Modelling chordwise-varying porosity to reduce aerofoil-turbulence interaction noise

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joint work with: Lorna Ayton, Thomas Geyer, Paruchuri Chaitanya, Ennes Sarradj

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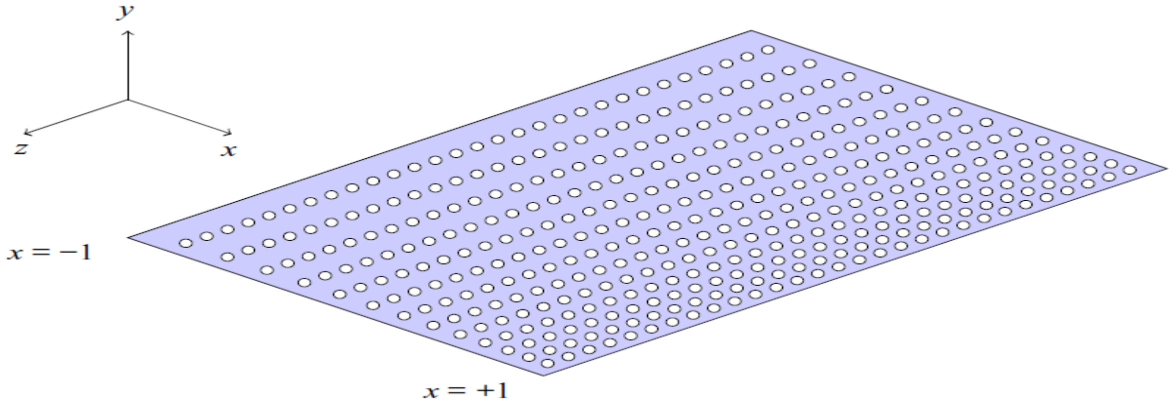
Related paper: “Reducing aerofoil–turbulence interaction noise through chordwise-varying porosity”, JFM (2021)

Mathematical Model



2D Helmholtz:

$$\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + k_0^2\right)\phi = 0,$$

scattered field

incident field

Impedance condition:

$$\left.\frac{\partial \phi}{\partial y}\right|_{y=0} + \left.\frac{\partial \phi_{\mathrm{I}}}{\partial y}\right|_{y=0} = -\mu(x)[\phi], \qquad |x| < 1$$

variable porosity parameter

Numerical Method

Idea: Separation of variables in elliptic coordinates:

$$x = \cosh(\nu)\cos(\tau), y = \sinh(\nu)\sin(\tau)$$

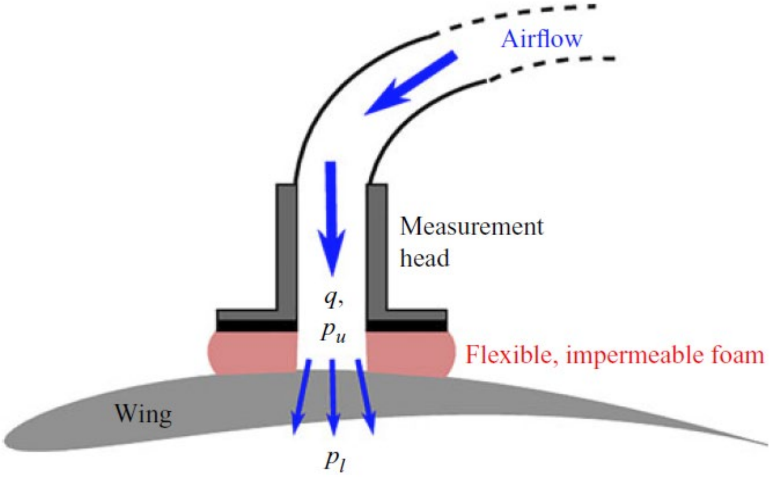
$$\phi(\nu, \tau) = \sum_{n=1}^{\infty} a_n \mathrm{se}_n(\tau)\mathrm{Hse}_n(\nu)$$

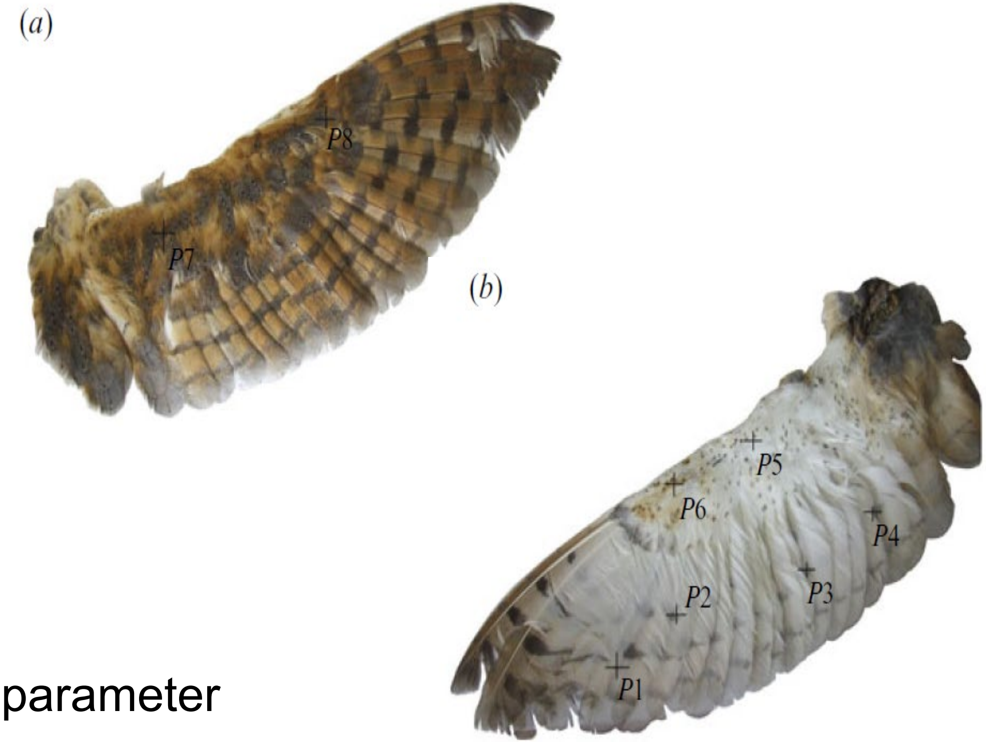
$$\Downarrow$$

$$\sum_{n=1}^{N} a_n \mathrm{se}_n\left(\cos^{-1}(x)\right)\left[1 - 2\mathrm{Hse}_n(0)\mu(x)\sqrt{1-x^2}\right] = -\sqrt{1-x^2} \cdot \frac{\partial \phi_{\mathrm{I}}}{\partial y}(x)$$

Collocation yields a linear system for the unknown coefficients.

Experimental Measurement of Permeability





Flow resistance → Porosity parameter

Convergence

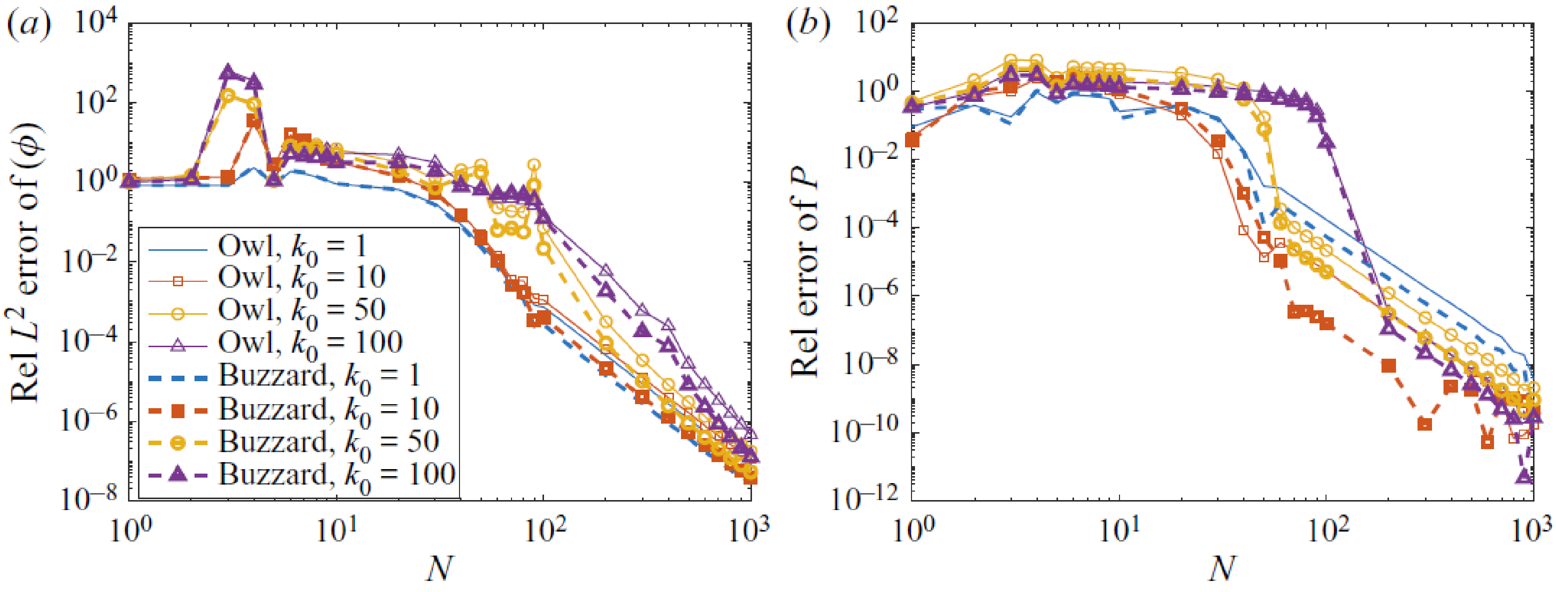


FIGURE 6. Relative errors for $[\phi]$ (a, L^2 norm error over $[-1, 1]$) and SPL (b). The method has a high order of algebraic convergence, allowing us to compute physical values to several significant figures.

Bio-inspired Results I

$$\phi_{\mathrm{I}} = \frac{\mathrm{i}k_0^2}{4r_0^2}(x - x_0)(y - y_0)H_2^{(1)}(k_0 r_0)$$

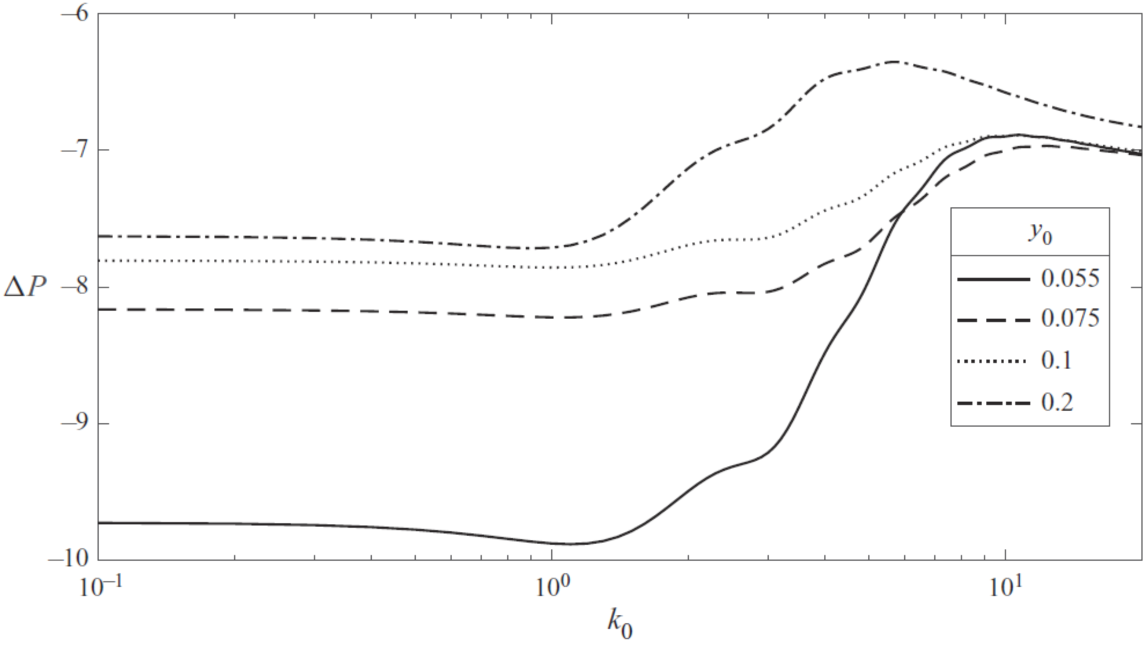


FIGURE 9. Value of ΔP for a near-field quadrupole source at $x_0 = 0.95$ and various y_0. Negative values indicate the owl is quieter than the buzzard by that many dB.

Bio-inspired Results II

$$\partial\phi_{\mathrm{I}}/\partial y|_{y=0} = -e^{\mathrm{i}\delta x}, \text{ where } \delta = k_1/\sqrt{1-M^2}, \text{ and } k_1 = \sqrt{1-M^2}k_0/M$$

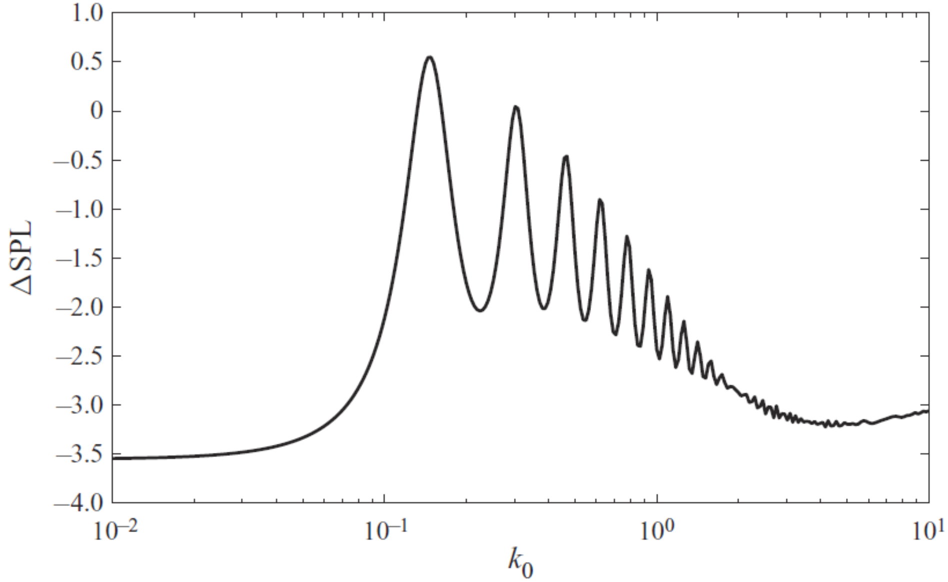
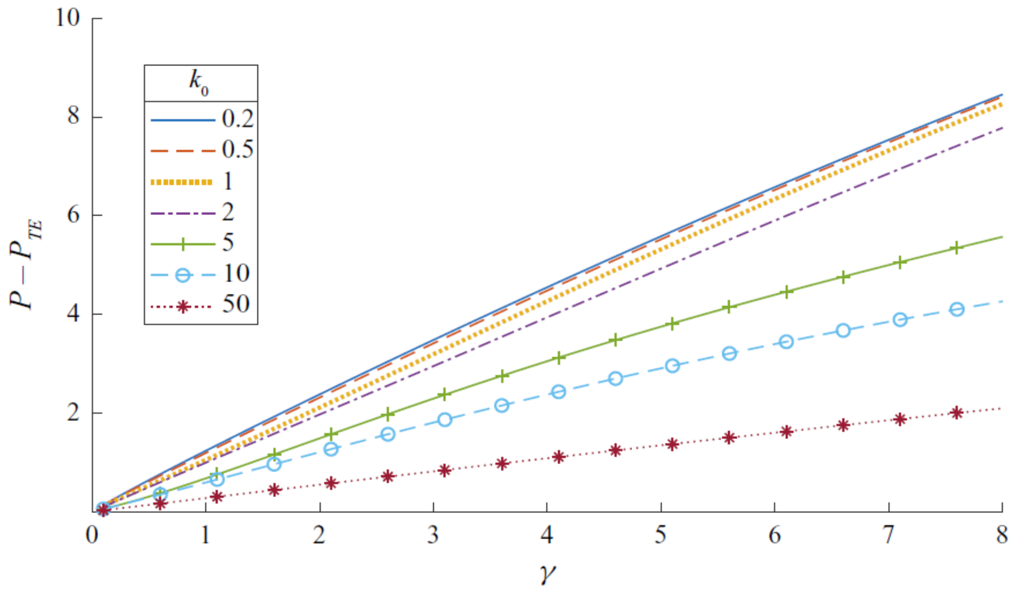


FIGURE 11. Value of ΔP for an incident gust. Negative values indicate the owl is quieter than the buzzard by that many dB.

Monotonic Distributions

$$\alpha_H(x) = \alpha_L + (\alpha_T - \alpha_L)\left(\frac{x}{2} + \frac{1}{2}\right)^{\gamma}, \quad \mu = \alpha_H K_R/(\pi r^2)$$



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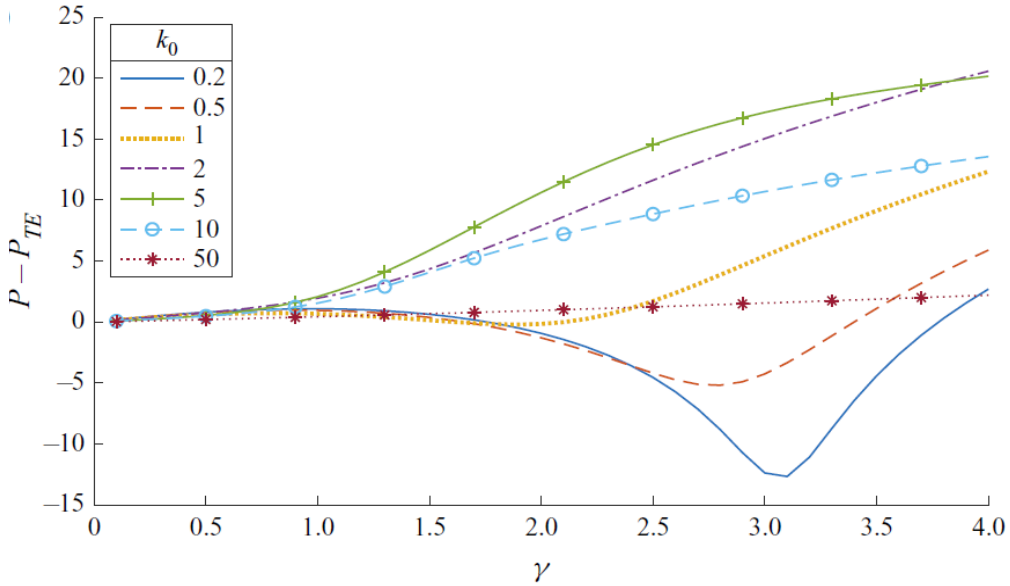

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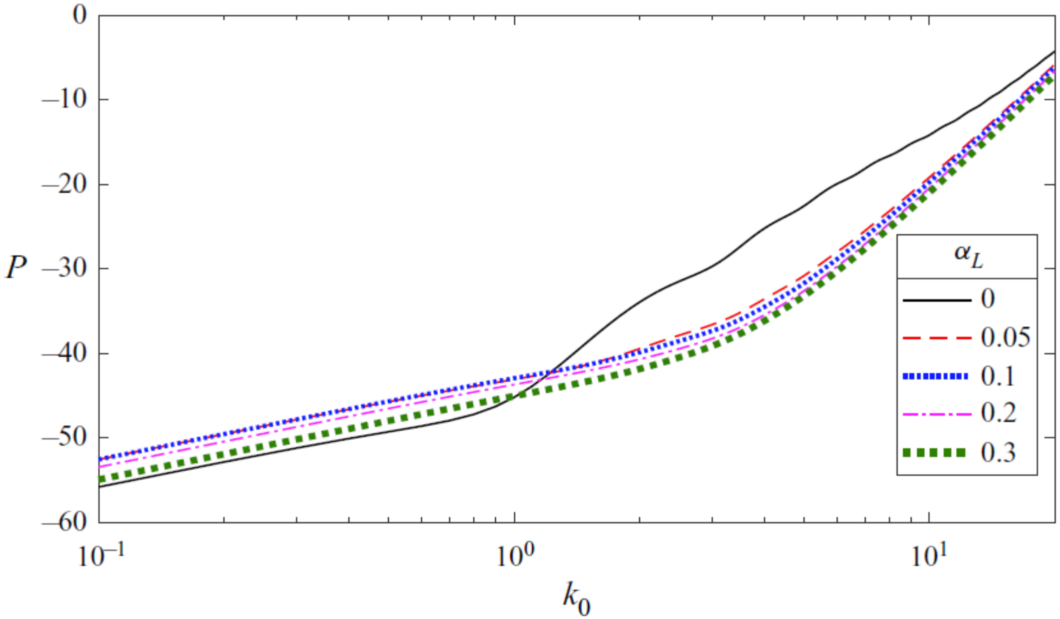

Conclusion

- Measured distributions from two species – owls and common buzzard - and matched their chordwise-varying air flow.
- Plate with a porosity mimicking the owl produces much less noise, both for trailing and leading edges.
- For low frequencies, monotonic variation from porous trailing edge to impermeable leading edge can be more acoustically beneficial than plate with constant trailing-edge porosity.
- Varying porosity to induce a destructively interfering acoustic field has the potential to benefit both acoustics and aerodynamics.

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