Perturbative and non-perturbative matching in HQET

ALPHA Collaboration

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Modern perspectives in lattice QCD: Quantum field theory and high performance computing. Proceedings, International School, 93rd Session, Les Houches, France, August 3-28, 2009.

Introduction

- Common practice in renormalization and matching of HQET
  - Wilson coefficients = matching coefficients from PT
  - Matrix elements non-perturbatively from
    – experiment
    – or lattice
  - quantities such as

$$\overline{\Lambda} = m_{\mathrm{B}} - m_{\mathrm{b}}\,, \quad \lambda_1 = \langle B|\overline{\psi}_{\mathrm{h}}\mathbf{D}^2\psi_{\mathrm{h}}|B\rangle$$

depend on a scheme; but one specifies the scheme and ...

- Strictly speaking none of this is correct.
- Does it matter?
- It is not necessary!

HQET Lagrangian

In the rest fram of a b-hadron ("velocity zero")

$$\overline{\psi}\{D_\mu\gamma_\mu + m\}\psi \quad \Rightarrow \quad \mathscr{L}_{\mathrm{h}}^{\mathrm{stat}} - \frac{1}{2m}(\mathcal{O}_{\mathrm{kin}} + \mathcal{O}_{\mathrm{spin}}) + \mathscr{L}_{\text{anti-quark}}$$

$$\mathscr{L}_{\mathrm{h}}^{\mathrm{stat}} = \overline{\psi}_{\mathrm{h}}\, D_0\, \psi_{\mathrm{h}}\,, \quad P_+\psi_{\mathrm{h}} = \psi_{\mathrm{h}}\,, \quad \overline{\psi}_{\mathrm{h}} P_+ = \overline{\psi}_{\mathrm{h}}\,, \quad P_\pm = \frac{1\pm\gamma_0}{2}$$

$$\mathcal{O}_{\mathrm{kin}}(x) = \overline{\psi}_{\mathrm{h}}(x)\,\mathbf{D}^2\,\psi_{\mathrm{h}}(x)\,,\ \mathcal{O}_{\mathrm{spin}}(x) = \overline{\psi}_{\mathrm{h}}(x)\,\boldsymbol{\sigma}\cdot\mathbf{B}(x)\,\psi_{\mathrm{h}}(x)\,,$$

Expanding correlation functions.

$$W_{\rm HQET} \equiv \exp(-a^4 \sum_x [\mathscr{L}_{\rm light}(x) + \mathscr{L}_{\rm h}^{\rm stat}(x)]) \times \left\{ 1 + a^4 \sum_x (\omega_{\rm kin}\, \mathcal{O}_{\rm kin}(x) + \omega_{\rm spin}\, \mathcal{O}_{\rm spin}(x)) \right\}$$

This yields

$$\begin{aligned} \langle \mathcal{O} \rangle &= \langle \mathcal{O} \rangle_{\rm stat} + \omega_{\rm kin} a^4 \sum_x \langle \mathcal{O}\mathcal{O}_{\rm kin}(x) \rangle_{\rm stat} + \omega_{\rm spin} a^4 \sum_x \langle \mathcal{O}\mathcal{O}_{\rm spin}(x) \rangle_{\rm stat} \\ &\equiv \langle \mathcal{O} \rangle_{\rm stat} + \omega_{\rm kin} \langle \mathcal{O} \rangle_{\rm kin} + \omega_{\rm spin} \langle \mathcal{O} \rangle_{\rm spin} \,, \end{aligned}$$

with

$$\langle \mathcal{O} \rangle_{\rm stat} = \frac{1}{\mathcal{Z}} \int_{\rm fields} \mathcal{O} \exp(-a^4 \sum_x [\mathscr{L}_{\rm light}(x) + \mathscr{L}_{\rm h}^{\rm stat}(x)])$$

Expanding correlation functions.

$$\langle \mathcal{O} \rangle \;=\; \langle \mathcal{O} \rangle_{\rm stat} + \omega_{\rm kin} \langle \mathcal{O} \rangle_{\rm kin} + \omega_{\rm spin} \langle \mathcal{O} \rangle_{\rm spin} \,,$$

also fields in correlation functions need to be expanded:

$$\mathcal{O}_{\rm QCD} \;=\; \sum_{\mathbf{x}} A_0(x) A_0^\dagger(0)$$

$$A_0(x) \;\rightarrow\; A_0^{\rm HQET}(x) = Z_{\rm A}^{\rm HQET}\,[A_0^{\rm stat}(x) + \sum_{i=1}^{4} c_{\rm A}^{(i)} A_0^{(i)}(x)]\,,$$

$$A_0^{(1)}(x) = \overline{\psi}_{\rm l}(x)\frac{1}{2}\gamma_5\gamma_i(\nabla_i^{\rm S} - \overleftarrow{\nabla}_i^{\rm S})\psi_{\rm h}(x)\,, \quad A_0^{(2)}(x) = -\widetilde{\partial}_i\, A_i^{\rm stat}\,,$$

$$c_{\rm A}^{(i)} = {\rm O}(1/m) \quad [A_0^{(i)}(x)] = 4$$

symmetric derivatives:

$$\widetilde{\partial}_i = \tfrac{1}{2}(\partial_i + \partial_i^*)\,, \quad \overleftarrow{\nabla}_i^{\rm S} = \tfrac{1}{2}(\overleftarrow{\nabla}_i + \overleftarrow{\nabla}_i^*)\,, \quad \nabla_i^{\rm S} = \tfrac{1}{2}(\nabla_i + \nabla_i^*)\,.$$

$A_0^{(3)}$, $A_0^{(4)}$ are here not needed

Expanding correlation functions.

Example

$$C^{\rm QCD}_{\rm AA,R}(x_0) = Z_{\rm A}^2\, a^3 \sum_{\mathbf{x}} \left\langle A_0(x) A_0^\dagger(0) \right\rangle_{\rm QCD}$$

its HQET expansion

$$\begin{aligned} C^{\rm QCD}_{\rm AA}(x_0) &= {\rm e}^{-m\,x_0} (Z^{\rm HQET}_{\rm A})^2 \Big[C^{\rm stat}_{\rm AA}(x_0) + c^{(1)}_{\rm A}\, C^{\rm stat}_{\delta{\rm AA}}(x_0) \\ &\qquad + \omega_{\rm kin}\, C^{\rm kin}_{\rm AA}(x_0) + \omega_{\rm spin}\, C^{\rm spin}_{\rm AA}(x_0) \Big] \\ &\equiv {\rm e}^{-m x_0} (Z^{\rm HQET}_{\rm A})^2\, C^{\rm stat}_{\rm AA}(x_0) \Big[1 + c^{(1)}_{\rm A}\, R^{\rm stat}_{\delta A}(x_0) \\ &\qquad + \omega_{\rm kin}\, R^{\rm kin}_{\rm AA}(x_0) + \omega_{\rm spin}\, R^{\rm spin}_{\rm AA}(x_0) \Big] \end{aligned}$$

with

$$\begin{aligned} C^{\rm stat}_{\delta{\rm AA}}(x_0) &= a^3 \sum_{\mathbf{x}} \langle A_0^{\rm stat}(x) (A_0^{(1)}(0))^\dagger \rangle_{\rm stat} + a^3 \sum_{\mathbf{x}} \langle A_0^{(1)}(x) (A_0^{\rm stat}(0))^\dagger \rangle_{\rm stat} \,, \\ C^{\rm kin}_{\rm AA}(x_0) &= a^3 \sum_{\mathbf{x}} \langle A_0^{\rm stat}(x) (A_0^{\rm stat}(0))^\dagger \rangle_{\rm kin} \\ C^{\rm spin}_{\rm AA}(x_0) &= a^3 \sum_{\mathbf{x}} \langle A_0^{\rm stat}(x) (A_0^{\rm stat}(0))^\dagger \rangle_{\rm spin} \,. \end{aligned}$$

Expanding correlation functions.

$$C_{\mathrm{AA}}^{\mathrm{QCD}}(x_0) = \mathrm{e}^{-m\,x_0}(Z_{\mathrm{A}}^{\mathrm{HQET}})^2 \Big[C_{\mathrm{AA}}^{\mathrm{stat}}(x_0) + c_{\mathrm{A}}^{(1)}\, C_{\delta\mathrm{AA}}^{\mathrm{stat}}(x_0) + \omega_{\mathrm{kin}}\, C_{\mathrm{AA}}^{\mathrm{kin}}(x_0) + \omega_{\mathrm{spin}}\, C_{\mathrm{AA}}^{\mathrm{spin}}(x_0) \Big]$$

- parameters in the effective theory

$$(\omega_1, \ldots \omega_5) = (m_0 = m + \delta m\,,\ \ln(Z_{\mathrm{A}}^{\mathrm{HQET}})\,,\ c_{\mathrm{A}}^{(1)}\,,\ \omega_{\mathrm{kin}}\,,\ \omega_{\mathrm{spin}})$$

$\omega_i = \omega_i(g_0, aM_{\mathrm{b}})$ bare parameters

- renormalization
  keep M_{b} fixed change $g_0 \to 0,\ a \to 0$: all divergences (logarithmic and power) absorbed in ω_i
- matching
  finite parts of the ω_i by matching to QCD

$$\Phi_i^{\mathrm{HQET}}(\{\omega_i\}) = \Phi_i^{\mathrm{QCD}}(M_{\mathrm{b}})$$

Expansion of energies...

$$m_{\rm B} = -\lim_{x_0\to\infty} \widetilde{\partial}_0 \ln C_{\rm AA}^{\rm QCD}(x_0) = \ldots$$
$$= m_{\rm bare} + E^{\rm stat} + \omega_{\rm kin} E^{\rm kin} + \omega_{\rm spin} E^{\rm spin} \,,$$

$$E^{\rm stat} = -\lim_{x_0\to\infty} \widetilde{\partial}_0 \ln C_{\rm AA}^{\rm stat}(x_0)$$
$$E^{\rm kin} = = -\frac{1}{2}\langle B|\mathcal{O}_{\rm kin}(0)|B\rangle_{\rm stat}$$
$$E^{\rm spin} = -\frac{1}{2}\langle B|\mathcal{O}_{\rm spin}(0)|B\rangle_{\rm stat} \,,$$

... and matrix elements

$$
\begin{aligned}
F_{\rm B}\sqrt{m_{\rm B}} &= \lim_{x_0\to\infty}\left\{2\exp(m_{\rm B}x_0)\, C_{\rm AA}^{\rm QCD}(x_0)\right\}^{1/2} \\
&= Z_{\rm A}^{\rm HQET}\,\Phi^{\rm stat}\,\lim_{x_0\to\infty}\Big\{1+\tfrac{1}{2}x_0\left[\omega_{\rm kin}E^{\rm kin}+\omega_{\rm spin}E^{\rm spin}\right] \\
&\quad +\tfrac{1}{2}c_{\rm A}^{(1)}R^{\rm stat}_{\delta A}(x_0)+\tfrac{1}{2}\omega_{\rm kin}R^{\rm kin}_{\rm AA}(x_0)+\tfrac{1}{2}\omega_{\rm spin}R^{\rm spin}_{\rm AA}(x_0)\Big\}\,, \\
\Phi^{\rm stat} &= \lim_{x_0\to\infty}\left\{2\exp(E^{\rm stat}x_0)\, C_{\rm AA}^{\rm stat}(x_0)\right\}^{1/2} \\
&= \langle B|A_0^{\rm stat}(0)|0\rangle
\end{aligned}
$$

Issues

- power divergences $\frac{g_0^{2L}}{a^n} \quad n = 1, 2$ need NP subtraction
- power corrections $(\alpha(m))^L \overset{m\to\infty}{\gg} \frac{\Lambda_{\rm QCD}}{m}$ need NP leading terms to define power corrections

- With a non-perturbative matching

$$\overline{\Lambda} = m_{\rm B} - m_{\rm b} \,, \quad \lambda_1 = \langle B|\overline{\psi}_{\rm h} \mathbf{D}^2 \psi_{\rm h}|B\rangle$$

have ambiguities of order

$$\Lambda_{\rm QCD} \,, \qquad \frac{\Lambda_{\rm QCD}^2}{m}$$

ambiguities $\equiv$ dependence on matching conditions

Beyond the classical theory: Renormalization and Matching at leading order in $1/m$

a matrix element of A_0:

QCD	HQET in static approx.
$Z_{\rm A}\,\langle f\vert A_0(x)\vert i\rangle_{\rm QCD}$	$Z_{\rm A}^{\rm stat}(\mu)\langle f\vert A_0^{\rm stat}(x)\vert i\rangle_{\rm stat}$
$\Phi^{\rm QCD}(m)$	$\Phi(\mu)$

- m: mass of heavy quark (b) in some definition (all other masses zero for simplicity)
- μ: arbitrary renormalization scale
- matching (equivalence):

$$\begin{aligned} \Phi^{\rm QCD}(m) &= \widetilde{C}_{\rm match}(m,\mu)\times\Phi(\mu) + \mathrm{O}(1/m) \\ \widetilde{C}_{\rm match}(m,\mu) &= 1 + c_1(m/\mu)\bar{g}^2(\mu) + \ldots \end{aligned}$$

Physical observables, such as $F_{\rm B_s}$, are independent of renormalization scheme, scale.
$\Rightarrow$ switch to Renormalization Group Invariants

Better: change to RGI's

see e.g. [R.S., arXiv:1008.0710]

$$\Phi_{\rm RGI} = \exp\left\{-\int^{\bar g(\mu)} {\rm d}x\, \frac{\gamma(x)}{\beta(x)}\right\} \Phi(\mu) = \underbrace{Z_{\rm RGI}(g_0)}_{\text{known, ALPHA Collaboration}} \times \underbrace{\Phi(g_0)}_{\text{bare ME}}$$

β : beta-fct

$$\equiv \left[\,2b_0\bar g(\mu)^2\,\right]^{-\gamma_0/2b_0} \exp\left\{-\int_0^{\bar g(\mu)} {\rm d}x \left[\frac{\gamma(x)}{\beta(x)} - \frac{\gamma_0}{b_0 x}\right]\right\} \Phi(\mu)$$

$$\Phi^{\rm QCD} = C_{\rm PS}(M/\Lambda) \times \Phi_{\rm RGI}$$

γ : AD in HQET

$$C_{\rm PS}(M/\Lambda) = \exp\left\{\int^{g_\star(M/\Lambda)} {\rm d}x\, \frac{\gamma_{\rm match}(x)}{\beta(x)}\right\}$$

with

Λ : Lambda-para

$$\frac{\Lambda}{M} = \exp\left\{-\int^{g_\star(M/\Lambda)} {\rm d}x\, \frac{1-\tau(x)}{\beta(x)}\right\}, \qquad \rightarrow \qquad g_\star(M/\Lambda)$$

M : RGI quark mass

$\gamma_{\rm match}$: describes the mass dependence

$g_\star$: $\mu = m_\star = \overline{m}(m_\star)\,,\; g_\star = \bar g(m_\star)$

Matching and RGI's

$$\frac{M}{\Phi}\frac{\partial\Phi}{\partial M}\bigg|_{\Lambda} = \frac{M}{C_{\mathrm{PS}}}\frac{\partial C_{\mathrm{PS}}}{\partial M}\bigg|_{\Lambda} = \frac{\gamma_{\mathrm{match}}(g_\star)}{1-\tau(g_\star)}\,, \quad g_\star = g_\star(M/\Lambda)\,.$$

and with

$$\gamma_{\mathrm{match}}(g_\star) \overset{g_\star\to 0}{\sim} -\gamma_0 g_\star^2 - \gamma_1^{\mathrm{match}} g_\star^4 + \ldots\,, \qquad \beta(\bar{g}) \overset{\bar{g}\to 0}{\sim} -b_0\bar{g}^3 + \ldots$$

we can give the leading large mass behaviour

$$C_{\mathrm{PS}} \quad \overset{M\to\infty}{\sim} \quad (2b_0 g_\star^2)^{-\gamma_0/2b_0} \sim [\log(M/\Lambda)]^{\gamma_0/2b_0}$$

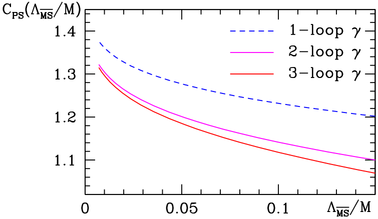
The present knowledge

For $\gamma_{\rm match}$ at l loops need
$\gamma_{\overline{\rm MS}} = \gamma$: l loops;
$C_{\rm match}(g_\star)$: $l-1$ loops

γ_0 [Shifmann& Voloshin; Politzer& Wise]

...

$\gamma_{\overline{\rm MS},2}$ [Chetyrkin & Grozin, 2003]

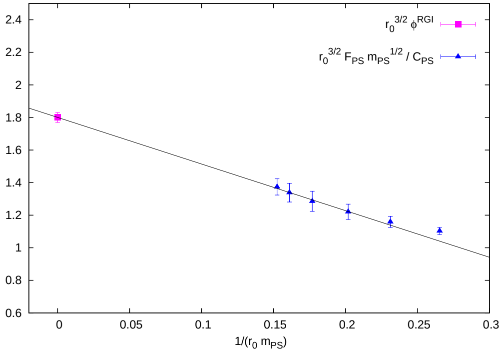


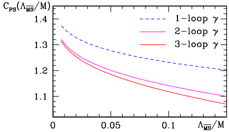
$C_{\rm match}(g_\star)$ to 3 loops [S. Bekavac, A.G. Grozin, P. Marquard, J.H. Piclum, D. Seidel, M. Steinhauser, 2009]

An application

[ALPHA Collaboration]

quenched





this looks good; one may interpolate to the physical point ... but

What is the accuracy of perturbation theory?

$C_{\mathrm{match}}(g_\star)$ to 3 loops [Bekavac et al, 2009] also for various bilinears

γ_{match}

$$\mathcal{O}_\Gamma = \overline{\psi}_{\mathrm{l}}(x)\Gamma\psi_{\mathrm{h}}(x)$$

Γ	notation
$\gamma_0\gamma_5$	A_0
γ_5	P
γ_k	V_k
γ_{kl}	T

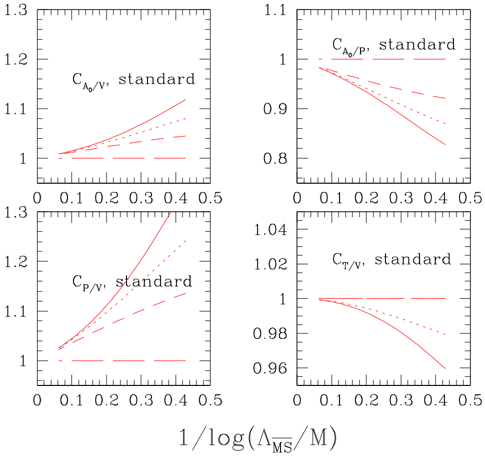
$$\Phi_\Gamma^{\mathrm{QCD}} = C_{\mathrm{match}}^\Gamma(g_\star) \times \Phi(\mu) = C_{\mathrm{match}}(g_\star) \exp\left\{\int^{g_\star} \mathrm{d}x \frac{\gamma(x)}{\beta(x)}\right\} \Phi_{\mathrm{RGI}}^\Gamma$$

$$\equiv \exp\left\{\int^{g_\star} \mathrm{d}x \frac{\gamma_{\mathrm{match}}^\Gamma(x)}{\beta(x)}\right\} \Phi_{\mathrm{RGI}}$$

$\gamma_{\mathrm{match}}^\Gamma$: 3-loops $\gamma_{\mathrm{match}}^\Gamma - \gamma_{\mathrm{match}}^{\Gamma'}$: 4-loops

[chiral symmetry of light quarks ($N_{\mathrm{light}} > 1$): $\gamma_{\mathrm{match}}^{\Gamma\gamma_5} = \gamma_{\mathrm{match}}^\Gamma$]

Compare different orders



We actually show
$C_{\Gamma/\Gamma'} = C^{\Gamma}_{\rm match}(m,\mu)/C^{\Gamma'}_{\rm match}(m,\mu)$

B-physics: $\Lambda_{\overline{\rm MS}}/M_{\rm b} \approx 0.04$
$-1/\log(\Lambda_{\overline{\rm MS}}/M_{\rm b}) \approx 0.3$

Perturbation theory is badly behaved
for charm quarks very badly
$-1/\log(\Lambda_{\overline{\rm MS}}/M_{\rm c}) \approx 0.5$

Different orders of PT

the normal behavior for one-scale quantities is

$$\mathcal{O} = o_0 + o_1\,\alpha + o_2\,\alpha^2 + \ldots \qquad \alpha = \bar{g}^2/(4\pi)$$
$$|o_i| \lesssim 1$$

(o_0 suitably normalized)

examples: $\mu\frac{\partial\bar{g}}{\partial\mu} = -\bar{g}\left\{b_0\alpha + b_1\alpha^2 + \ldots\right\}$

$$\frac{\mu}{\bar{m}}\frac{\partial\bar{m}}{\partial\mu} = -d_0\,\alpha - d_1\alpha^2 + \ldots$$
$$\frac{\mu}{\Phi}\frac{\partial\Phi}{\partial\mu} = \gamma = -\gamma_0\alpha - \gamma_1\alpha^2 + \ldots$$

($N_\mathrm{f} = 3$)

$\overline{\mathrm{MS}}$	b_i	0.71620	0.40529	0.32445	0.47367
	d_i	0.63662	0.76835	0.80114	0.90881
	γ_i	-0.31831	-0.26613	-0.25917	

Perturbation theory is well behaved

Different orders of PT

($N_\mathrm{f} = 3$, well behaved $\overline{\mathrm{MS}}$ RG functions)

$\overline{\mathrm{MS}}$	b_i	0.71620	0.40529	0.32445	0.47367
	d_i	0.63662	0.76835	0.80114	0.90881
	γ_i	-0.31831	-0.26613	-0.25917	

but mass-dependence (matching anomalous dimensions):

$\gamma_\mathrm{match}(g_\star) = \frac{m_\star}{\Phi^\mathrm{QCD}} \frac{\partial \Phi^\mathrm{QCD}}{\partial m_\star} = -\gamma_0 \alpha - \gamma_1 \alpha^2 + \dots$

($N_\mathrm{f} = 3$)

$A_0,\ \gamma_i$	-0.31831	-0.57010	-0.94645	
$V_0,\ \gamma_i$	-0.31831	-0.87406	-3.12585	
$\dots$				
$A_0/V,\ \gamma_i$	0	0.30396	2.17939	14.803

Perturbation theory is ill behaved (applicable at very small α)

Changing the scale

we had

$$\Phi^{\mathrm{QCD}}(m) \quad = \quad \widetilde{C}_{\mathrm{match}}(m,\mu) \times \Phi(\mu)$$

- Chose a "convenient" scale: $\mu = m_\star = \overline{m}(m_\star)\,,\ g_\star = \bar{g}(m_\star)$
- may set more generally

$$\mu = s^{-1}\, m_\star = \overline{m}(m_\star)\,,\quad g_\star = \bar{g}(m_\star)$$

- note: in the effective theory
  – one does NOT INTEGRATE OUT DOFs ABOVE $\mu = m_\star$
  – one matches the physics BELOW

  $\rightarrow$ expect $s > 1$ is better
- the result is simply ($\hat{g} = \bar{g}(s^{-1} m_\star)$))

$$\begin{aligned}
\gamma_{\mathrm{match}}(g_\star) \quad &= \quad \hat{\gamma}_{\mathrm{match}}(\hat{g}) = -\hat{\gamma}_0\alpha - \hat{\gamma}_1\alpha^2 + \ldots \\
&\quad\ \ \hat{\gamma}_0 = \gamma_0\,,\ \hat{\gamma}_1 = \gamma_1 + 2b_0\gamma_0 \ldots \\
C_{\mathrm{PS}}(M/\Lambda) \quad &= \quad \exp\left\{ \int^{\hat{g}} \mathrm{d}x \frac{\hat{\gamma}_{\mathrm{match}}(x)}{\beta(x)} \right\}.
\end{aligned}$$

Changing the scale

$$\gamma_{\rm match}(g_\star) = \hat\gamma_{\rm match}(\hat g) = -\hat\gamma_0\alpha - \hat\gamma_1\alpha^2 + \ldots$$
$$\hat\gamma_0 = \gamma_0\,,\ \hat\gamma_1 = \gamma_1 + 2b_0\gamma_0 \ldots$$
$$C_{\rm PS}(M/\Lambda) = \exp\left\{\int^{\hat g} {\rm d}x\,\frac{\hat\gamma(x)}{\beta(x)}\right\}$$

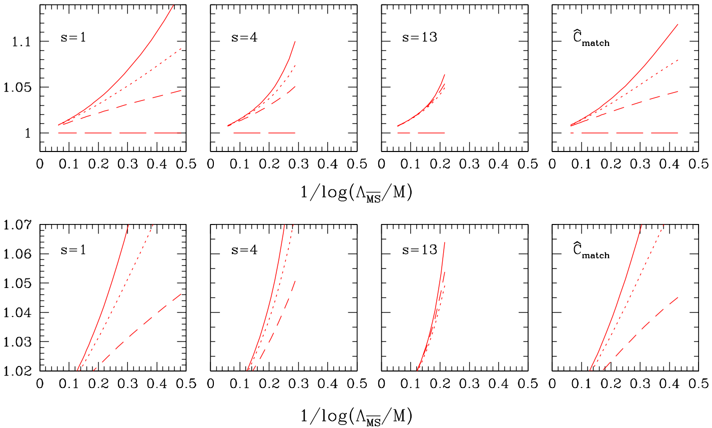
					s
$A_0,\ \hat\gamma_i$	-0.31831	-0.57010	-0.94645		1
	-0.31831	0	0.39720		3.4916
$V_0,\ \hat\gamma_i$	-0.31831	-0.87406	-3.12585		1
	-0.31831	0	-0.231121		6.8007
...					
$A_0/V,\ \hat\gamma_i$	0	0.30396	2.17939	14.803	1
	0	0.30396	0.972221	4.733	4
	0	0.30396	-0.05414	1.82678	13
	0	0.30396	-0.23495	1.85344	16

The behavior can be improved significantly but $s \gtrsim 4$ is required

$\alpha(m_{\rm b}/4)$ is not small!

[Very similar for $C_{\rm match}(m_{\rm Q},\mu)$ with $\mu = s^{-1}m_{\rm Q}$ expanded in $\alpha(\mu)$]

Conversion functions with and without scale optimization



The ratio $C_{\rm PS}/C_{\rm V}$, evaluated in the first column as described here. In columns two and three the expansion in $g_\star$ is generalized to an expansion in $\bar{g}(m_\star/s)$, see App. **??**. The last column contains the conventionally used $\hat{C}^{\rm PS}_{\rm match}(m_{\rm Q},m_{\rm Q},m_{\rm Q})/\hat{C}^{\rm V}_{\rm match}(m_{\rm Q},m_{\rm Q},m_{\rm Q})$, see App. **??**. For B-physics we have $\Lambda_{\overline{\rm MS}}/M_{\rm b} \approx 0.04$ and $-1/\ln(\Lambda_{\overline{\rm MS}}/M_{\rm b}) \approx 0.3$. The loop order changes from one-loop (long-dashes) up to 4-loop (full line) anomalous dimension.

A conclusion on perturbative matching

- is not easily drawn
- the effective scale seems well below $\mu = m$
- seems reliable only for masses beyond m_{b}, where it is of limited use for us
- a similar statement is found in [Bekavac et al, 2009]
- other ideas?
- in any case perturbative matching is only theoretically consistent at leading order in $1/m_{\mathrm{b}}$

$$\alpha^k(m) \sim \left[\frac{1}{2b_0 \log(m/\Lambda_{\mathrm{QCD}})}\right]^k \overset{m \gg \Lambda_{\mathrm{QCD}}}{\gg} 1/m$$

The problem

- originates from PT in QCD (in the limit of a large mass)
- has nothing to do with divergences of HQET
- **is absent when matching non-perturbatively**

Non-perturbative determination of parameters [ALPHA Collaboration ,2001 - 2010]

static parameters

$$\omega^{\rm stat} = (m_{\rm bare}^{\rm stat} , [\ln(Z_{\rm A})]^{\rm stat})^t , \quad N_{\rm HQET} = 2$$

parameters at first order

$$\omega^{\rm HQET} = (m_{\rm bare} , \ln(Z_{\rm A}^{\rm HQET}) , c_{\rm A}^{(1)} , \omega_{\rm kin} , \omega_{\rm spin})^t \quad N_{\rm HQET} = 5$$
$$\omega^{(1/m)} = \omega^{\rm HQET} - \omega^{\rm stat}$$

matching: $L_1 \approx 0.5\,\mathrm{fm}$

$$\Phi_i(L_1, M, a) = \Phi_i^{\rm QCD}(L_1, M, 0) , \; i = 1 \ldots N_{\rm HQET} .$$

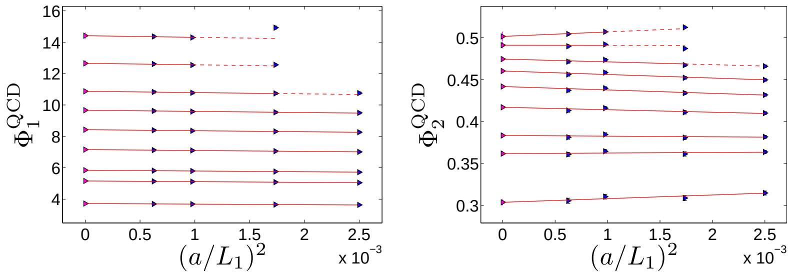
Why $L_1 = 0.5\mathrm{fm}$?

- $a = 0.012 \ldots 0.025\mathrm{fm}$ is accessible
  b-quark can be simulated, continuum limit can be taken
- $0.5\mathrm{fm} \approx 1/\Lambda_{\rm QCD}$ is a low energy scale

Non-perturbative determination of parameters [ALPHA Collaboration ,2001 - 2010]

$L_1 = 0.5$fm: $a = 0.012 \dots 0.025$fm:

b-quark can be simulated, continuum limit can be taken



[Blossier, Della Morte, Fritzsch, Garron, Heitger, Simma, S., Tantalo]

$\Phi_1 = L\, m_{\rm B}(L_1) = {\rm O}(z)$

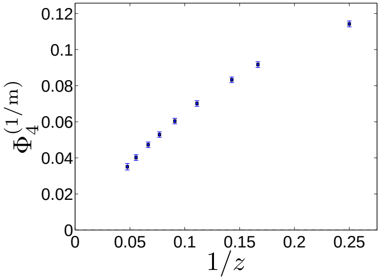
$\Phi_2 = \ln(L_1^{3/2}\, [F_{\rm B}\sqrt{m_{\rm B}}](L_1) = {\rm O}(z^0)$

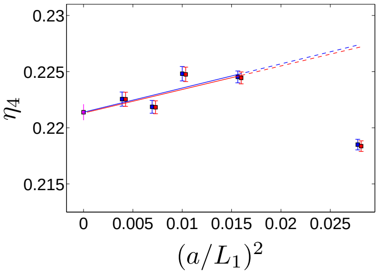
different z

Non-perturbative determination of parameters [ALPHA Collaboration ,2001 - 2010]

$L_1 = 0.5\text{fm}$: $a = 0.012 \ldots 0.025\text{fm}$:

b-quark can be simulated, continuum limit can be taken





$$\Phi_4 - \eta_4 = \omega_{\rm kin} \Phi_4^{\rm kin} = \mathrm{O}(1/z)$$

Non-perturbative determination of parameters [ALPHA Collaboration ,2001 - 2010]

$$\Phi_i(L_1, M, a) = \Phi_i^{\rm QCD}(L_1, M, 0)\,,\; i = 1 \ldots N_{\rm HQET}\,.$$

natural:

$$\begin{aligned} \Phi_1 &= L\Gamma^{\rm P} \equiv -L\widetilde{\partial}_0 \ln(-f_{\rm A}(x_0))_{x_0=L/2} \overset{L\to\infty}{\sim} Lm_{\rm B} \\ \Phi_2 &= \ln(Z_{\rm A}\frac{-f_{\rm A}}{\sqrt{f_1}}) \overset{L\to\infty}{\sim} \ln(L^{3/2}F_{\rm B}\sqrt{m_{\rm B}/2})\,, \end{aligned}$$

HQET expansion

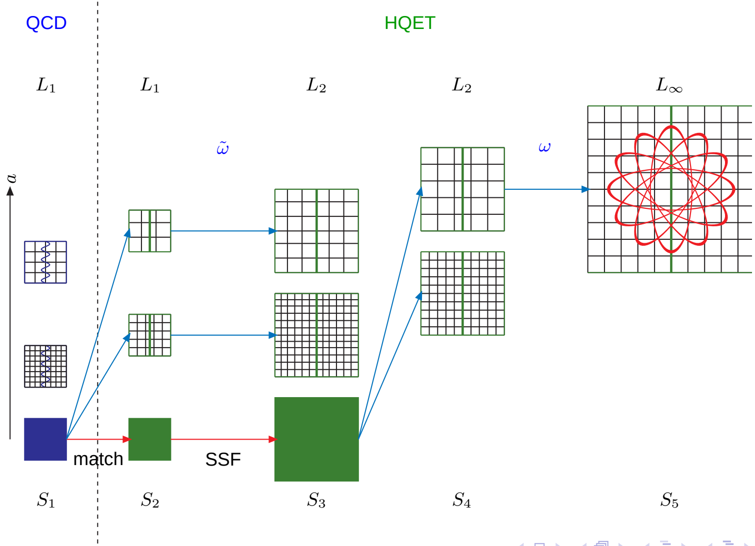
$$\begin{aligned} \Phi_1 &= L\,[m_{\rm bare} + \Gamma^{\rm stat}] + {\rm O}(1/m_{\rm b}) \\ \Phi_2 &= \ln(Z_{\rm A}^{\rm stat}) + \zeta_{\rm A} + {\rm O}(1/m_{\rm b}) \end{aligned}$$

in general

$$\begin{aligned} \Phi(L, M, a) &= \eta(L, a) + \phi(L, a)\,\omega(M, a) \\ \eta &= \begin{pmatrix} \Gamma^{\rm stat} \\ \zeta_{\rm A} \\ \ldots \end{pmatrix}, \quad \phi = \begin{pmatrix} L & 0 & \ldots \\ 0 & 1 & \ldots \\ \ldots & & \end{pmatrix} \end{aligned}$$

Full strategy to determine $\omega(M_{\rm b}, a)$, $a = 0.05\mathrm{fm} \ldots 0.1\mathrm{fm}$

(in



Independence of matching condition

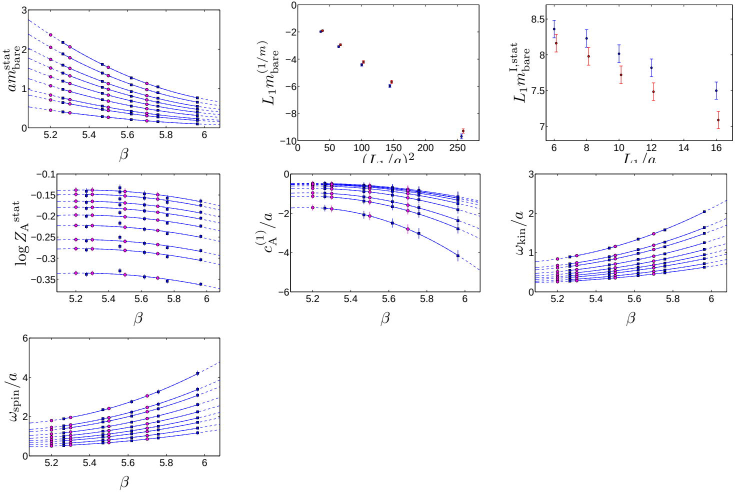
$$\Delta \log(Z_A^{\rm stat}) = \log(Z_A^{\rm stat}) - \log(Z_A^{\rm stat})|_{(\theta_0,\theta_1,\theta_2)=(0.5,0.5,1)}$$

$$\Delta \log(Z_A^{\rm HQET}) = \log(Z_A^{\rm HQET}) - \log(Z_A^{\rm HQET})|_{(\theta_0,\theta_1,\theta_2)=(0.5,0.5,1)}$$

	$\Delta \log(Z_A^{\rm stat})$	$\Delta \log(Z_A^{\rm HQET})$		
(θ_1,θ_2):		$(0, 0.5)$	$(0.5, 1)$	$(0, 1)$
$\theta_0 = 0.0$	0.014(2)	−0.046(47)	−0.001(6)	−0.012(14)
$\theta_0 = 0.5$	0	−0.048(49)	0	−0.012(12)
$\theta_0 = 1.0$	−0.046(2)	−0.055(60)	0.007(7)	−0.008(18)

HYP2 discretization at $\beta = 5.3$ and $z = 13$.

Parameters are now known for $N_{\mathrm{f}} = 2$



[Blossier, Della Morte, Fritzsch, Garron, Heitger, Simma, S., Tantalo]

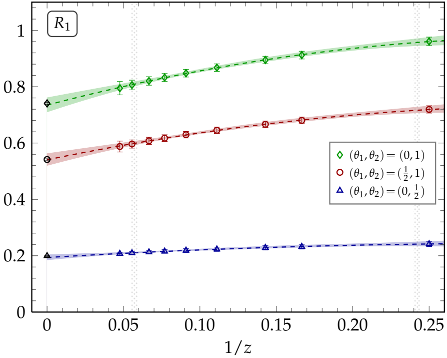
The end

Thank you for your attention.

Tests of HQET [Fritzsch, Jüttner, Heitger, S., Wennekers]

Example: SF boundary-to-boundary correlators

$$R_1 \quad = \quad \tfrac{1}{4}\left(\ln\left(\frac{f_1(\theta_1)k_1(\theta_1)^3}{f_1(\theta_2)k_1(\theta_2)^3}\right)\right) \qquad z = LM$$

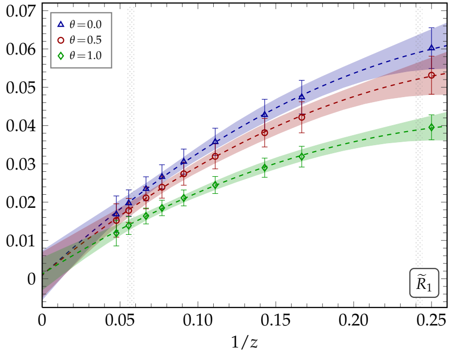


spin averaged

Tests of HQET [Fritzsch, Jüttner, Heitger, S., Wennekers]

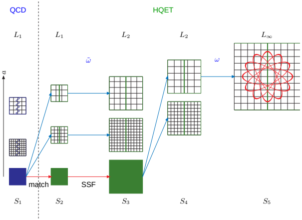
Example: SF boundary-to-boundary correlators

$$\widetilde{R}_1 \quad = \quad \frac{3}{4} \ln \left(\frac{f_1}{k_1} \right) \propto \omega_{\rm spin} \qquad z = LM$$



spin symmetry violating

Full strategy (1-2a)



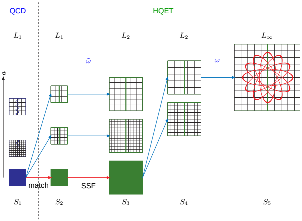
(1) continuum limit in QCD $a = 0.025\,\mathrm{fm} \dots 0.012\,\mathrm{fm}$

$$\Phi_i^{\mathrm{QCD}}(L_1, M, 0) = \lim_{a/L_1 \to 0} \Phi_i^{\mathrm{QCD}}(L_1, M, a)\,,$$

(2a) HQET parameters $a = 0.05\,\mathrm{fm} \dots 0.025\,\mathrm{fm}$

$$\begin{aligned} \tilde{\omega}(M, a) &\equiv \phi^{-1}(L_1, a)\,[\Phi(L_1, M, 0) - \eta(L_1, a)] \\ &= \begin{pmatrix} L_1^{-1}\Phi_1(L_1, M, 0) - \Gamma^{\mathrm{stat}}(L_1, a) \\ \Phi_2(L_1, M, 0) - \zeta_{\mathrm{A}}(L_1, a) \\ \dots \end{pmatrix} \\ L_1/a &\gg 1\,, \quad aM_{\mathrm{b}} \text{ irrelevant} \end{aligned}$$

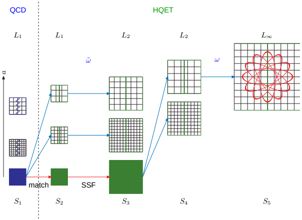
Full strategy



(2b) step scaling to $L_2 = 2L_1$ $a = 0.05\,\mathrm{fm} \ldots 0.025\,\mathrm{fm}$
Insert $\tilde{\omega}$ into $\Phi(L_2, M, a)$

$$\begin{aligned}
\Phi(L_2, M, 0) &= \lim_{a/L_2 \to 0} \{\eta(L_2, a) + \phi(L_2, a)\,\tilde{\omega}(M, a)\} \\
&= \lim_{a/L_2 \to 0} \begin{pmatrix} L_2\Gamma^{\rm stat}(L_2, a) + \frac{L_2}{L_1}\Phi_1(L_1, M, 0) - L_2\Gamma^{\rm stat}(L_1, a) \\ \zeta_{\rm A}(L_2, a) + \Phi_2(L_1, M, 0) - \zeta_{\rm A}(L_1, a) \\ \ldots \end{pmatrix} \\
&= \lim_{a/L_2 \to 0} \underbrace{\begin{pmatrix} L_2[\Gamma^{\rm stat}(L_2, a) - \Gamma^{\rm stat}(L_1, a)] \\ \zeta_{\rm A}(L_2, a) - \zeta_{\rm A}(L_1, a) \\ \ldots \end{pmatrix}}_{\text{finite HQET SSF's}} + \underbrace{\begin{pmatrix} \frac{L_2}{L_1}\Phi_1(L_1, M, 0) \\ \Phi_2(L_1, M, 0) \\ \ldots \end{pmatrix}}_{\text{QCD, mass dependence}} .
\end{aligned}$$

Full strategy



(3.) Repeat (2a.) for $L_1 \to L_2$:

$$\omega(M,a) \equiv \phi^{-1}(L_2,a)\,[\Phi(L_2,M,0) - \eta(L_2,a)]\,.$$

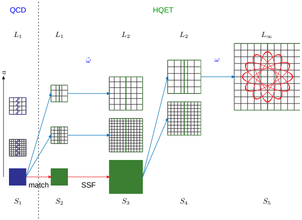
With same resolutions $L_2/a = 10\ldots20$ now $a = 0.1\,\mathrm{fm}\ldots0.05\,\mathrm{fm}$

(4.) insert ω into the expansion of large volume observables, e.g.

$$m_\mathrm{B} = \quad \omega_1 + E^\mathrm{stat}$$

from above: $m_\mathrm{B} = m_\mathrm{B}(M_\mathrm{b}) \to$ determine M_b

Full strategy



$m_{\rm B} = m_{\rm B}(M_{\rm b}) \rightarrow$ determine $M_{\rm b}$

explicitly in static approximation

$$
\begin{aligned}
m_{\rm B} = & \\
& \lim_{a\rightarrow 0}[E^{\rm stat} - \Gamma^{\rm stat}(L_2,a)] & a = 0.1{\rm fm} \ldots 0.05{\rm fm} & \quad [S_4,S_5] \\
& + \lim_{a\rightarrow 0}[\Gamma^{\rm stat}(L_2,a) - \Gamma^{\rm stat}(L_1,a)] & a = 0.05{\rm fm} \ldots 0.025{\rm fm} & \quad [S_2,S_3] \\
& + \frac{1}{L_1}\lim_{a\rightarrow 0}\Phi_1(L_1,M_{\rm b},a) & a = 0.025\,{\rm fm} \ldots 0.012\,{\rm fm} & \quad [S_1]\,.
\end{aligned}
$$