Some Constants and Units

- Planck's Constant: $\hbar = 1.05{\times}10^{-34}$ Js , or $h = 2\pi\hbar = 6.63{\times}10^{-34}$ Js
- Speed of light: $c = 3.00{\times}10^{8}$ ms^{-1} ; Wavelength of visible light (approx) $4{\times}10^{-7}$m to $7{\times}10^{-7}$m
- Unit of electric charge: $e = 1.60{\times}10^{-19}$ C ; Unit of energy: electron-volt, 1 eV $= 1.60{\times}10^{-19}$ J
- Fine structure constant: $\alpha = e^2/4\pi\epsilon_0\hbar c \approx 1/137$ (dimensionless)
- Electron mass: $m_e = 9.11{\times}10^{-31}$ kg ; Proton mass: $m_p = 1.67{\times}10^{-27}$ kg
- Bohr radius: $r_1 = 4\pi\epsilon_0\hbar^2/m_e e^2 = \hbar/m_e c\alpha = 5.29{\times}10^{-11}$ m

Wave Behaviour

We will refer to any real or complex valued function with periodicity in time and/or space as a *wave*. The following remarks summarise a few useful definitions and ideas.

- A function of time t obeying $f(t{+}T) = f(t)$ has *period* T, *frequency* $\nu = 1/T$, and *angular* or *circular frequency*

$$\omega = 2\pi\nu = 2\pi/T\ .$$

Familiar examples are $f(t) = \cos\omega t$, $\sin\omega t$ or $\exp \pm i\omega t$. It is also customary to refer to ω as the frequency, provided this leads to no confusion.

A function of position x (in one dimension) obeying $f(x{+}\lambda) = f(x)$ has *wavelength* λ and *wavenumber*

$$k = 2\pi/\lambda$$

Examples are $f(x) = \cos kx$, $\sin kx$ or $\exp \pm ikx$. The analogous functions of a position vector $\mathbf{x}$ with periodicity in three dimensions are $f(\mathbf{x}) = \exp i\mathbf{k}\cdot\mathbf{x}$ where $\mathbf{k}$ is the *wave vector*, and the wavelength is then $\lambda = 2\pi/|\mathbf{k}|$. We shall refer to such functions as *plane waves*.

- The wave equation in one dimension for a function $f(x,t)$ is

$$\frac{\partial^2 f}{\partial t^2} - c^2\frac{\partial^2 f}{\partial x^2} = 0$$

where c is some constant. This has solutions which are periodic in both position and time:

$$f_\pm(x,t) = A_\pm \exp(\pm ikx - i\omega t) \qquad (*)$$

provided that the wavelength and frequency are related by

$$\omega = ck \qquad \text{or} \qquad \lambda\nu = c\ .$$

Such solutions represent waves which move or *propagate* with speed c to the right or left, according to the sign in $(*)$ (assuming ω, $k > 0$). The constant $A_\pm$ is the *amplitude* of the wave.

In electromagnetic (EM) waves, the field components obey the three dimensional wave equation, obtained by replacing $\partial^2/\partial x^2$ by ∇^2 above. This has solutions of an analogous form

$$f(\mathbf{x},t) = A\exp(i\mathbf{k}{\cdot}\mathbf{x} - i\omega t) \qquad \text{with} \qquad \omega = c|\mathbf{k}|\ .$$

Such a wave propagates in the direction of $\mathbf{k}$, with speed c, now the speed of light.

• Other kinds of waves arise as solutions of other *governing equations* which may differ significantly from the standard wave equation. A function does not have to satisfy the standard wave equation in order to be usefully thought of as a wave! The *Schrödinger Equation* is one example of an alternative governing equation; it is the central equation in QM and we will study it in some depth. (In other physical applications, e.g. waves in real fluids, we should expect the wave equation to be modified by friction or dissipative terms.)

• Many different governing equations give rise to propagating solutions of the form $(*)$, provided the frequency is chosen to be a suitable function of the wavenumber, $\omega(k)$. Moreover, if the governing equation is *linear* in f, then any solutions f_1 and f_2 can be combined to give a new solution:

$$f = f_1 + f_2$$

This is the *Principle of Superposition* and it is responsible for much behaviour we tend to think of as *wave-like*.

• *Interference* or *diffraction* occurs when waves from different sources merge, or when parts of a wave recombine after passing around or through some obstacle. When a number of such waves are superposed, they may interfere *constructively*, increasing the size of the amplitude, or *destructively*, diminishing the amplitude. The result is an interference or diffraction pattern which depends on the sources or the obstacles.

When light is passed through a number of narrow slits, the resulting diffraction pattern provides conclusive evidence that light is a wave. Passing higher energy waves, such as X-rays, through matter gives a way of determining the crystalline arrangement of atoms from the resulting diffraction patterns.

A Few Historical Highlights

• 1801-03: Interference/diffraction experiments by Young show that light is a wave

• 1862-4: Maxwell identifies light as an EM (electromagnetic) wave

• 1897: Thompson discovers the electron, the first elementary particle

• 1900: Planck introduces the energy-frequency relation, with h as a new physical constant, and derives the *black body spectrum* (the distribution of energy with frequency for EM radiation in thermal equilibrium)

• 1905: Einstein imparts clearer physical meaning to photons, using them to explain the *photoelectric effect*, and other experimental results

• 1909: In a version of the double slit experiment, G.I. Taylor demonstrates that light produces a wave-like interference pattern on photographic film even when the light source is filtered so that only one photon at a time is recorded by the film.

• 1911: Based on scattering experiments, Rutherford proposes a model of the atom with most of its mass concentrated in a small, compact *nucleus*

• 1913: Bohr proposes an atomic model with electrons orbiting a nucleus and with quantisation of their angular momentum, using this to derive observed line spectra

• 1923: Compton scattering of X-rays on electrons confirms that photons are relativistic particles of zero rest mass

• 1923-24: de Broglie proposes *wave-particle duality* for matter, as for radiation

• 1925-30: The emergence of *Quantum Mechanics*, through work of Heisenberg, Born, Jordan, Dirac, Pauli, Schrödinger, and others

• 1927-28: Diffraction experiments of Davisson, Germer and Thompson confirm that electrons behave as waves as well as particles