Heavy quark expansion in QCD and $b \to c$ near zero recoil

N. Uraltsev

Introduction; HQE in QCD

The $1/m_Q$ expansion in QCD can be constructed for a few important cases where we now know quite a bit about its terms

The key information is provided by the Small Velocity heavy quark sum rules, including spin sum rules first established for heavy quarks in QCD

Allowed to predict values of $\overline{\Lambda}$, μ_π^2, ρ_D^3, ... based on the hyperfine mass splitting $M_{B^*} - M_B \approx 47\,\text{MeV}$

A similar analysis has been extended motivated by the formfactor $F(0)$ in $B \to D^* \ell\nu$ near zero recoil   Gambino, Mannel, N.U. arXiv:1004.2859 [hep-ph]

Arrived at three apparently isolated, yet linked through the HQE, observations:

- Large negative *inelastic* corrections to $F(0)$ driving it down to $F(0) \approx 0.86$
- Large value of the nonlocal correlators of $\bar{Q}\vec{\pi}^2 Q$ and $\bar{Q}\vec{\sigma}\vec{B}Q$ in B mesons from the hyperfine splitting ΔM^2 in B vs. D

  The enhanced negative corrections in $F(0)$ are related to the 'discrepancy' in the hyperfine splitting ratio between charm and beauty mesons

- This implies enhanced inclusive yield of *radials* in $b \to c\,\ell\nu$

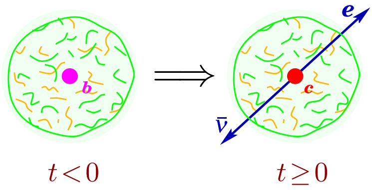
  Resolve '$\frac{1}{2} > \frac{3}{2}$' paradox
  Account for the missing semileptonic channels
  Predict significance of the $\frac{3}{2}^+$ 'D-wave'

As a byproduct we find significant corrections to the ground-state factorization; relevant for precision inclusive decays

V_{cb} at zero recoil

$$\mathrm{d}w\,(B \to D^* + \ell\bar{\nu}) \;\sim\; G_F^2 \cdot |V_{cb}|^2 \cdot |\vec{p}\,| \cdot |F_{B\to D^*}(\vec{p}\,)|^2$$

$|V_{cb}|$ requires $F_{B\to D^*}(\vec{p}\,)$ – it is shaped by bound-state physics



At $\vec{p}=0$ $(\,\vec{p}_e = -\vec{p}_{\bar{\nu}}\,)$

almost nothing happened!

Without *isotopic* effects (in the heavy quark limit) $F_{\vec{p}=0} = 1$:

$$F_{n/p}(0) \;=\; 1 + \frac{0}{m_{c,b}} + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{m^2_{c,b}}\right) + \mathcal{O}\left(\frac{\Lambda^3_{\mathrm{QCD}}}{m^3_{c,b}}\right) + \ldots$$

No $1/m_{b,c}$-corrections

(cf. Ademollo-Gatto)

1986 Voloshin, Shifman
1990 Luke

Challenge to theory: corrections to $F(0)=1$ are driven by $1/m_c$, potentially significant!

Originally (before 05/1994) were thought to be only about -0.02 Neubert

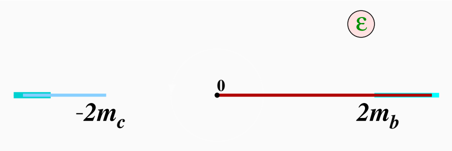
In fact, deviations from the symmetry limit in QCD are *considerably larger* in QCD Shifman, N.U., Vainshtein

There have been folklore around *sum rules for F_{D^*}*
often incorrect...

The QCD approach

$$T^{\rm zr}(q_0) = \int \mathrm{d}^3x \int \mathrm{d}x_0 \; e^{-iq_0x_0} \frac{1}{2M_B} \langle B| \tfrac{1}{3}\; iT\;\{\bar{c}\gamma_k\gamma_5 b(x)\;\bar{b}\gamma_k\gamma_5 c(0)\}|B\rangle$$

$$q_0 = M_B - M_D{}^* - \epsilon$$



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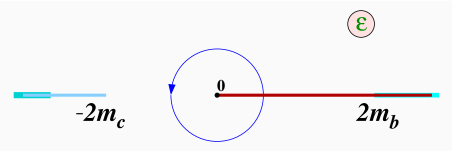

$$I_0(\mu) = -\frac{1}{2\pi i} \oint\limits_{|\epsilon|=\mu} T^{\rm zr}(\epsilon)\,\mathrm{d}\epsilon$$

$T^{\rm zr}(\epsilon)$ can be calculated in the short-distance expansion at $|\epsilon| \gg \Lambda_{\rm QCD}$, expansion parameters are $\frac{\mu_{\rm hadr}}{\epsilon}$, $\frac{\mu_{\rm hadr}}{2m_c+\epsilon}$, $\frac{\mu_{\rm hadr}}{2m_b-\epsilon}$ and α_s at the related scale

hence OPE for $I_0(\mu)$

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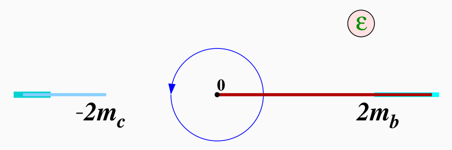

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Using analytic properties we can shrink the contour onto the physical cut $\epsilon > 0$, then by optical theorem

$$I_0(\mu) = \frac{1}{\pi} \int_0^\mu \mathrm{Im}\; T^{\rm zr}(\epsilon)\, \mathrm{d}\epsilon = |F_{D^*}|^2 + \sum_{\epsilon<\mu} |F_{B\to n}|^2$$

OPE in QCD

$$\sum_{f \neq D^*}^{\epsilon_f < \mu} \left| F_{B \to f} \right|^2 \;\equiv\; w_{\rm inel}(\mu)$$

$$F_{D^*} = \sqrt{I_0(\mu) - w_{\rm inel}(\mu)} \;<\; \sqrt{I_0(\mu)}$$

OPE:

$$I_0(\mu) = \xi_A^{\rm pert}(\mu) - \Delta_{\frac{1}{m_Q^2}} - \Delta_{\frac{1}{m_Q^3}} - \Delta_{\frac{1}{m_Q^4}} - \ldots$$

$$\Delta_{\frac{1}{m_Q^2}} = \frac{\mu_G^2}{3m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right)$$

$$\Delta_{\frac{1}{m_Q^3}} = \frac{\rho_D^3 - \frac{1}{3}\rho_{LS}^3}{4m_c^3} + \frac{\rho_D^3 + \rho_{LS}^3}{6m_c m_b}\left(\frac{1}{m_c} - \frac{1}{2m_b}\right)$$

$\ldots$

Brief synopsis

$$F_{D^*} = \sqrt{\xi_A^{\rm pert} - \Delta_{\rm power} - w_{\rm inel}}$$

$$\sqrt{\xi_A^{\rm pert}} \simeq 0.98 \quad \text{at } \mu \simeq 0.8\,\mathrm{GeV}, \qquad -\Delta_{\frac{1}{m_Q^2}} - \Delta_{\frac{1}{m_Q^3}} \simeq -0.13$$

$$F_{D^*} \le 0.92 \quad \text{– upper bound}$$

It turns out the sum of the excitation probabilities (*wavefunction overlap deficit*) should be quite significant, N.U. hep-ph/0312001

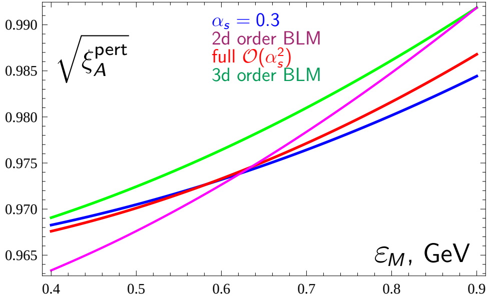
$$w_{\rm inel} \gtrsim 0.14 \qquad\qquad F_{D^*} \approx 0.86 \quad \text{– prediction}$$

How well is the OPE under control?

- Perturbative corrections: small numerically

$$\sqrt{\xi_A} \simeq 1 - 0.022 + (0.005 - 0.004) + 0.0022 - 0.0014 + \ldots$$

applies only to Wilsonian $\xi_A^{\rm pert}(\mu)$!



Assume $\mu = \varepsilon_M$ around $0.8\,\mathrm{GeV}$

$\sqrt{\xi_A} \simeq 0.98$ at $\alpha_s(m_b) = 0.22$

uncertainty 1% seems reasonably conservative

Convergence

- Power corrections to I_0:

Take the low-end expectation values $\mu_\pi^2 \simeq 0.4\,\mathrm{GeV}^2$, $\rho_D^3 \simeq 0.15\,\mathrm{GeV}^3$:

$-\Delta_{\frac{1}{m_Q^2}} \simeq -0.095$ $\quad -\Delta_{\frac{1}{m_Q^3}} \simeq -0.028$ $\quad -\Delta_{\frac{1}{m_Q^4}} \simeq 0.02$ $\quad -\Delta_{\frac{1}{m_Q^5}} \simeq 0.01$

- α_s-corrections to the coefficients of power-suppressed terms

Calculated for μ_π^2, correction is extremely small. Expect mild effect for μ_G^2; in $1/m_Q^3$ even a 30% renormalization would not produce a significant change

$\sqrt{1-\Delta^A} \lesssim 0.94 \pm 0.01$ $\qquad$ the upper bound seems safe at 1% level

Would expect formfactor about 0.92 *if no overlap deficit were there*

Sum of inelastic probabilities

Wavefunction overlap deficit in the language of Quantum Mechanics

Required to turn the upper bound for F_{D^*} into an estimate

$$I_1(\mu) = -\frac{1}{2\pi i} \oint\limits_{|\epsilon|=\mu} T^{\rm zr}(\epsilon)\, \epsilon \, {\rm d}\epsilon = \sum_{\epsilon<\mu} \epsilon_n |F_{B\to n}|^2$$

$$w_{\rm inel}(\mu) = \frac{I_1(\mu)}{\bar\epsilon(\mu)}$$

$\bar\epsilon(\mu)$ – 'average excitation energy' (up to μ)

$I_1(\mu)$ is calculated in the OPE similar to $I_0(\mu)$:

$$I_1 = \underbrace{\frac{-(\rho_{\pi G}^3+\rho_A^3)}{3m_c^2}}_{\text{BPS limit}} + \underbrace{\frac{-2\rho_{\pi\pi}^3-\rho_{\pi G}^3}{3m_c m_b}}_{(\delta\,\text{BPS})^1} + \underbrace{\frac{\rho_{\pi\pi}^3+\rho_{\pi G}^3+\rho_S^3+\rho_A^3}{4}}_{(\delta\,\text{BPS})^2} \left(\frac{1}{m_c^2}+\frac{2}{3m_c m_b}+\frac{1}{m_b^2}\right) + \mathcal{O}\left(\frac{1}{m_Q^3}\right)$$

Inelastic piece

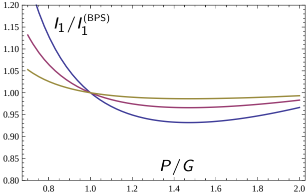
$$I_1^{(\mathrm{BPS})} = \frac{-(\rho_{\pi G}^3 + \rho_A^3)}{3m_c^2} + \mathcal{O}\left(\frac{1}{m_c^3}\right)$$

$(\rho_{\pi G}^3 + \rho_A^3) - \rho_{LS}^3$ determines hyperfine splitting to order $1/m_Q^2$

Extract comparing B and D mesons

$\delta^{(2)} I_1$ is positive; $\delta^{(1)} I_1$ comes with small coefficient $1/3m_c m_b$ and the minimum is very shallow:

$$I_1/I_1^{(\mathrm{BPS})} = 1 - \underbrace{(1-\nu_{3/2})\frac{m_c^2}{m_b^2}}_{<0.07} + [\ldots]^2 \qquad \nu_{3/2} > 0$$



$$w_{\mathrm{inel}} = \frac{0.48\,\mathrm{GeV}^3 + \kappa\, 0.35\,\mathrm{GeV}^3}{3m_c^2\, \epsilon_{\mathrm{rad}}} \simeq 14\%$$

$|\kappa| \lesssim 0.15$

A 6% decrease in F_{D^*}

The way to evaluate $\sum |F_{B\to n}|^2$ through I_1 in the 't Hooft model yields almost exact number

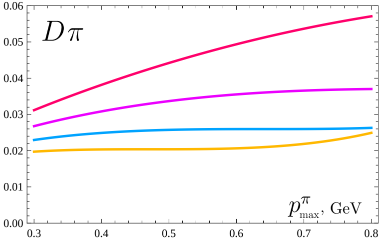
Continuum

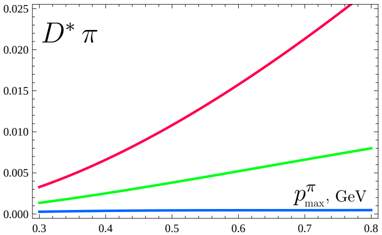
Excited states should predominantly be resonances (*radial* excitations)

Continuum is $1/N_c$-suppressed and is usually smaller

$D^{(*)}\pi$ inelastic contributions can be estimated for soft pions

The complete estimate yields a significant contribution:





$g_{D^*D\pi} = 4.9\,\text{GeV}^{-1}$ $\quad (\Gamma_D = 96\,\text{KeV})$

$g_{B^*B\pi}/g_{D^*D\pi} = 1,\ 0.8,\ 0.6$ and 0.4

$g_{B^*B\pi}/g_{D^*D^*\pi} = 1,\ 0.8,\ 0.6$

Altogether we expect $D^{(*)}\pi$ piece to yield around 4% in $\sum |F_{B\to n}|^2$;

about a fourth of the resonance-based estimate $\qquad \delta F_{D^*} \simeq -2\%$

F_{D^*} summary

QCD lower bound:

$F_{D^*} < 0.92$ $\qquad$ $F_{D^*} < 0.9$ including continuum estimate

The unbiased predicted value

$$F_{D^*} \lesssim 0.86$$

The central number has about 2% accuracy it may lower if μ_π^2 turns out larger

Central value goes down for increasing μ_π^2 yet the corrections from higher power terms also increase

Experimentally challenging theoretically advantageous

N.U. 2003

$$\langle D(p_2)|\bar{c}\gamma_\nu b|B(p_1)\rangle = f_+(p_1+p_2)_\nu + f_-(p_1-p_2)_\nu$$

$f_\pm \equiv f_\pm(\vec{q}^{\,2})$

A single amplitude $J_0 = (M_B+M_D)f_+(0) + (M_B-M_D)f_-(0)$ at $\vec{q}=0$

HQ limit: $f_+ = \frac{M_B+M_D}{2\sqrt{M_B M_D}}, \qquad f_- = -\frac{M_B-M_D}{M_B+M_D}\, f_+$

$$\frac{J_0}{2\sqrt{M_B M_D}} = 1 - a_2\left(\frac{1}{m_c}-\frac{1}{m_b}\right)^2 - a_3\left(\frac{1}{m_c}-\frac{1}{m_b}\right)^2\left(\frac{1}{m_c}+\frac{1}{m_b}\right) + \ldots$$

Power corrections are well under control and small

Any amplitude with massless leptons depends, however solely on f_+,
(only the combination of f_+ and f_- has no $1/m$ corrections)

$F_+ \equiv \frac{2\sqrt{M_B M_D}}{M_B+M_D}\, f_+$ has $1/m_Q$ corrections since nothing forbids this in $\vec{J}$

Not a drawback in the era of dynamics

$$F_+ = 1 + \left(\frac{\overline{\Lambda}}{2} - \overline{\Sigma}\right)\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\frac{M_B - M_D}{M_B + M_D} - \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

$$\overline{\Lambda} = M_B - m_b, \quad \overline{\Sigma} = \ldots$$

From inclusive decays and exact sum rules we know

$\frac{\overline{\Lambda}}{2} - \overline{\Sigma}$ (positive, but small $\propto \frac{\mu_\pi^2 - \mu_G^2}{3\mu_{\rm hadr}}$)

Moreover, we know `all` power corrections here are small

at small $\mu_\pi^2 - \mu_G^2$

$$\boxed{\frac{2\sqrt{M_B M_D}}{M_B + M_D}\, f_+(0) = 1.04 \pm 0.01 \pm 0.01}$$

N.U. 2003

All orders in $1/m$ in 'BPS', to $1/m^2 \cdot 1/\mathrm{BPS}^2$, α_s^1

The bulk 3% is the perturbative factor, only 1% comes from power terms

Summary

Comprehensive heavy quark expansion can say much for nonperturbative effects in certain cases

Inequalities or positivity properties are essential, require a physical renormalization scheme. Conceptually similar to the lattice, yet may differ in details

Exploit physical behavior of the correlators in Minkowsky domain

- $F_{D^*} \approx 0.86$; uncertainty about 2% at known μ_π^2, ρ_D^3 plus effect of higher-order power terms. Central value goes down for larger μ_π^2, yet higher power corrections become significant
- Large inelastic contribution decreasing F_{D^*} by 6%

  Close to the BPS estimate
- Large nonlocal correlators $\rho^3_{...}$ from hyperfine splittings
  'Discrepancy' in the splitting for B and D is settled
- Large inclusive yield of '*radials*', $7\div10\%$ of $\Gamma_{\rm sl}$
  Resolve $\frac{1}{2}$ vs. $\frac{3}{2}$ problem
  May provide missing semileptonic channels


RECYCLE

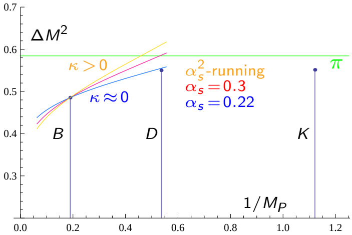
Hyperfine splitting in D vs. B

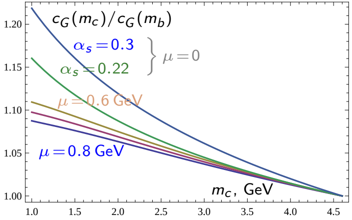
$$M_{B^*}^2 - M_B^2 \simeq M_{D^*}^2 - M_D^2 \simeq M_{K^*}^2 - M_K^2 \simeq M_\rho^2 - M_\pi^2$$

If these were exact and if perturbative corrections could be discarded

$$-(-\rho_{LS}^3 + \rho_{\pi G}^3 + \rho_A^3) \simeq 2\overline{\Lambda}\mu_G^2\,(1+\kappa)$$

$$-(\rho_{\pi G}^3 + \rho_A^3) \approx 0.48\,\mathrm{GeV}^3$$





The final outcome is $\kappa \approx -0.15$ and $-(\rho_{\pi G}^3 + \rho_A^3) \approx 0.48\,\mathrm{GeV}^3$

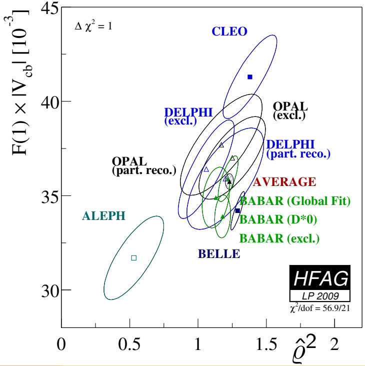
Experimental determination

We can take V_{cb} extracted from inclusive decays and calculate F_{D^*}

Shifman, N.U. 1994

$$
\begin{aligned}
F_{D^*} \simeq 0.810 \pm 0.007 \pm 0.026 \pm \delta_{\rm incl} \qquad & \text{BaBar 2008} \\
0.849 \pm 0.011 \pm \delta_{\rm incl} \qquad & \text{HFAG Average 2009} \\
0.836 \pm 0.015 \pm \delta_{\rm incl} \qquad & \text{HFAG 08, CLEO / ALEPH excluded}
\end{aligned}
$$

(inclusive value taken without error bars)

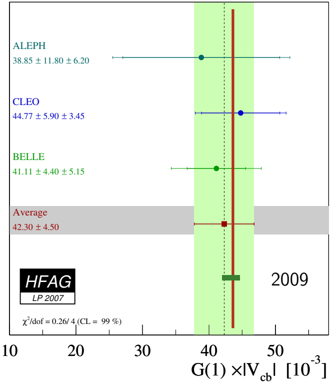


Finally F_{D^*} moved practically to the value obtained in the heavy quark expansion in QCD, 0.86±0.02

For $B \to D\,\ell\nu$:

$$F_+(0) \simeq 1.021 \pm 0.019 \pm 0.041 \pm \delta_{\rm incl}$$

Good agreement with the dynamic heavy quark expansion in QCD



Using $|V_{cb}|$ from $\Gamma_{\rm sl}(B)$ I predicted

$$|V_{cb}|G(1) = 43.\underset{5}{\cancel{7}} \cdot 10^{-3}$$

There is no tension between *inclusive* and *exclusive* V_{cb}

rather a remarkable agreement