Chapter 9. Stress relaxation

Stress relaxation is a special property of non-Newtonian fluids

- not in elastic solids
- not in viscous fluids

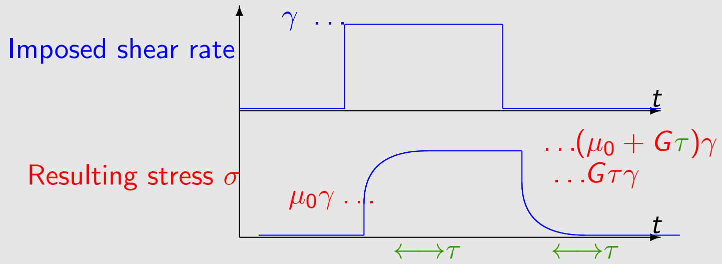
Hence

$$\text{non-Newtonian} \neq \tfrac{1}{2}\,\text{elastic solid} + \tfrac{1}{2}\,\text{viscous fluid}$$

Important relaxation time τ of stress/microstructure.

Linear visco-elasticity – common to all fluid models

μ_0 solvent viscosity, G elastic modulus, τ relaxation time.



- Early viscosity μ_0
- Steady state viscosity $\mu_0 + G\tau$
- Takes τ to build up to steady state
- steady deformation = shear rate γ × memory time τ

NB steady flows are unsteady Lagrangian.

E.G. linear visco-elasticity for Oldroyd-B

Microstructure A:

$$\frac{DA}{Dt} - \nabla u^T \cdot A - A \cdot \nabla u + \frac{1}{\tau}(A - I) = 0$$

Stress σ:

$$\sigma = -pI + 2\mu_0 E + G\,(A - I)$$

Weak flow: $\nabla u \ll \frac{1}{\tau}$, so $A = I + a$ with $|a| \ll 1$

$$\frac{Da}{Dt} + \frac{1}{\tau} a = 2E$$

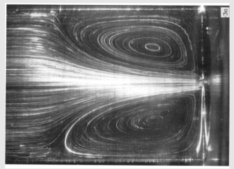
Start up:

$$a = \dot\gamma\tau\left(1 - e^{-t/\tau}\right) \qquad \sigma = \mu_0\dot\gamma + G\dot\gamma\tau\left(1 - e^{-t/\tau}\right)$$

Stopping:

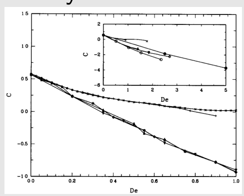
$$a = \dot\gamma\tau\, e^{-t/\tau} \qquad \sigma = G\dot\gamma\tau\, e^{-t/\tau}$$

Contraction flow – Lagrangian unsteady



Numerical Oldroyd-B

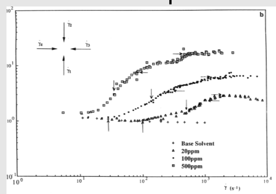
$\frac{\Delta P}{\text{Stokes}}$



$U\tau/L$

Experiments

$\frac{\Delta P}{\text{Stokes}}$



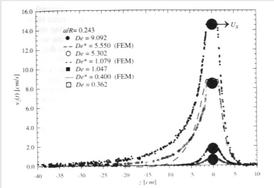
$U\tau/L$

Δp scaled by Stokes using steady-state viscosity $\mu_0 + G\tau$.

But if flow fast, lower pressure drop from early-time viscosity μ_0.

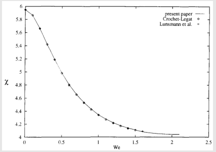
Oldroyd-B has no big increase in Δp, and no big upstream vortex

Flow past a sphere – Lagrangian unsteady



Numerical Oldroyd-B

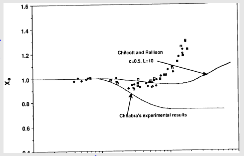
$\frac{\text{Drag}}{\text{Stokes}}$



$U\tau/L$

Experiments

$\frac{\text{Drag}}{\text{Stokes}}$



$U\tau/L$

Drag scaled by Stokes using steady-state viscosity $\mu_0 + G\tau$.

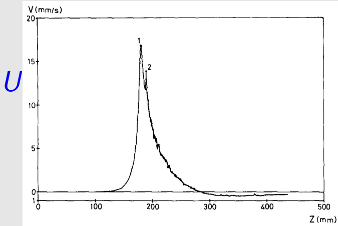
But if flow fast lower, lower drag from early-time viscosity μ_0.

Oldroyd-B has no big increase in drag, and no big wake

. . . and negative wakes

Experiment

U



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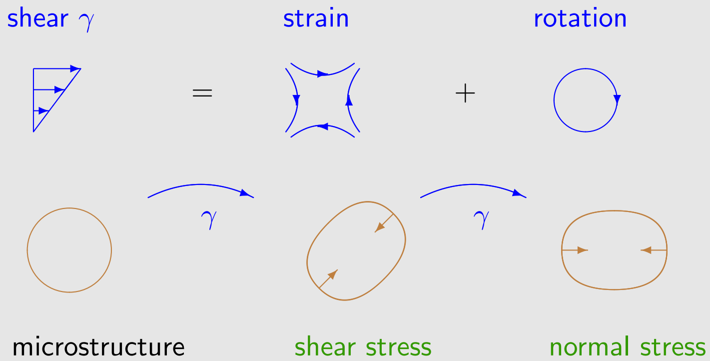
Bisgaard 1983 JNNFM

Unrelaxed elastic stress in wake, cancelled by negative viscous flow.

Tension in streamlines

– relaxation + slightly nonlinear effect



Shear stress $= G \times (\text{rate} = \gamma) \times (\text{memory time} = \tau)$

Normal stress (tension in streamlines) = shear stress $\times \gamma\tau$.

Tension in streamlines

- Rod climbing
- Secondary circulation
- Migration into chains
- Migration to centre of pipe
- Falling rods align with gravity
- Stabilisation of jets
- Co-extrusion instability
- Taylor-Couette instability