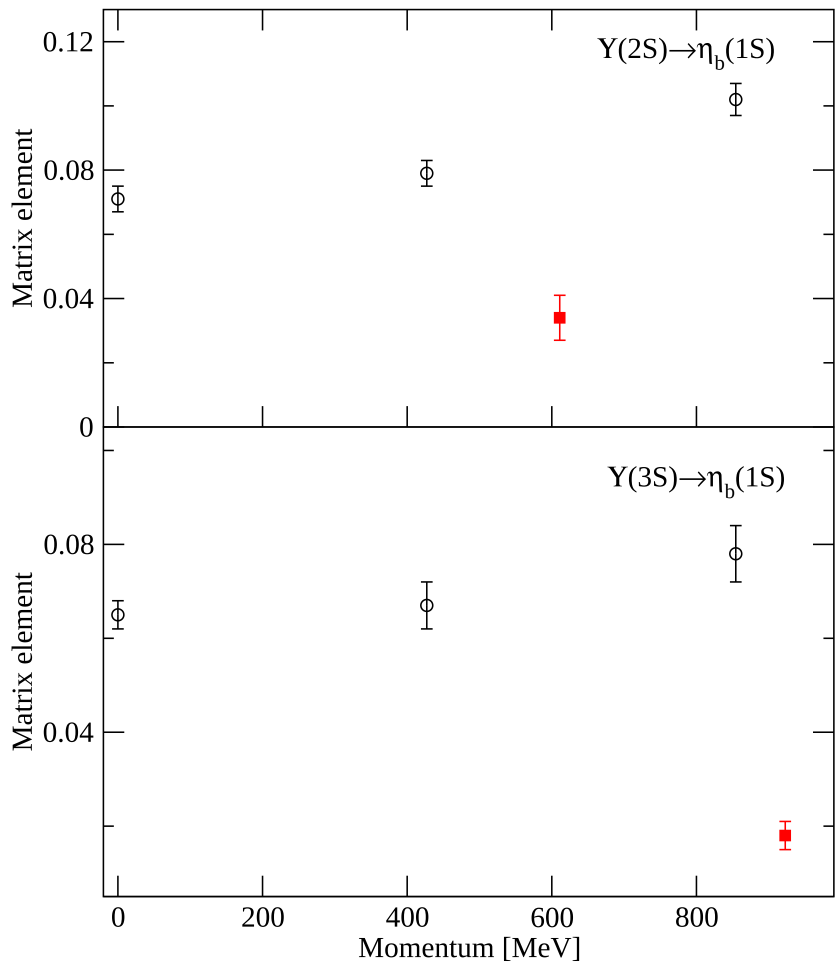
Small and negative.

Qualitative success



Quantitative problem

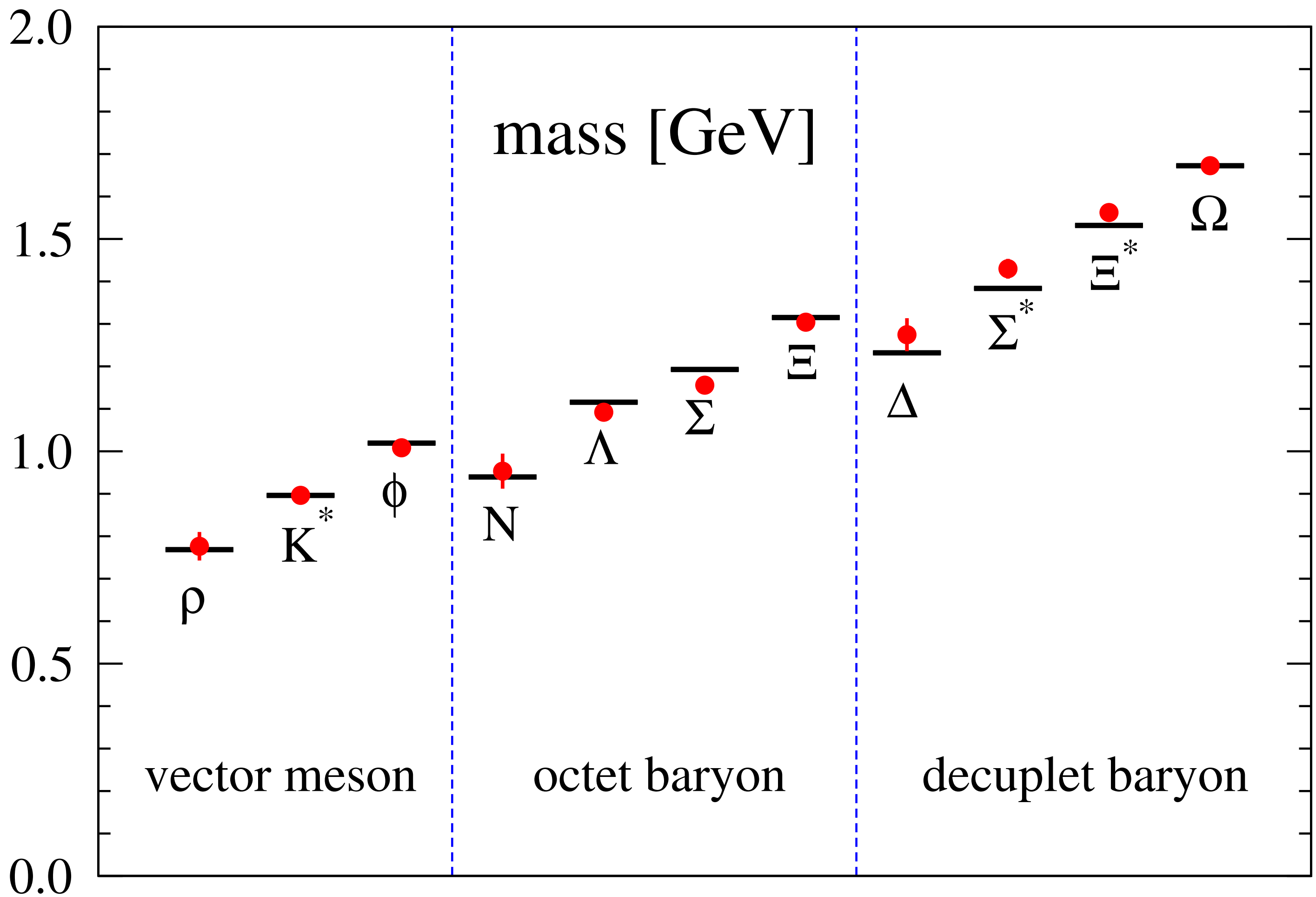
BABAR, PRL 101, 071801 (2008) and *BABAR, PRL 103, 161801 (2009)*



The PACS-CS configurations

S. Aoki *et al.* Phys. Rev. D79, 034503 (2009).

- Iwasaki+clover improved action. We use one ensemble of 192 configurations.
- m_u=$m_d \gtrsim$ physical (m_π=156 MeV) and $m_s \gtrsim$ physical (m_K=553 MeV).
- $32^3 \times 64$ lattices with $\beta = 1.90 \Rightarrow$ a=0.0907(14) fm and $L = 32a = 2.9$ fm.
- Parameters are set using m_π, m_K and m_Ω as input.



Tadpole-improved NRQCD action

$$H = \frac{-\Delta^{(2)}}{2M_0} - c_1\frac{(\Delta^{(2)})^2}{8{M_0}^3} + \frac{c_2}{{U_0}^4}\frac{ig}{8{M_0}^2}(\Delta\cdot\mathbf{E} - \mathbf{E}\cdot\Delta)$$
$$-\frac{c_3}{{U_0}^4}\frac{g}{8{M_0}^2}\boldsymbol{\sigma}\cdot(\Delta\times\mathbf{E} - \mathbf{E}\times\Delta) - \frac{c_4}{{U_0}^4}\frac{g}{2M_0}\boldsymbol{\sigma}\cdot\mathbf{B}$$
$$+c_5\frac{a^2\Delta^{(4)}}{24M_0} - c_6\frac{a(\Delta^{(2)})^2}{16n{M_0}^2} + O(v^6)$$

The stability parameter n is algorithmic not physical; we use $n = 4$.

Tadpole improvement via average link in Landau gauge: $U_0 = 0.8463$.

We use tadpole-improved leading order: $c_i = 1$ for all i.

The bottom quark bare mass $M_0 = 1.945$ is set by fitting the experimental η_b mass. Specifically, the η_b kinetic energy is used: $E(p) - E(0) = \sqrt{p^2 + {M_0}^2} - M_0$ with the three smallest lattice momenta.

Bottomonium propagation

- 16 random U(1) wall sources per configuration.

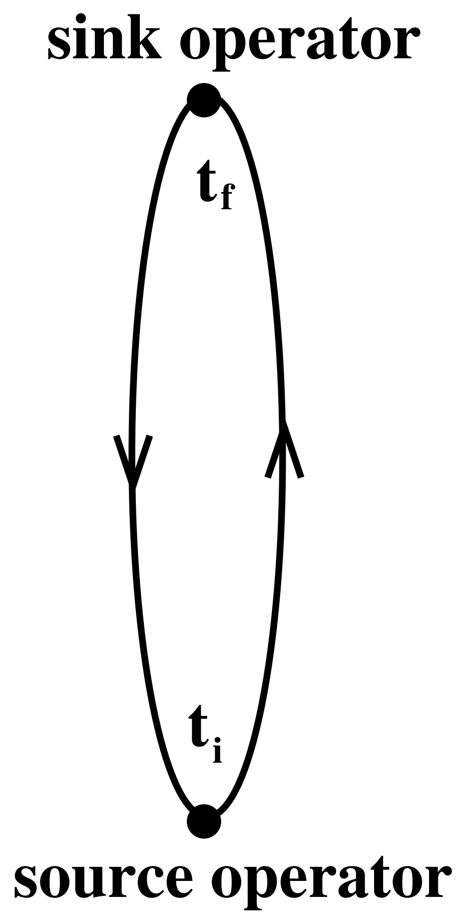
- Smearing in Coulomb gauge:
(l)ocal, (s)meared, (d)oubly-smeared.

$$O_{\eta_b} = \sum_y \chi(x)\phi(x-y)\psi(y)$$
$$O_{\Upsilon} = \sum_y \chi(x)\sigma_3\phi(x-y)\psi(y)$$
$$\phi(r) = \left(1-\frac{r}{2a_0}\right)\exp\left(\frac{-r}{2a_0}\right)$$

with $a_0 = 1.4$ (lattice units).

- Constrained multi-exponential fitting to all times except the source:

$$g_{oo'}(t) = \sum_{n=1}^{N} c_{o'}(n)c_o(n)e^{-E_n(t_f-t_i)}$$



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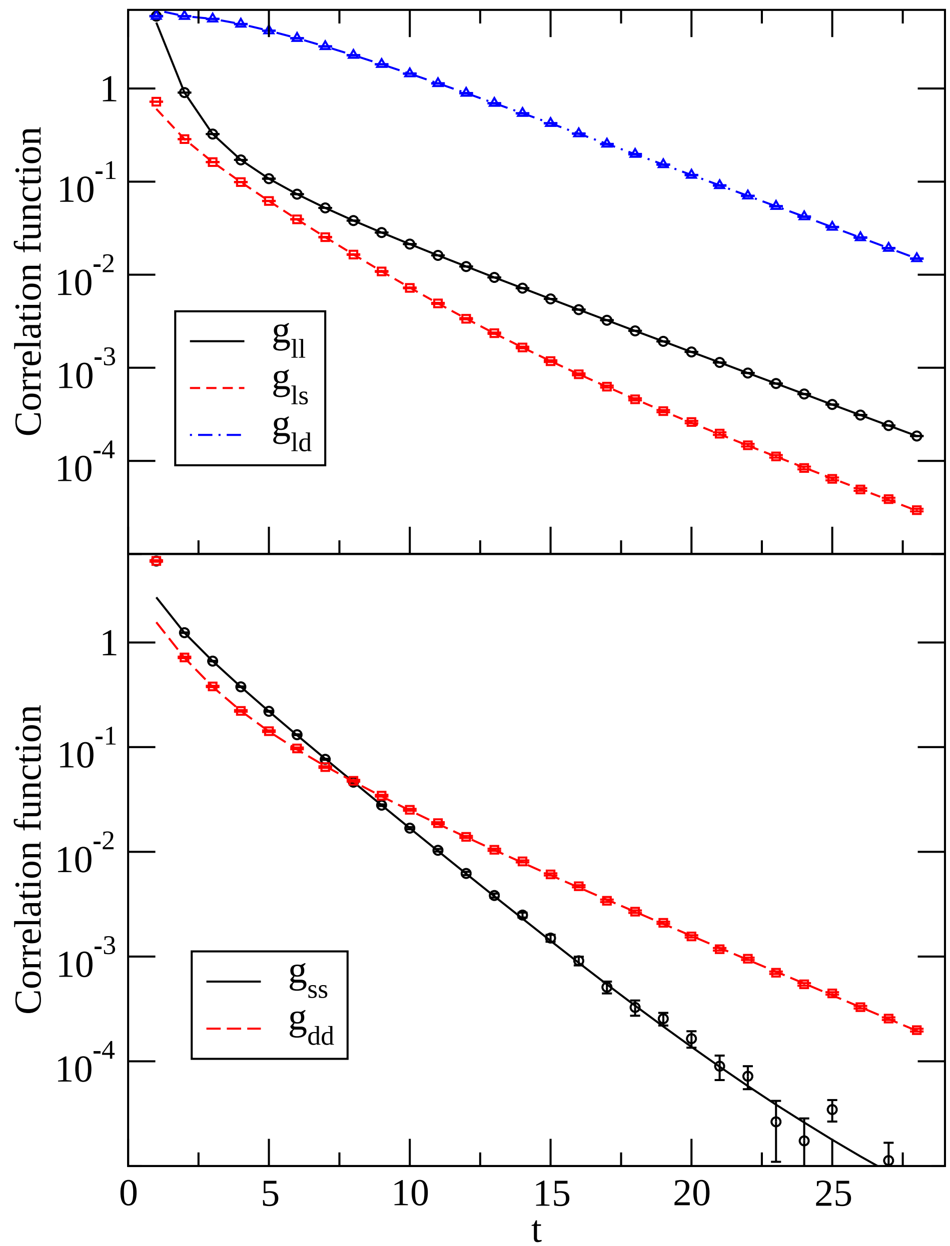
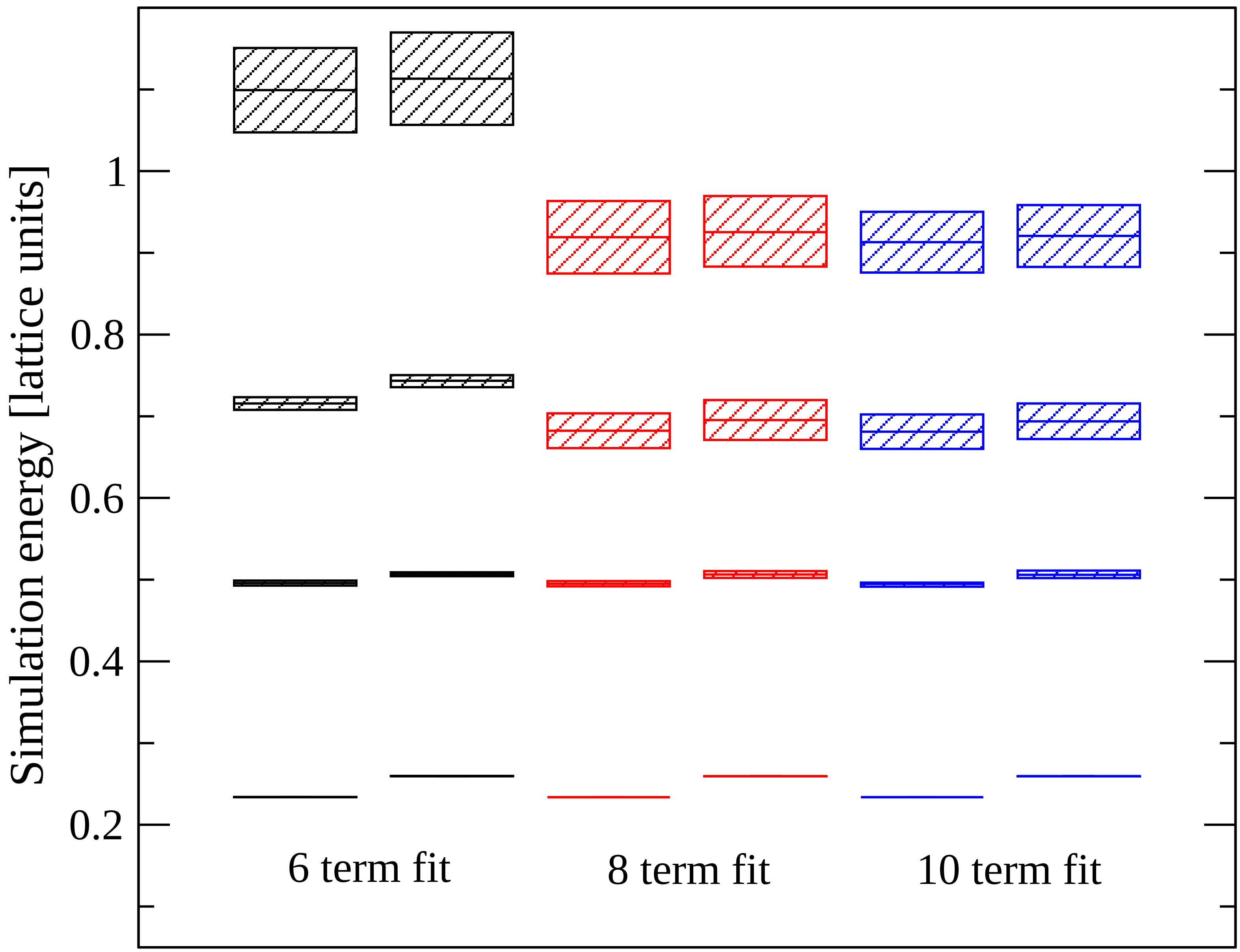
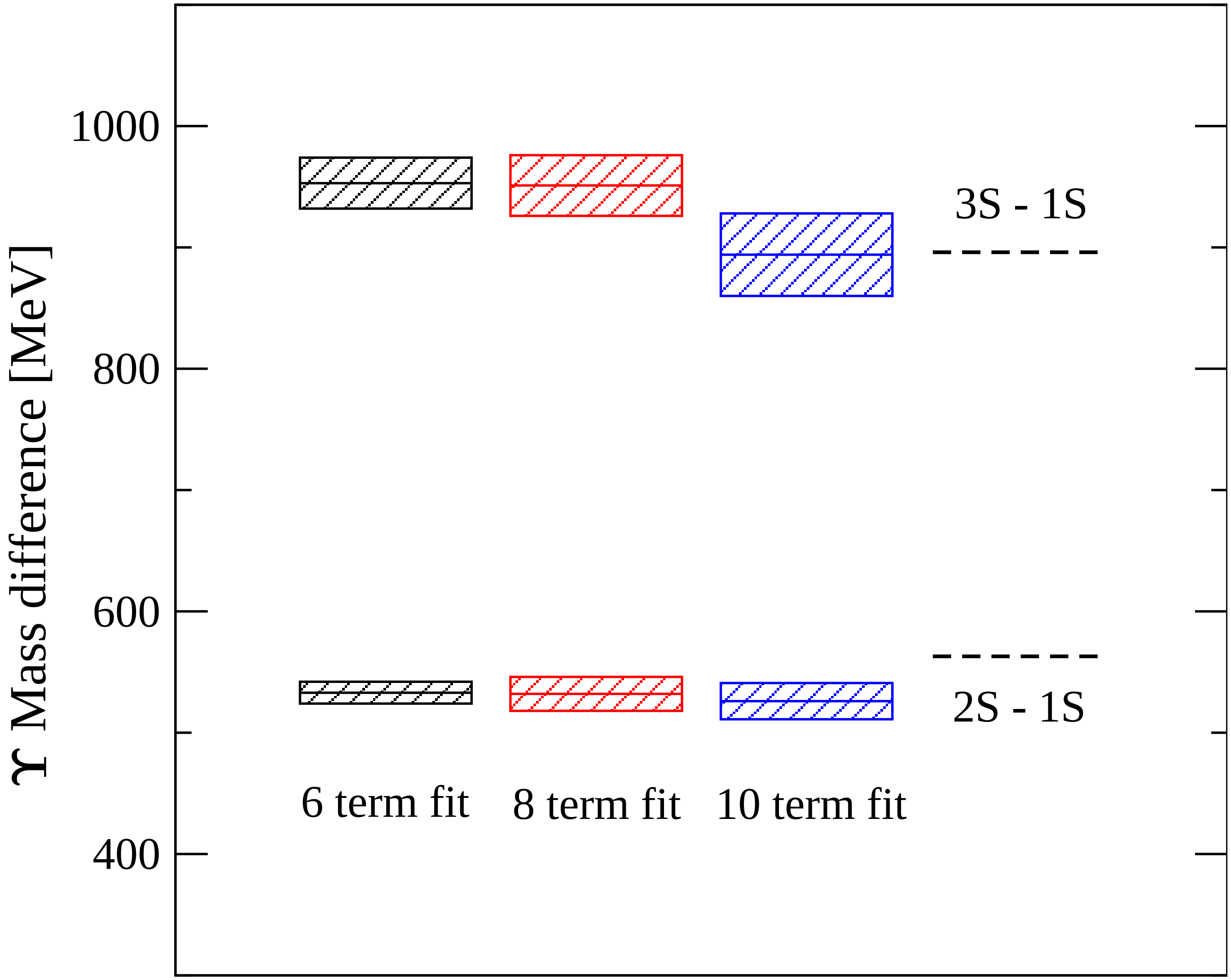
some Υ, Υ' correlators
(N=10 terms per fit)



Stability of η_b and Υ mass fits



Stability and accuracy of mass differences



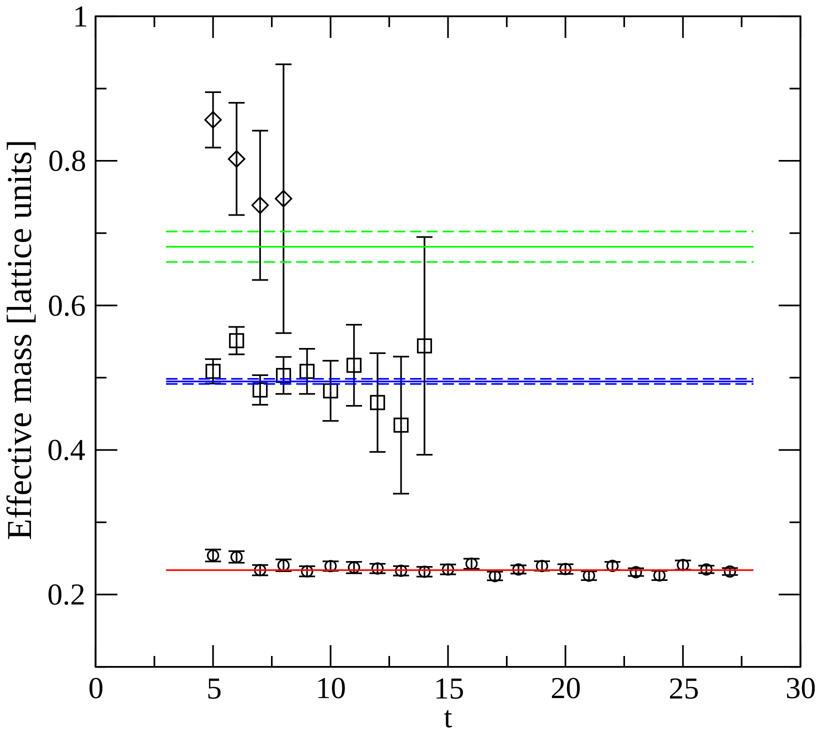
We find $m_{\Upsilon} - m_{\eta_b} = 56 \pm 1$ MeV (statistical error only).

The PDG average is 69.8 ± 2.8 MeV; the recent Belle result is $59.3 \pm 1.9^{+2.4}_{-1.4}$ MeV.

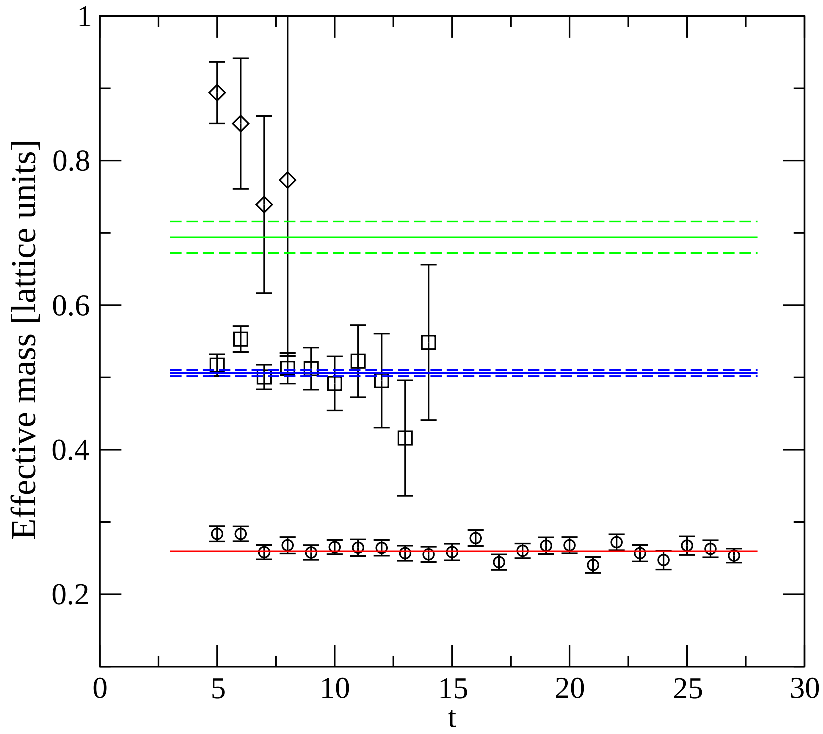
Agreement with the variational method

The variational method solves the eigenvalue problem on each time step.
$g(t)f_k(t) = \lambda_k(t)g(t_0)f_k(t)$ where $g(t)$ is the correlator matrix.

pseudoscalar



vector



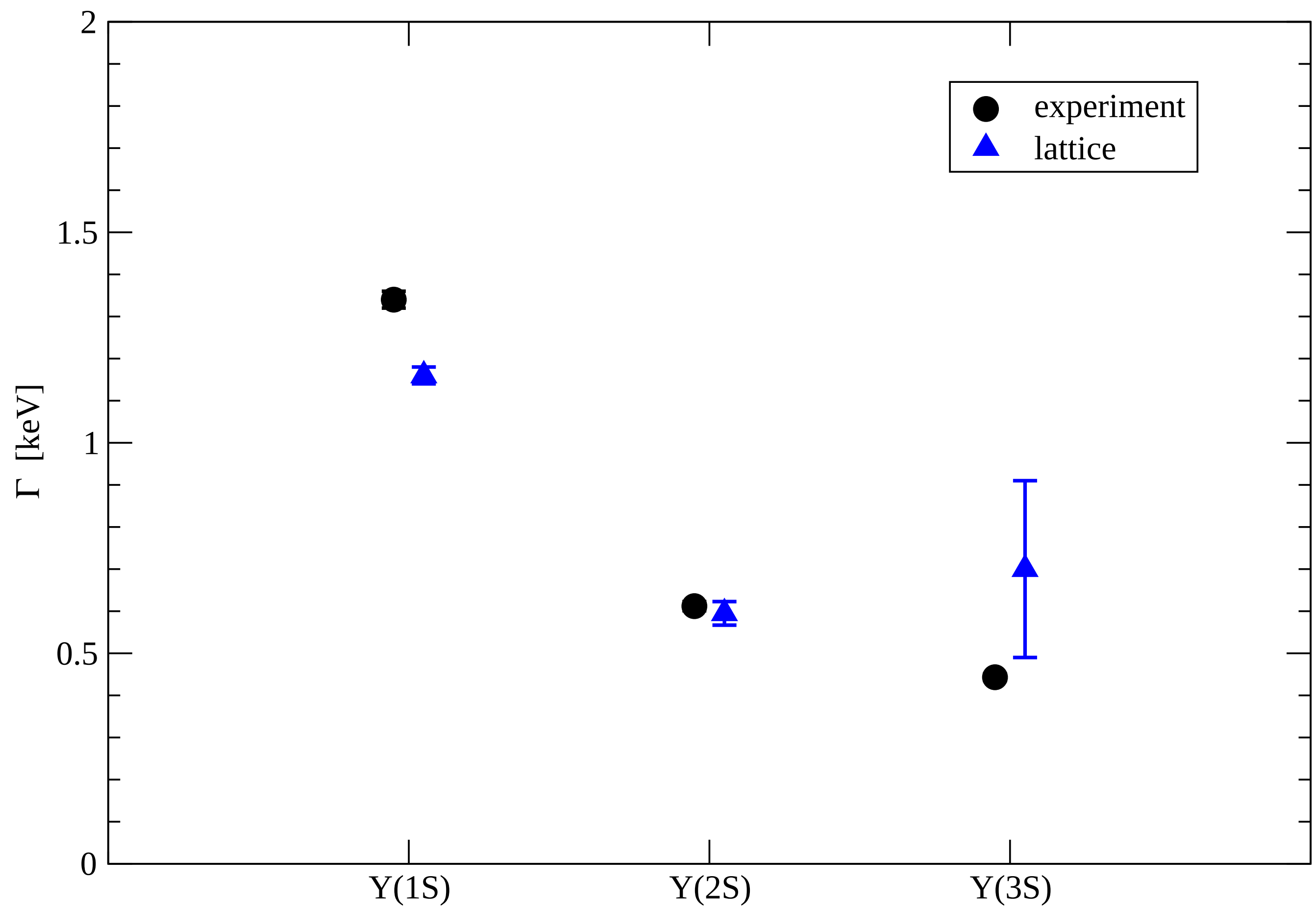
Black symbols are variational. Horizontal lines are 10-term multi-state fits.

Variational results would become more precise with more operators.

Leptonic decay of Υ

$$\Gamma[\Upsilon(nS) \to e^+e^-)] \quad = \quad \frac{16\pi\alpha}{9}\frac{|\Psi_n(0)|^2}{M^2_{\Upsilon(nS)}} Z^2_{\rm match} \quad \approx \quad \frac{16\pi\alpha}{9}\frac{c^2_{\rm local}}{6M^2_{\Upsilon(nS)}}$$

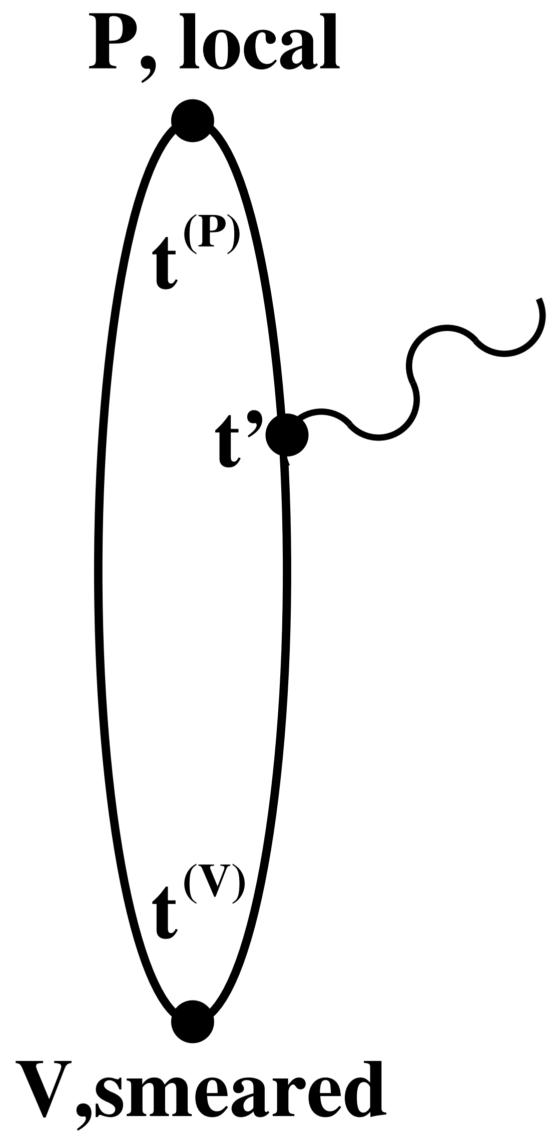
where $\Upsilon_n(0)$ denotes the wave function at the origin
and $Z_{\rm match}$ relates the lattice vector current to the renormalized continuum current.



Three-point functions

$$\sum_n \sum_{n'} c_s^{(V)}(n) A_{nn'}^{(VP)} c_l^{(P)}(n') \exp\left(-E_n^{(V)}(t' - t^{(V)})\right) \exp\left(-E_{n'}^{(P)}(t^{(P)} - t')\right)$$

- $A_{nn'}^{(VP)}$ is the matrix element of interest.
- Two-point c and E values are retained.
- Source is V or P, and is l or s or d.
  Likewise for sink.
  6 "source,sink" used: ll, ls, sl, ss, ld, dl.
- P momentum is (0,0,0), (1,0,0) or (2,0,0).
  V momentum is always zero.
- Current insertion is just a Pauli matrix
  (i.e. leading nonrelativistic term).



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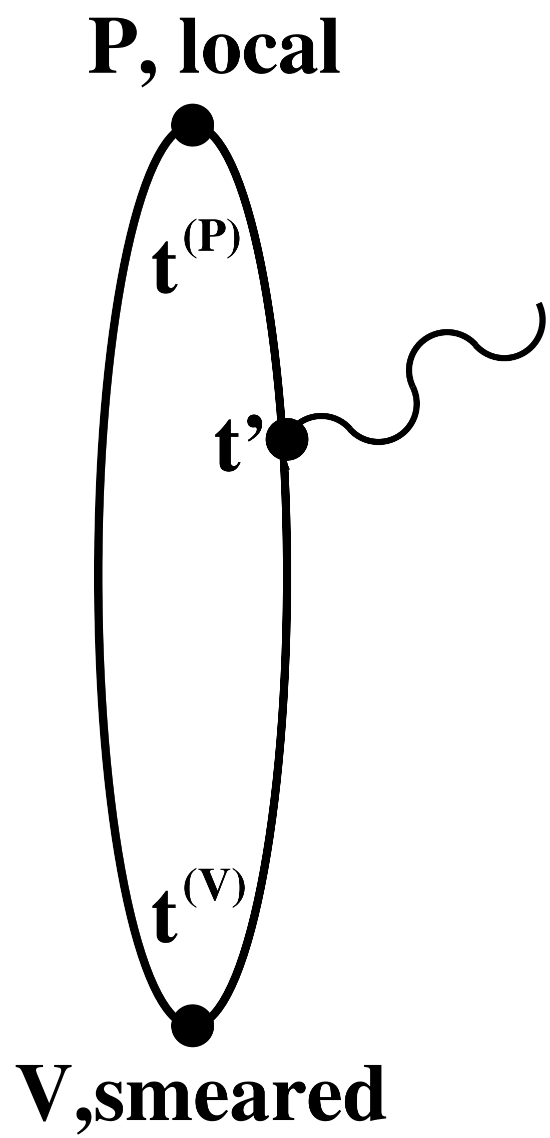
- Three source-sink time separations:
  Δt=19 and 27 and 15 (since publication).

- 10-term fits not required.
  We use nn' =11, 12, 21, 13, 31, 22.

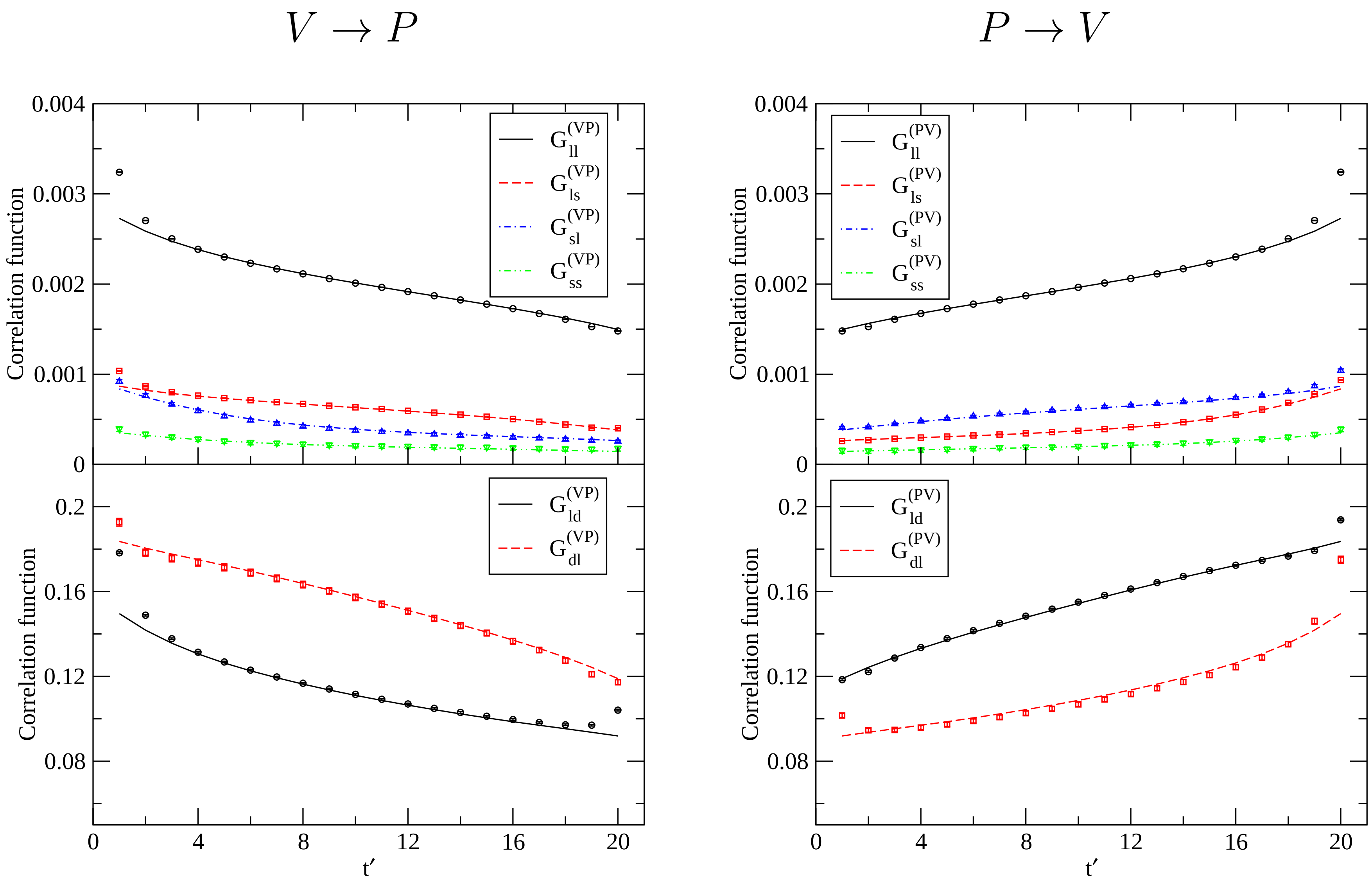
- Excluding nn' =22 causes
  Δt=19 and 27 to disagree.

- The time fit range is
  $t_{\rm src} + 2 < t' < t_{\rm snk} - 2$.



Three-point functions

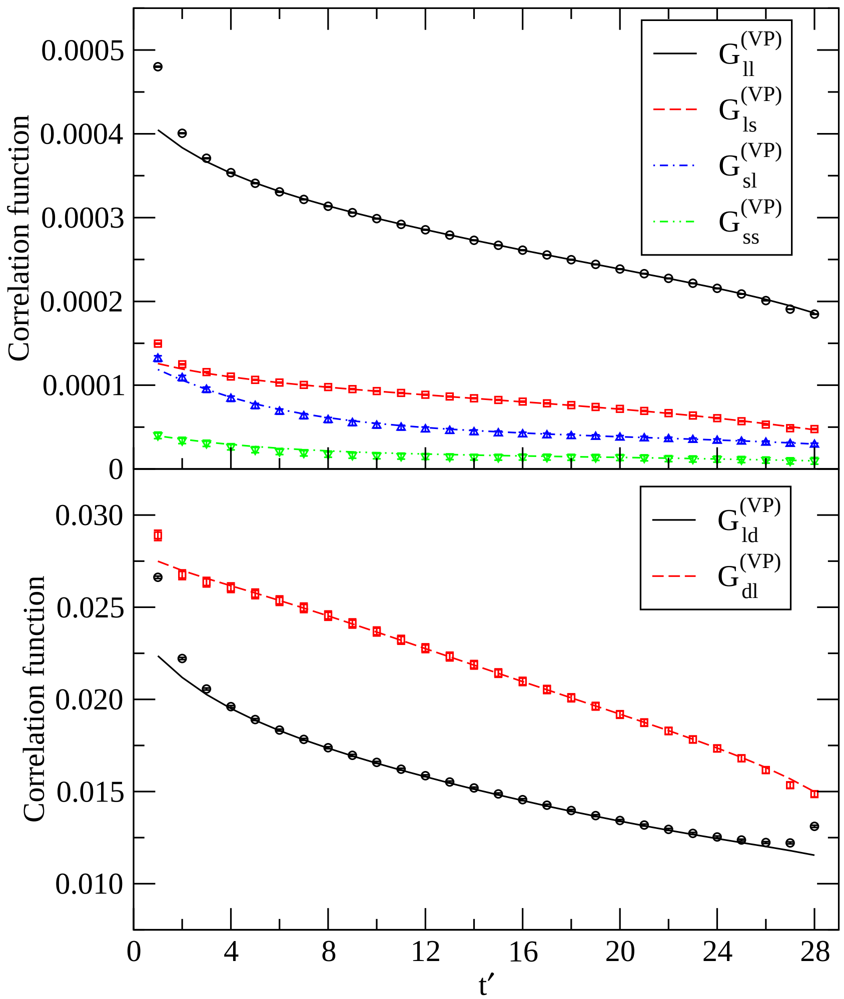
with $\Delta t = 19$

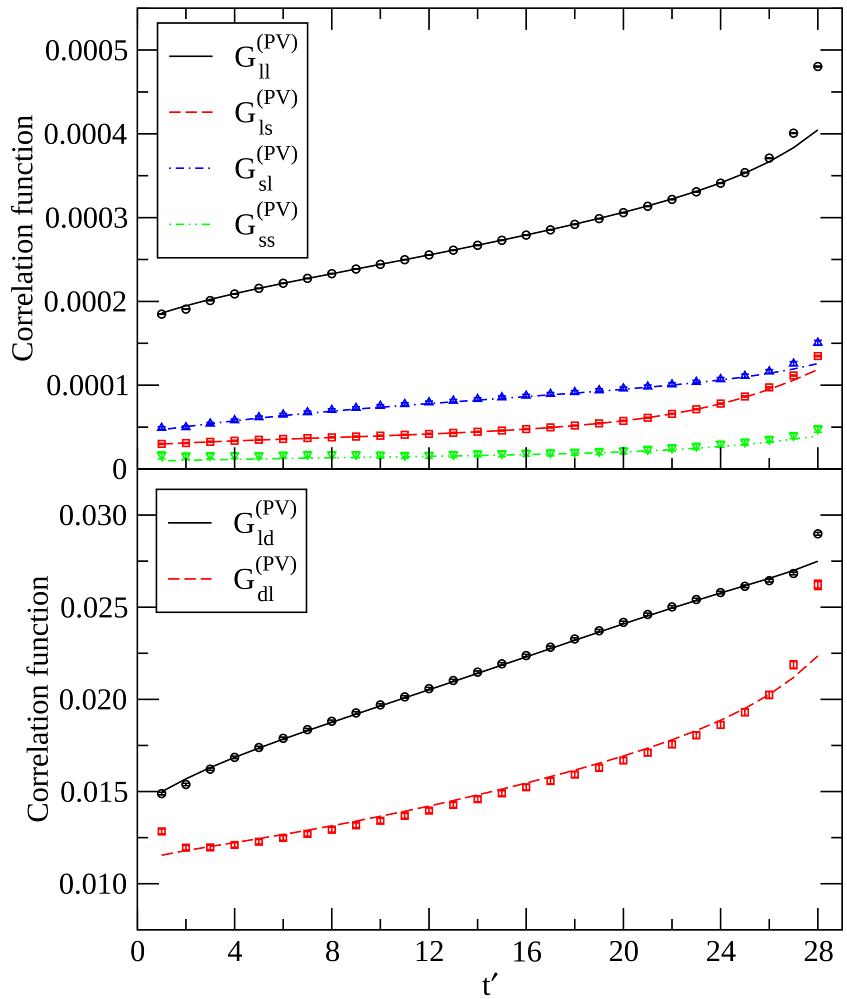


Three-point functions

with $\Delta t = 27$

$V \to P$ $P \to V$

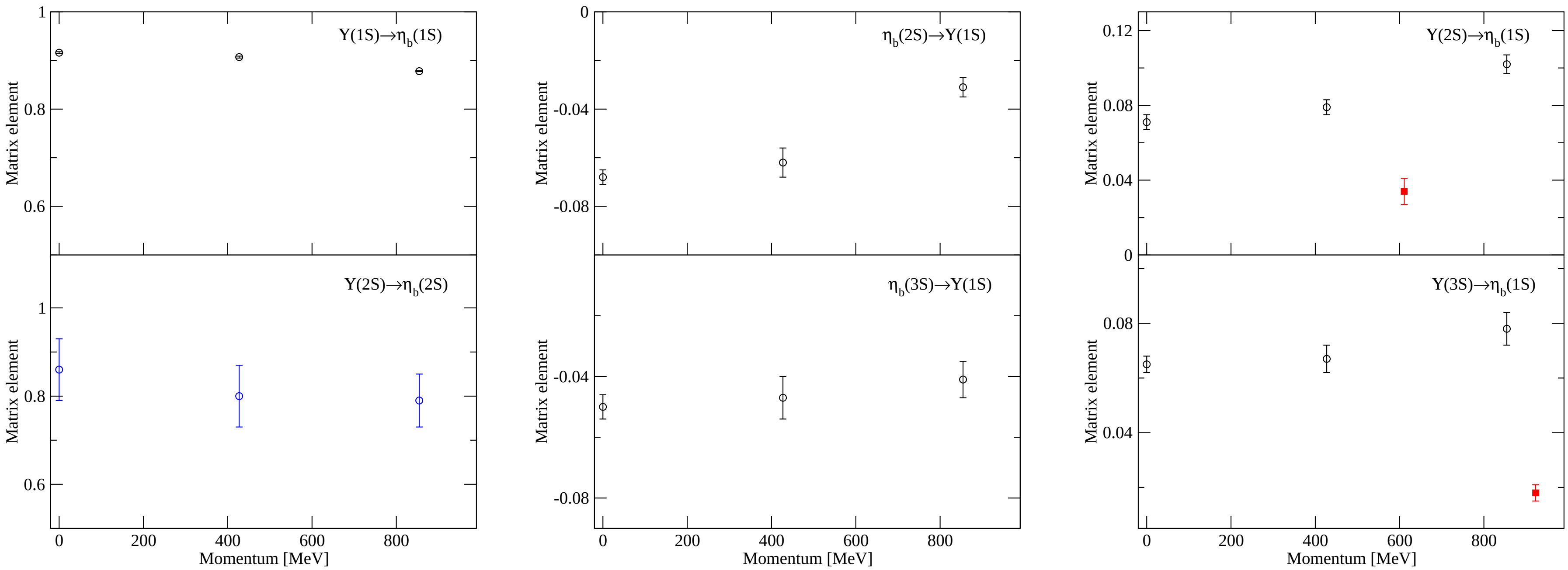




Stability of $A_{nn'}^{(VP)}$ fits

recoil momentum	Δt	N_{cf}	$A_{11}^{(VP)}$	$A_{12}^{(VP)}$	$A_{13}^{(VP)}$	$A_{21}^{(VP)}$	$A_{31}^{(VP)}$	$A_{22}^{(VP)}$
(0,0,0)	19	10	0.916(2)	-0.043(7)	-0.069(6)	0.090(7)	0.052(5)	
		10	0.915(2)	-0.068(2)	-0.050(4)	0.072(4)	0.065(3)	1.11(31)
		12	0.915(2)	-0.068(3)	-0.050(4)	0.071(4)	0.065(3)	1.11(23)
	27	10	0.916(2)	-0.062(7)	-0.056(7)	0.075(7)	0.059(6)	
		10	0.916(2)	-0.068(3)	-0.050(6)	0.071(3)	0.062(4)	2.1(2.2)
		12	0.916(2)	-0.068(3)	-0.051(6)	0.071(4)	0.062(4)	1.9(1.8)
(1,0,0)	19	10	0.908(1)	-0.042(8)	-0.060(8)	0.095(7)	0.057(5)	
		10	0.907(1)	-0.062(6)	-0.047(7)	0.079(4)	0.068(5)	0.92(27)
		12	0.907(1)	-0.062(6)	-0.047(7)	0.079(5)	0.067(5)	0.95(21)
	27	10	0.908(2)	0.057(8)	-0.052(9)	0.082(6)	0.063(6)	
		10	0.907(2)	0.061(5)	-0.048(8)	0.079(4)	0.066(6)	1.6(1.9)
		12	0.907(2)	0.061(5)	-0.048(8)	0.079(5)	0.066(6)	1.6(1.5)
(2,0,0)	19	10	0.878(1)	-0.010(6)	-0.055(6)	0.116(7)	0.066(6)	
		10	0.877(1)	-0.030(4)	-0.041(6)	0.101(5)	0.078(6)	1.01(25)
		12	0.877(1)	-0.031(4)	-0.041(6)	0.102(5)	0.078(6)	1.02(20)
	27	10	0.878(1)	-0.026(6)	-0.041(8)	0.104(6)	0.066(8)	
		10	0.878(2)	-0.031(4)	-0.037(8)	0.101(5)	0.070(6)	1.9(1.8)
		12	0.878(2)	-0.029(5)	-0.039(7)	0.100(5)	0.068(6)	1.0(1.6)

Qualitative success, quantitative problem



Possible improvements

- matching of the vector current: lattice to renormalized continuum.
- relativistic corrections to the transition operator.
- $O(v^6)$ terms.
- radiative corrections to coefficients in the NRQCD Hamiltonian.
- multiple lattice spacings and a continuum limit.

Other issues

- Light quarks are close to their physical values.
- The lattice volume is large compared to the physical system.

Masses of higher angular momentum states of bottomonium

- Which J^{PC} states appear as "ground states" on a lattice?
- Which of those states are accessible with present-day methods and existing configurations?

Creation operators for "ground states"

Λ^{PC}	J^{PC}	${}^{2S+1}L_J$	Ω
A_1^{-+}	0^{-+}	${}^1S_0$	1
T_1^{--}	1^{--}	${}^3S_1$	$\{\sigma_1, \sigma_2, \sigma_3\}$
T_1^{+-}	1^{+-}	${}^1P_1$	$\{\Delta_1, \Delta_2, \Delta_3\}$
A_1^{++}	0^{++}	${}^3P_0$	$\Delta_1\sigma_1 + \Delta_2\sigma_2 + \Delta_3\sigma_3$
T_1^{++}	1^{++}	${}^3P_1$	$\{\Delta_2\sigma_3 - \Delta_3\sigma_2, \Delta_3\sigma_1 - \Delta_1\sigma_3, \Delta_1\sigma_2 - \Delta_2\sigma_1\}$
E^{++}	2^{++}	${}^3P_2$	$\{(\Delta_1\sigma_1 - \Delta_2\sigma_2)/\sqrt{2}, (\Delta_1\sigma_1 + \Delta_2\sigma_2 - 2\Delta_3\sigma_3)/\sqrt{6}\}$
T_2^{++}	2^{++}	${}^3P_2$	$\{\Delta_2\sigma_3 + \Delta_3\sigma_2, \Delta_3\sigma_1 + \Delta_1\sigma_3, \Delta_1\sigma_2 + \Delta_2\sigma_1\}$
E^{-+}	2^{-+}	${}^1D_2$	$\{(D_{11} - D_{22})/\sqrt{2}, (D_{11} + D_{22} - 2D_{33})/\sqrt{6}\}$
T_2^{-+}	2^{-+}	${}^1D_2$	$\{D_{23}, D_{31}, D_{12}\}$
E^{--}	2^{--}	${}^3D_2$	$\{(D_{23}\sigma_1 - D_{13}\sigma_2)/\sqrt{2}, (D_{23}\sigma_1 + D_{31}\sigma_2 - 2D_{12}\sigma_3)/\sqrt{6}\}$
T_2^{--}	2^{--}	${}^3D_2$	$\{(D_{22} - D_{33})\sigma_1 + D_{13}\sigma_3 - D_{12}\sigma_2, (D_{33} - D_{11})\sigma_2 + D_{21}\sigma_1 - D_{23}\sigma_3,$
			$(D_{11} - D_{22})\sigma_3 + D_{32}\sigma_2 - D_{31}\sigma_1\}$
A_2^{--}	3^{--}	${}^3D_3$	$D_{12}\sigma_3 + D_{23}\sigma_1 + D_{31}\sigma_2$
A_2^{+-}	3^{+-}	${}^1F_3$	D_{123}
T_2^{+-}	3^{+-}	${}^1F_3$	$\{D_{122} - D_{133}, D_{233} - D_{211}, D_{311} - D_{322}\}$
A_2^{++}	3^{++}	${}^3F_3$	$(D_{221} - D_{331})\sigma_1 + (D_{332} - D_{112})\sigma_2 + (D_{113} - D_{223})\sigma_3$
T_1^{-+}	4^{-+}	${}^1G_4$	$\{D_{2223} - D_{3332}, D_{3331} - D_{1113}, D_{1112} - D_{2221}\}$
A_1^{--}	4^{--}	${}^3G_4$	$(D_{2223} - D_{3332})\sigma_1 + (D_{3331} - D_{1113})\sigma_2 + (D_{1112} - D_{2221})\sigma_3$
E^{+-}	5^{+-}	${}^1H_5$	$\{(D_{23111} - D_{13222})/\sqrt{2}, (D_{23111} + D_{13222} - 2D_{12333})/\sqrt{6}\}$
A_2^{-+}	6^{-+}	${}^1I_6$	$D_{112222} + D_{223333} + D_{331111} - D_{221111} - D_{332222} - D_{113333}$
A_1^{+-}	9^{+-}	${}^1L_9$	$D_{122233333} + D_{233311111} + D_{311122222} - D_{133322222} - D_{211133333} - D_{322211111}$

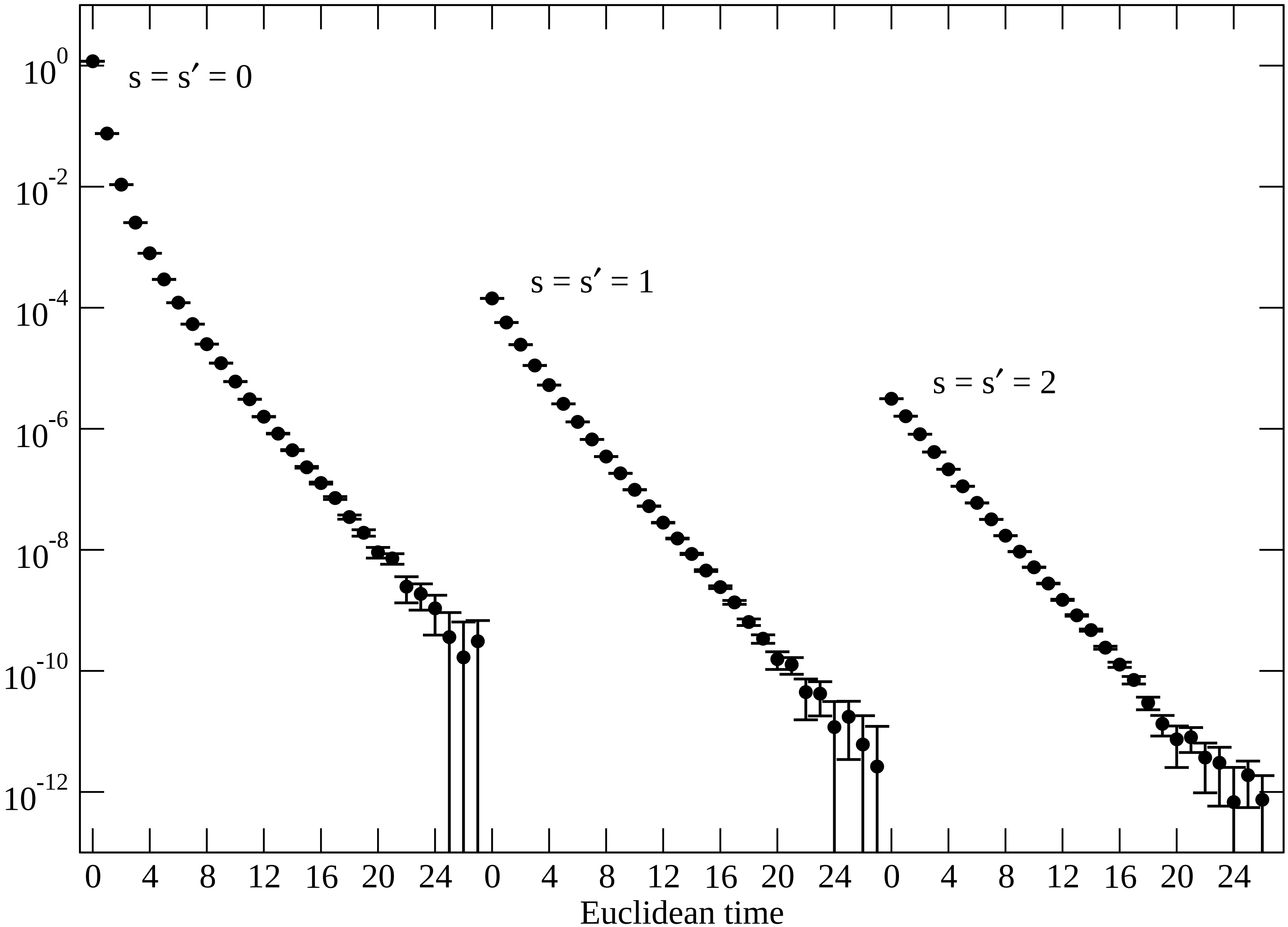
Simulation details

- same PACS-CS ensemble (198 configurations)
- 64 random-U(1) wall sources per configuration
- gauge-invariant smearing: $\psi(x) \to \left(1 + 0.15\Delta^2\right)^{8s} \psi(x)$ with $s = 0, 1, 2$
- stout links (Morningstar&Peardon,2004) for F-wave operators
- a generalized multi-exponential fit:

$$g(t - t_0) = \sum_{n=1}^{N'} \sum_{s=0}^{2} \sum_{s'=0}^{2} f_s(n) f_{s'}(n) e^{-E_n(t-t_0)} + \sum_{n=N'+1}^{N} \sum_{s=0}^{2} \sum_{s'=0}^{2} f_{s,s'}(n) e^{-E_n(t-t_0)}$$

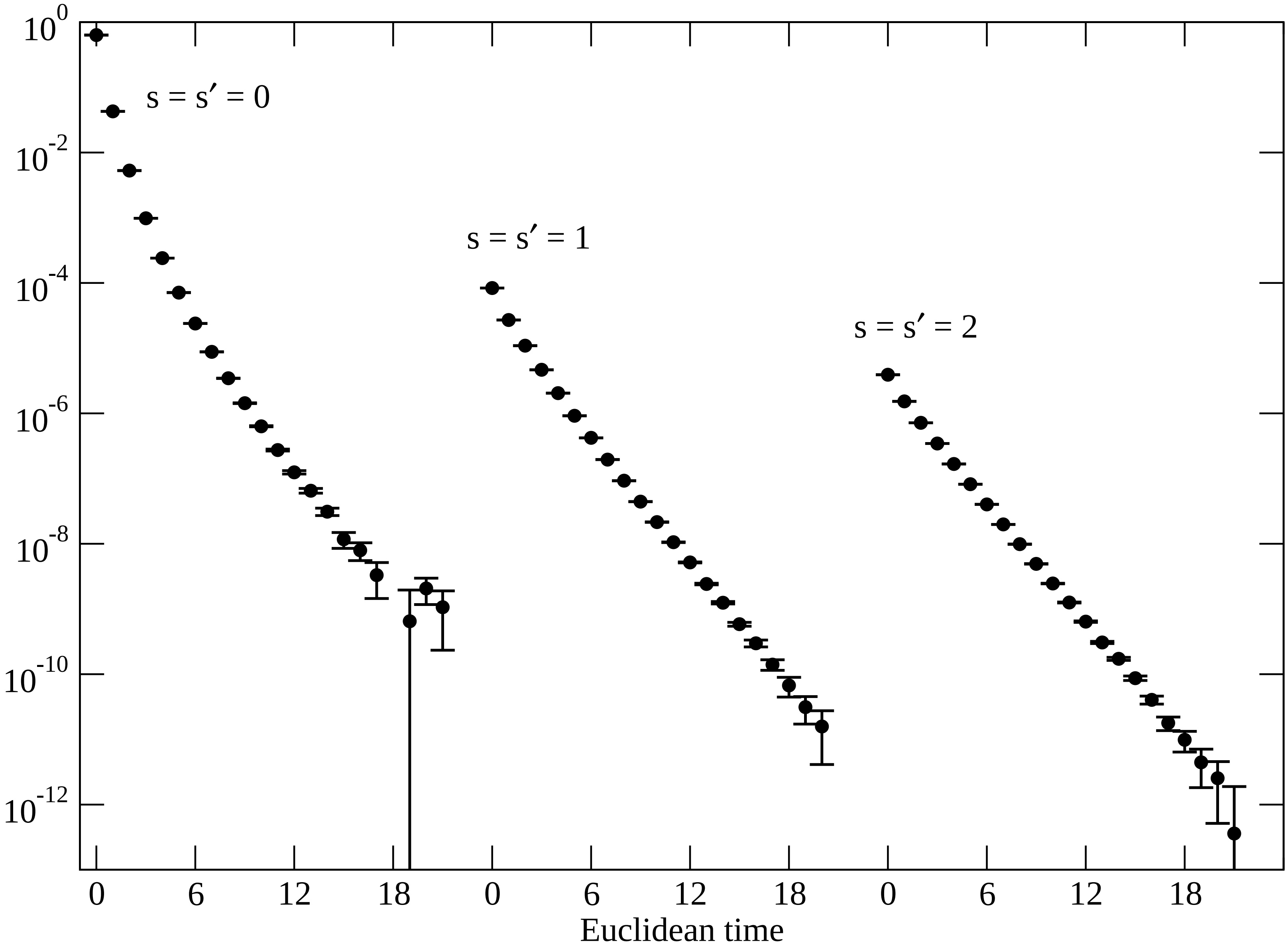
Sample E^{--} correlation functions.

(The lightest meson is ${}^3D_2$.)



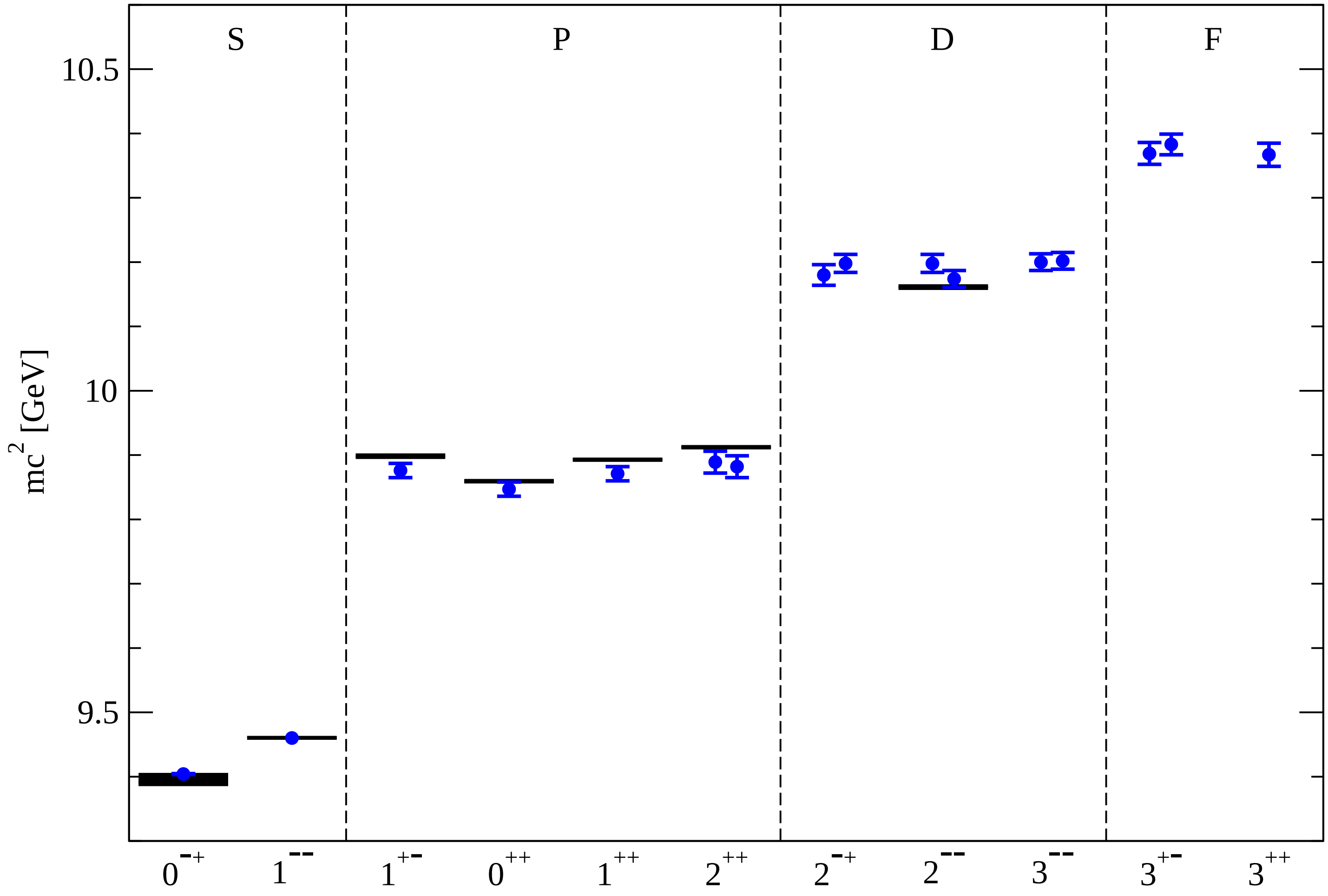
Sample T_2^{+-} correlation functions.

(The lightest meson is ${}^1F_3$.)



“ground state” bottomonium spectrum

Lattice (with statistical errors) and experiment.



preliminary G wave result: mass(T_1^{-+}) = 10.75±0.07 GeV/c^2

quark model expectation: mass(G wave) = 10.52 GeV/c^2

(Quarkonium Working Group, hep-ph/0412158, figure 4.10.)

Conclusions

masses:

• A set of quark-antiquark operators for all lattice irreps, Λ^{PC}, has been constructed. These correspond to the 16 bottomonium "ground states" for a lattice simulation, so they are a natural starting point for numerical studies.

• S, P, D and F waves are observed. A first look at a G wave suggests it is also within reach with present-day methods and existing gauge configurations.

M1 transitions:

• These decays are sensitive to a variety of small effects and are thus a valuable challenge for lattice simulations.

• The observed qualitative success is encouraging.

• The observed quantitative discrepancies (relative to experiment) provide the opportunity for future progress.