Spatial pattern formation in non-equilibrium condensates. Part II.

Natalia Berloff
DAMTP
University of Cambridge

Outline

- Introduction: Exciton–polariton condensates
- Gross-Pitaevskii equation with loss and gain
  - Radially symmetric stationary states
  - Spiral vortex states
  - Vortex lattices
- Non-equilibrium spinor system: interplay between interconversion and detuning
  - Stability of cross–polarized vortices
  - Synchronisation/desynchronisation
- Controllable half-vortex lattices
- Turbulence in nonequilibrium condensates

Outline

- Introduction: Exciton–polariton condensates
- Gross-Pitaevskii equation with loss and gain
  - Radially symmetric stationary states
  - Spiral vortex states
  - Vortex lattices
- Non-equilibrium spinor system: interplay between interconversion and detuning
  - Stability of cross–polarized vortices
  - Synchronisation/desynchronisation
- Controllable half-vortex lattices
- Turbulence in nonequilibrium condensates

Outline

- Introduction: Exciton–polariton condensates
- Gross-Pitaevskii equation with loss and gain
  - Radially symmetric stationary states
  - Spiral vortex states
  - Vortex lattices
- Non-equilibrium spinor system: interplay between interconversion and detuning
  - Stability of cross–polarized vortices
  - Synchronisation/desynchronisation
- Controllable half-vortex lattices
- Turbulence in nonequilibrium condensates

Acknowledgements

Magnus Borgh
Southampton University

Jonathan Keeling
St Andrews University

M.Borgh, J.Keeling and N.G.Berloff, PRB, **81**, 235302 (2010)
N.G.Berloff, arXiv:1010.5225 (2010)
J.Keeling and N.G.Berloff, arXiv:1102.5302 (2011).

Motivation

Polariton condensates as sustained non-equilibrium system with nontrivial spatiotemporal properties.
Advantage: source term can be made to vary in space and time to control pattern formation.
Recent experiments study

- quantised vortices [Lagoudakis et al Nature Phys 2008]
- solitary waves propagation [Amo et al Nature 2009]
- pattern in 1D samples [Wertz et al Nature Phys 2010]

Polarization degree of freedom allows to create topological textures more complicated than simple vortices.

How to control the transition between different states?
Phase transition from linear to elliptical to circular polarisations?

Mean-field model of a non-equilibrium BEC of exciton-polaritons

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}} + U|\psi|^2 + i(\gamma_{\text{net}} - \Gamma|\psi|^2)\right]\psi,$$

V_{ext} is an external trapping potential, $= \frac{1}{2}m\omega^2 r^2$, γ_{net}– net gain, Γ – effective loss, U – effective (pseudo-) interaction potential.

Length in units of oscillator length $\sqrt{\hbar/m\omega}$, energies in units of $\hbar\omega$, and $\psi \to \sqrt{\hbar\omega/2U}\psi$, yields:

$$i\partial_t\psi = \left[-\nabla^2 + r^2 + |\psi|^2 + i\left(\alpha - \sigma|\psi|^2\right)\right]\psi.$$

Two parameters: $\alpha = 2\gamma_{\text{net}}/\hbar\omega$ (gain), and $\sigma = \Gamma/U$ (loss).
Estimate from experiments: $0 \leq \alpha \leq 10$ and $\sigma \sim 0.3$
[Keeling and NB, PRL, **100**,250401 (2008)]

Gross-Pitaevskii equation with loss and gain

Mean-field model of a non-equilibrium BEC of exciton-polaritons

$$i\hbar\partial_t\psi = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\rm ext} + U|\psi|^2 + i(\gamma_{\rm net} - \Gamma|\psi|^2)\right]\psi,$$

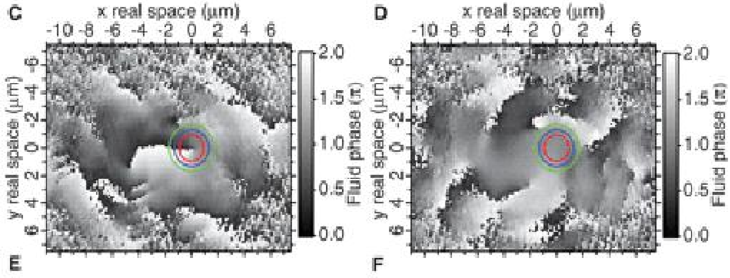
$V_{\rm ext}$ is an external trapping potential, $= \frac{1}{2}m\omega^2 r^2$, $\gamma_{\rm net}$– net gain, Γ – effective loss, U – effective (pseudo-) interaction potential. Length in units of oscillator length $\sqrt{\hbar/m\omega}$, energies in units of $\hbar\omega$, and $\psi \to \sqrt{\hbar\omega/2U}\psi$, yields:

$$\mathrm{i}\partial_t\psi = \left[-\nabla^2 + r^2 + |\psi|^2 + i\left(\alpha - \sigma|\psi|^2\right)\right]\psi.$$

Two parameters: $\alpha = 2\gamma_{\rm net}/\hbar\omega$ (gain), and $\sigma = \Gamma/U$ (loss).
Estimate from experiments: $0 \le \alpha \le 10$ and $\sigma \sim 0.3$
[Keeling and NB, PRL, **100**,250401 (2008)]

[Lagoudakis et al, Science, November 2009]:

Phase maps of left- and right-circular polarized polariton states



Observed all possible $(\pm 1, \pm 1)$ vortex states.

- Include spin degree of freedom: left- and right-circular polariton states ψ_L and ψ_R.
- For weakly-interacting dilute Bose gas model:

$$H = \frac{\hbar^2 |\nabla \psi_L|^2}{2m} + \frac{\hbar^2 |\nabla \psi_R|^2}{2m} + \frac{U_0}{2}\left(|\psi_L|^2 + |\psi_R|^2 \right)^2$$

- Tendency to biexciton formation $\rightarrow U_1$. Magnetic field: Ω_B
- J_2 Circular symmetry broken – two equivalent axes.
  J_1 preferred direction – inequivalent axes.

Polariton spin degree of freedom

- Include spin degree of freedom: left- and right-circular polariton states ψ_L and ψ_R.
- For weakly-interacting dilute Bose gas model:

$$H = \frac{\hbar^2|\nabla\psi_L|^2}{2m} + \frac{\hbar^2|\nabla\psi_R|^2}{2m} + \frac{U_0}{2}\left(|\psi_L|^2 + |\psi_R|^2\right)^2$$

- Tendency to biexciton formation $\rightarrow U_1$. Magnetic field: Ω_B
- J_2 Circular symmetry broken – two equivalent axes.
  J_1 preferred direction – inequivalent axes.

- Include spin degree of freedom: left- and right-circular polariton states ψ_L and ψ_R.
- For weakly-interacting dilute Bose gas model:

$$
\begin{aligned}
H &= \frac{\hbar^2|\nabla\psi_L|^2}{2m} + \frac{\hbar^2|\nabla\psi_R|^2}{2m} + \frac{U_0}{2}\left(|\psi_L|^2 + |\psi_R|^2\right)^2 \\
&- 2U_1|\psi_L|^2|\psi_R|^2 + \Omega_B\left(|\psi_L|^2 - |\psi_R|^2\right)
\end{aligned}
$$

- Tendency to biexciton formation $\rightarrow$ U_1 . Magnetic field: Ω_B
- J_2 Circular symmetry broken – two equivalent axes.
  J_1 preferred direction – inequivalent axes.

Polariton spin degree of freedom

- Include spin degree of freedom: left- and right-circular polariton states ψ_L and ψ_R.
- For weakly-interacting dilute Bose gas model:

$$
\begin{aligned}
H &= \frac{\hbar^2|\nabla\psi_L|^2}{2m} + \frac{\hbar^2|\nabla\psi_R|^2}{2m} + \frac{U_0}{2}\left(|\psi_L|^2 + |\psi_R|^2\right)^2 \\
&- 2U_1|\psi_L|^2|\psi_R|^2 + \Omega_B\left(|\psi_L|^2 - |\psi_R|^2\right) \\
&+ J_1\left(\psi_L^\dagger\psi_R + H.c\right) + J_2\left(\psi_L^\dagger\psi_R + H.c.\right)^2
\end{aligned}
$$

- Tendency to biexciton formation $\rightarrow$ U_1 . Magnetic field: Ω_B
- J_2 Circular symmetry broken – two equivalent axes.
  J_1 preferred direction – inequivalent axes.

Non-equilibrium spinor system

Spinor Gross-Pitaevskii equation: [Borgh et al PRB, **81**, 235302 (2010)]

$$i\hbar\partial_t\psi_L = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\mathrm{ext}}(r) + \frac{\Omega_B}{2} + U_0|\psi_L|^2 + (U_0 - 2U_1)|\psi_R|^2 + i\left(\gamma_{\mathrm{net}} - \Gamma|\psi_L|^2\right)\right]\psi_L + J_1\psi_R$$

Similarly for ψ_R with $\psi_L \leftrightarrow \psi_R$ and $\Omega_B \to -\Omega_B$.

Dimensionless cGPE:

$$i\partial_t\psi_L = \left[-\nabla^2 + v(r) + |\psi_L|^2 + (1-u_a)|\psi_R|^2 + \frac{\Delta}{2} + i\left(\alpha - \sigma|\psi_L|^2\right)\right]\psi_L + J\psi_R.$$

If $v(r) = r^2$ then take $\alpha \to \alpha\Theta(r_0 - r)$ as before.

Questions:

1. Normal modes of uniform model: diffusive, linear, gapped.
2. Effect of Δ and J on vortices?
3. How does interconversion J interact with currents?
4. Synchronization/desynchronization.

Non-equilibrium spinor system

Spinor Gross-Pitaevskii equation: [Borgh et al PRB, **81**, 235302 (2010)]

$$i\hbar\partial_t\psi_L = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}}(r) + \frac{\Omega_B}{2} + U_0|\psi_L|^2 + (U_0 - 2U_1)|\psi_R|^2 + i\left(\gamma_{\text{net}} - \Gamma|\psi_L|^2\right)\right]\psi_L + J_1\psi_R$$

Similarly for ψ_R with $\psi_L \leftrightarrow \psi_R$ and $\Omega_B \to -\Omega_B$.
Dimensionless cGPE:

$$i\partial_t\psi_L = \left[-\nabla^2 + v(r) + |\psi_L|^2 + (1-u_a)|\psi_R|^2 + \frac{\Delta}{2} + i\left(\alpha - \sigma|\psi_L|^2\right)\right]\psi_L + J\psi_R.$$

If $v(r) = r^2$ then take $\alpha \to \alpha\Theta(r_0 - r)$ as before.
Questions:

1. Normal modes of uniform model: diffusive, linear, gapped.
2. Effect of Δ and J on vortices?
3. How does interconversion J interact with currents?
4. Synchronization/desynchronization.

Non-equilibrium spinor system

Spinor Gross-Pitaevskii equation: [Borgh et al PRB, **81**, 235302 (2010)]

$$i\hbar\partial_t\psi_L = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}}(r) + \frac{\Omega_B}{2} + U_0|\psi_L|^2 + (U_0 - 2U_1)|\psi_R|^2 \right.$$
$$\left. + i\left(\gamma_{\text{net}} - \Gamma|\psi_L|^2\right)\right]\psi_L + J_1\psi_R$$

Similarly for ψ_R with $\psi_L \leftrightarrow \psi_R$ and $\Omega_B \to -\Omega_B$.
Dimensionless cGPE:

$$i\partial_t\psi_L = \left[-\nabla^2 + v(r) + |\psi_L|^2 + (1-u_a)|\psi_R|^2 + \frac{\Delta}{2} + i\left(\alpha - \sigma|\psi_L|^2\right)\right]\psi_L + J\psi_R.$$

If $v(r) = r^2$ then take $\alpha \to \alpha\Theta(r_0 - r)$ as before.

Questions

1. Normal modes of uniform model: diffusive, linear, gapped.
2. Effect of Δ and J on vortices?
3. How does interconversion J interact with currents?
4. Synchronization/desynchronization.

Spinor Gross-Pitaevskii equation: [Borgh et al PRB, **81**, 235302 (2010)]

$$i\hbar\partial_t\psi_L = \left[-\frac{\hbar^2\nabla^2}{2m} + V_{\text{ext}}(r) + \frac{\Omega_B}{2} + U_0|\psi_L|^2 + (U_0 - 2U_1)|\psi_R|^2 + i\left(\gamma_{\text{net}} - \Gamma|\psi_L|^2\right)\right]\psi_L + J_1\psi_R$$

Similarly for ψ_R with $\psi_L \leftrightarrow \psi_R$ and $\Omega_B \to -\Omega_B$.
Dimensionless cGPE:

$$i\partial_t\psi_L = \left[-\nabla^2 + v(r) + |\psi_L|^2 + (1-u_a)|\psi_R|^2 + \frac{\Delta}{2} + i\left(\alpha - \sigma|\psi_L|^2\right)\right]\psi_L + J\psi_R.$$

If $v(r) = r^2$ then take $\alpha \to \alpha\Theta(r_0 - r)$ as before.

Questions:

1. Normal modes of uniform model: diffusive, linear, gapped.
2. Effect of Δ and J on vortices?
3. How does interconversion J interact with currents?
4. Synchronization/desynchronization.

$J = 0$: All $(\pm 1, 0)$ and $(\pm 1, \pm 1)$ vortex complexes are dynamically stable.

$J \neq 0, \Delta = 0$: Solutions $(+1, +1)$ are stable, $(\pm 1, 0)$ and $(+1, -1)$ are unstable.

$J = 0$: All $(\pm1, 0)$ and $(\pm1, \pm1)$ vortex complexes are dynamically stable.

$J \neq 0, \Delta = 0$: Solutions $(+1, +1)$ are stable, $(\pm1, 0)$ and $(+1, -1)$ are unstable.

$J = 0$: All $(\pm 1, 0)$ and $(\pm 1, \pm 1)$ vortex complexes are dynamically stable.

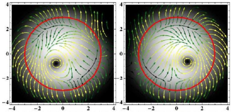
$J \neq 0, \Delta = 0$: Solutions $(+1, +1)$ are stable, $(\pm 1, 0)$ and $(+1, -1)$ are unstable.

Outcome of instability $\Delta = 0$

$J = 0$: All $(\pm 1, 0)$ and $(\pm 1, \pm 1)$ vortex complexes are dynamically stable.

$J \neq 0, \Delta = 0$: Solutions $(+1, +1)$ are stable, $(\pm 1, 0)$ and $(+1, -1)$ are unstable.

Outcome of instability $\Delta = 0$



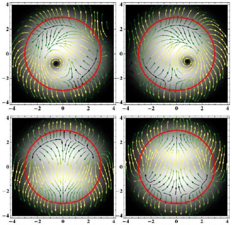
$J = 0.5$

Stability of cross-polarized vortices

$J = 0$: All $(\pm 1, 0)$ and $(\pm 1, \pm 1)$ vortex complexes are dynamically stable.

$J \neq 0, \Delta = 0$: Solutions $(+1, +1)$ are stable, $(\pm 1, 0)$ and $(+1, -1)$ are unstable.

Outcome of instability $\Delta = 0$



$J = 0.5$

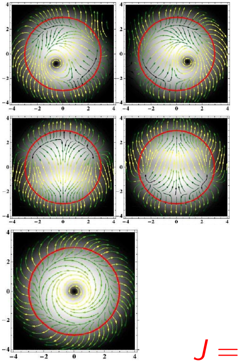
$J = 1$

Stability of cross-polarized vortices

$J = 0$: All $(\pm 1, 0)$ and $(\pm 1, \pm 1)$ vortex complexes are dynamically stable.

$J \neq 0, \Delta = 0$: Solutions $(+1, +1)$ are stable, $(\pm 1, 0)$ and $(+1, -1)$ are unstable.

Outcome of instability $\Delta = 0$



$J = 0.5$

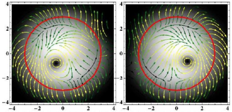
$J = 1$

$J = 1.5$

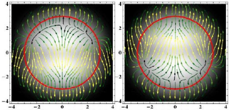
$J = 0$: All $(\pm 1, 0)$ and $(\pm 1, \pm 1)$ vortex complexes are dynamically stable.

$J \neq 0, \Delta = 0$: Solutions $(+1, +1)$ are stable, $(\pm 1, 0)$ and $(+1, -1)$ are unstable.

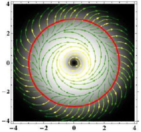
Outcome of instability $\Delta = 0$



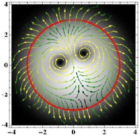
$J = 0.5$



$J = 1$



$J = 1.5$



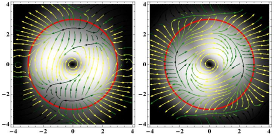
$J = 2$

Stability of cross-polarized vortices

$J = 0$: All $(\pm 1, 0)$ and $(\pm 1, \pm 1)$ vortex complexes are dynamically stable.

$J \neq 0, \Delta = 0$: Solutions $(+1, +1)$ are stable, $(\pm 1, 0)$ and $(+1, -1)$ are unstable.

$J \neq 0, \Delta \neq 0$: For a given J, any sufficiently large Δ allows the vortex complexes $(+1, -1)$ and $(\pm 1, 0)$ to stabilize.



$J = 1, \Delta = 8$

Two-mode system

Neglect $v(r)$ and spatial variations, write

$$\psi_{L,R} = \sqrt{\rho_{L,R}}\, e^{i(\phi \pm \theta/2)}, \quad R = \frac{\rho_L + \rho_R}{2}, \quad z = \frac{\rho_L - \rho_R}{2},$$

$$\dot{\theta} = -\Delta - 2u_a z + \frac{2Jz\cos(\theta)}{\sqrt{R^2 - z^2}}$$

$$\dot{z} = 2(\alpha - 2\sigma R)z - 2J\sqrt{R^2 - z^2}\sin(\theta)$$

$$\dot{R} = 2\sigma\left(\frac{\alpha}{\sigma}R - R^2 - z^2\right).$$

Two-mode system

Neglect $v(r)$ and spatial variations, write

$$\psi_{L,R} = \sqrt{\rho_{L,R}}e^{i(\phi\pm\theta/2)}, \quad R = \frac{\rho_L + \rho_R}{2}, \quad z = \frac{\rho_L - \rho_R}{2},$$

$$\dot{\theta} = -\Delta - 2u_a z + \frac{2Jz\cos(\theta)}{\sqrt{R^2 - z^2}}$$

$$\dot{z} = 2(\alpha - 2\sigma R)z - 2J\sqrt{R^2 - z^2}\sin(\theta)$$

$$\dot{R} = 2\sigma\left(\frac{\alpha}{\sigma}R - R^2 - z^2\right).$$

Neglect $v(r)$ and spatial variations, write

$$\psi_{L,R} = \sqrt{\rho_{L,R}} e^{i(\phi \pm \theta/2)}, \quad R = \frac{\rho_L + \rho_R}{2}, \quad z = \frac{\rho_L - \rho_R}{2},$$

$$\dot{\theta} = -\Delta - 2u_a z + \frac{2Jz\cos(\theta)}{\sqrt{R^2 - z^2}}$$

$$\dot{z} = 2(\alpha - 2\sigma R)z - 2J\sqrt{R^2 - z^2}\sin(\theta)$$

$$\dot{R} = 2\sigma\left(\frac{\alpha}{\sigma}R - R^2 - z^2\right).$$

Josephson regime $J \ll u_a R$ &
$z \ll R$;

Neglect $v(r)$ and spatial variations, write

$$\psi_{L,R} = \sqrt{\rho_{L,R}} e^{i(\phi \pm \theta/2)}, \quad R = \frac{\rho_L + \rho_R}{2}, \quad z = \frac{\rho_L - \rho_R}{2},$$

$$\dot{\theta} = -\Delta - 2u_a z + \frac{2Jz\cos(\theta)}{\sqrt{R^2 - z^2}}$$

$$\dot{z} = 2(\alpha - 2\sigma R)z - 2J\sqrt{R^2 - z^2}\sin(\theta)$$

$$\dot{R} = 2\sigma\left(\frac{\alpha}{\sigma}R - R^2 - z^2\right).$$

Josephson regime $J \ll u_a R$ &
$z \ll R$;

Neglect $v(r)$ and spatial variations, write

$$\psi_{L,R} = \sqrt{\rho_{L,R}} e^{i(\phi \pm \theta/2)}, \quad R = \frac{\rho_L + \rho_R}{2}, \quad z = \frac{\rho_L - \rho_R}{2},$$

$$\dot{\theta} = -\Delta - 2u_a z + \frac{2Jz\cos(\theta)}{\sqrt{R^2 - z^2}}$$

$$\dot{z} = 2(\alpha - 2\sigma R)z - 2J\sqrt{R^2 - z^2}\sin(\theta)$$

$$\dot{R} = 2\sigma\left(\frac{\alpha}{\sigma}R - R^2 - z^2\right).$$

Josephson regime $J \ll u_a R$ & $z \ll R$; $R = R_0 = \alpha/\sigma$

Neglect $v(r)$ and spatial variations, write

$$\psi_{L,R} = \sqrt{\rho_{L,R}}e^{i(\phi\pm\theta/2)}, \quad R = \frac{\rho_L + \rho_R}{2}, \quad z = \frac{\rho_L - \rho_R}{2},$$

$$\dot\theta = -\Delta - 2u_a z + \frac{2Jz\cos(\theta)}{\sqrt{R^2 - z^2}}$$

$$\dot z = 2(\alpha - 2\sigma R)z - 2J\sqrt{R^2 - z^2}\sin(\theta)$$

$$\dot R = 2\sigma\left(\frac{\alpha}{\sigma}R - R^2 - z^2\right).$$

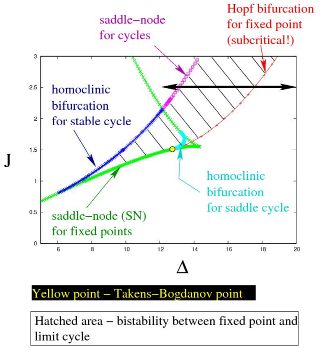
Josephson regime $J \ll u_a R$ & $z \ll R$; $R = R_0 = \alpha/\sigma$
Equation for a driven, damped pendulum

$$\ddot\theta + 2\alpha\dot\theta = -2\alpha\Delta + 4u_a J\frac{\alpha}{\sigma}\sin(\theta).$$

Neglect $v(r)$ and spatial variations, write

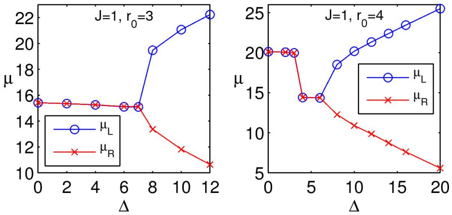
$$\psi_{L,R} = \sqrt{\rho_{L,R}}e^{i(\phi \pm \theta/2)}, \quad R = \frac{\rho_L + \rho_R}{2}, \quad z = \frac{\rho_L - \rho_R}{2},$$

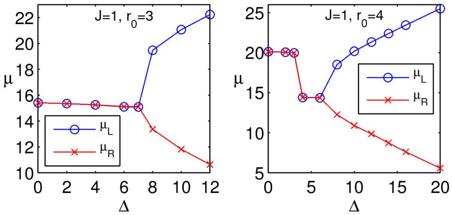
$$\dot{\theta} = -\Delta - 2u_a z + \frac{2Jz\cos(\theta)}{\sqrt{R^2 - z^2}}$$
$$\dot{z} = 2(\alpha - 2\sigma R)z - 2J\sqrt{R^2 - z^2}\sin(\theta)$$
$$\dot{R} = 2\sigma\left(\frac{\alpha}{\sigma}R - R^2 - z^2\right).$$



[with Balanov and Janson]

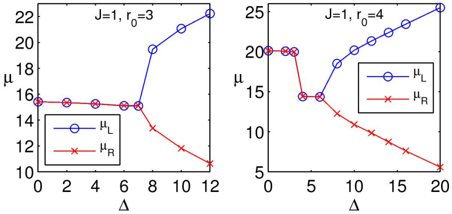
Trapped spinor system: $\mu_{L,R} = \mathrm{i}\partial_t \langle \ln \psi_{L,R} \rangle$ vs Δ.





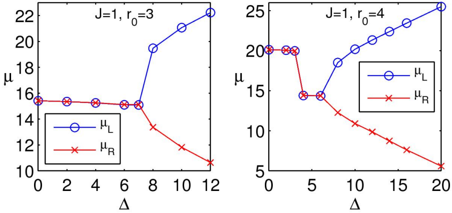
Simple case no vortices; $r_0 < r_{TF}$.

Trapped spinor system: $\mu_{L,R} = i\partial_t \langle \ln \psi_{L,R} \rangle$ vs Δ.



Simple case no vortices; $r_0 < r_{TF}$.

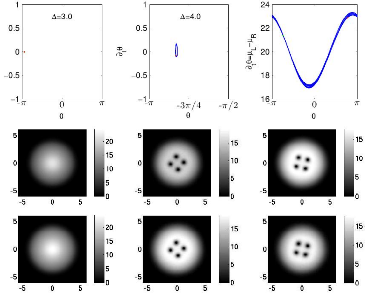
Marginal case $r_0 \sim r_{TF}$.
Δ causes $R(L)$ to grow (shrink).



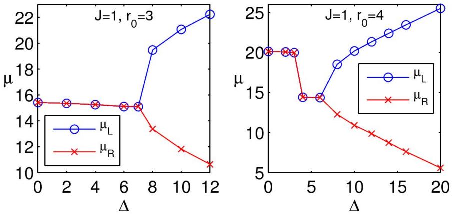
Simple case no vortices; $r_0 < r_{TF}$.

Marginal case $r_0 \sim r_{TF}$.

Δ causes $R(L)$ to grow (shrink).

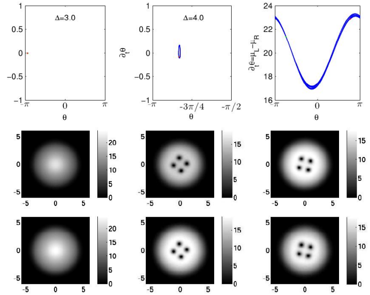


Trapped spinor system: $\mu_{L,R} = \mathrm{i}\partial_t \langle \ln \psi_{L,R} \rangle$ vs Δ.

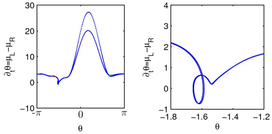


Simple case no vortices; $r_0 < r_{TF}$.

Marginal case $r_0 \sim r_{TF}$.
Δ causes $R(L)$ to grow (shrink).

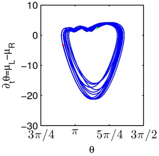


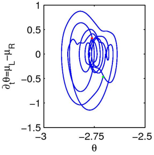
"Simple case" not so simple: retrograde loop

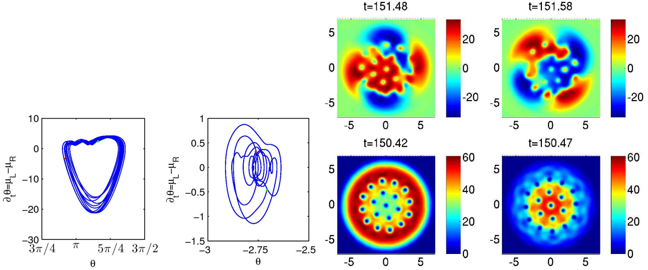


Full two-component model

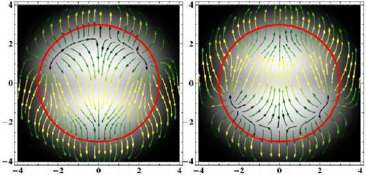
- Full model with a trap confirms the predictions of two-mode model, but has richer behaviour:

- Full model with a trap confirms the predictions of two-mode model, but has richer behaviour:
    - Phase portraits: fixed points, limit cycles with winding 0, 1, 2; retrograde loops, quasi-periodic and chaotic behaviours
    - 



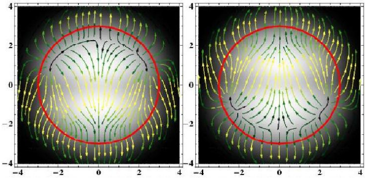
- Full model with a trap confirms the predictions of two-mode model, but has richer behaviour:
  - Phase portraits: fixed points, limit cycles with winding 0, 1, 2; retrograde loops, quasi-periodic and chaotic behaviours
  - Counter-rotating lattices; spatially non-uniform interconversions...
  - 

Stationary density depletion for intermediate J and small Δ



$\Delta = 0$

Stationary density depletion for intermediate J and small Δ



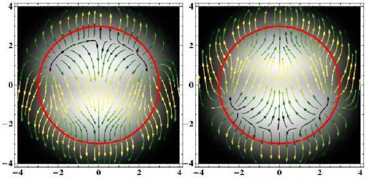
$\Delta = 0$

Density depletions appear in trapped and uniform equilibrium condensates: dark/black/grey solitons; rarefaction waves;

Travelling hole solutions of the complex Ginzburg–Landau equations: e.g. Nozaki–Bekki solutions

Are these relevant?

Stationary density depletion for intermediate J and small Δ



$\Delta = 0$

Density depletions appear in trapped and uniform equilibrium condensates: dark/black/grey solitons; rarefaction waves;
Travelling hole solutions of the complex Ginzburg–Landau equations: e.g. Nozaki–Bekki solutions

Are these relevant?

From simulations $\psi_L(x, y) = \psi_R(x, -y)$, so this stationary state satisfies

$$i\partial_t\psi = [-\nabla^2 + r^2 + |\psi|^2 + \mathrm{i}(\alpha\Theta(r_0 - r) - \sigma|\psi|^2)]\psi + J\psi(x, -y).$$

One-dimensional modified GL equation

Consider solutions of a modified GL equation without trap

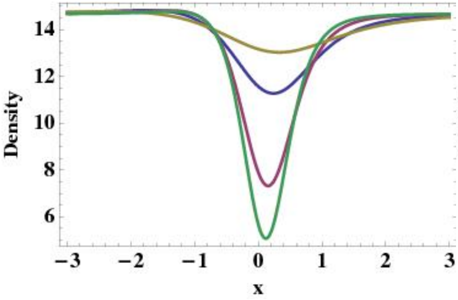
$$i\partial_t \psi = -\psi_{xx} + |\psi|^2\psi + \mathrm{i}(\alpha - \sigma|\psi|^2)\psi + J\psi(-x).$$

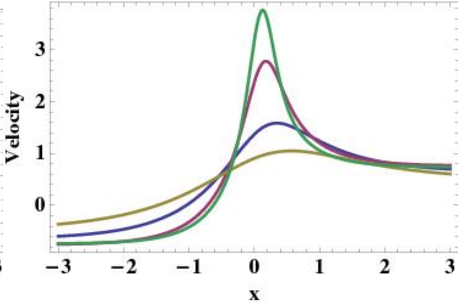
One-dimensional modified GL equation

Consider solutions of a modified GL equation without trap

$$i\partial_t\psi = -\psi_{xx} + |\psi|^2\psi + \mathrm{i}(\alpha - \sigma|\psi|^2)\psi + J\psi(-x).$$

Stationary solutions exist for $0 < J < J_{cr}$. Black soliton evolves into these states.



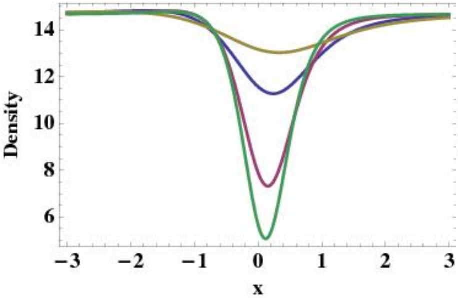


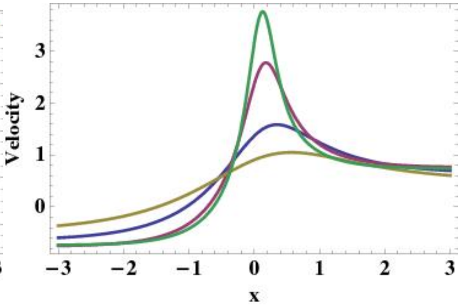
One-dimensional modified GL equation

Consider solutions of a modified GL equation without trap

$$i\partial_t\psi = -\psi_{xx} + |\psi|^2\psi + \mathrm{i}(\alpha - \sigma|\psi|^2)\psi + J\psi(-x).$$

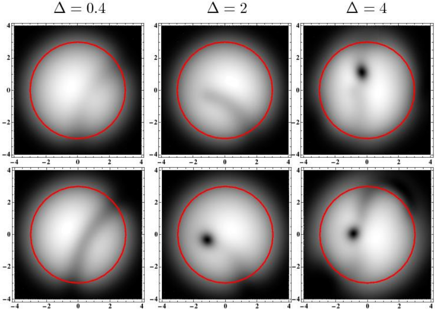
Stationary solutions exist for $0 < J < J_{cr}$. Black soliton evolves into these states.





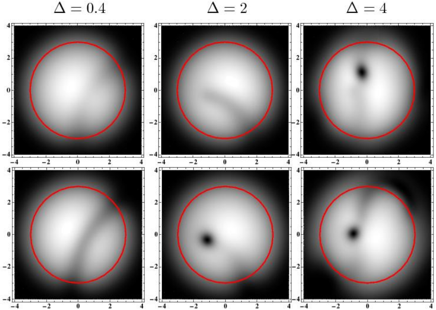
Note: For Nozaki–Bekki holes $J = 0$ but one needs diffusion $i\psi_{xx}$ (spectral filtering to stabilize the central frequency of the pulse)

Densities of L and R components for $J = 1$

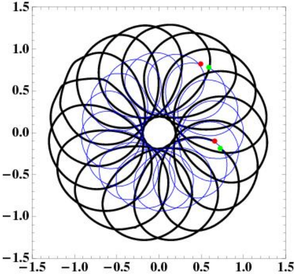


Similarly complicated cycloid trajectories of vortices are known for two-layer fluids with one vortex in each layer — e.g. in models of tropical vortices. Reaction difusion equations may lead to spiral wave dynamics.

Densities of L and R components for $J = 1$



Trajectories for $\Delta = 4$



Spirographs (epitrochoids/hypotrochoid)

Similarly complicated cycloid trajectories of vortices are known for two-layer fluids with one vortex in each layer — e.g. in models of tropical vortices. Reaction difusion equations may lead to spiral wave dynamics.

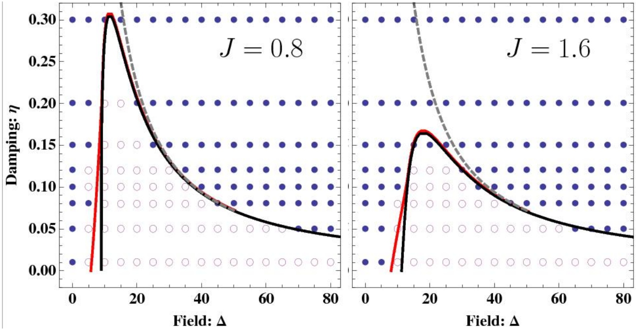
[J.Keeling and NB, arXiv:1102.5302]
Vortex patterns generated by superposition of fluxes.
Spinor complex Ginzburg-Landau equation:

$$2i\partial_t\psi_{l,r} = \left[\pm\frac{\Delta}{2} - \nabla^2 + v(r) + |\psi_{l,r}|^2 + (1-u_a)|\psi_{r,l}|^2 \right.$$
$$\left. +i\left(\alpha - 2i\eta\partial_t - \sigma|\psi_{l,r}|^2 - \tau|\psi_{r,l}|^2\right)\right]\psi_{l,r} + J\psi_{r,l}.$$

η – energy relaxation [Wouters and Savona arXiv:1007.5431 (2010)];
τ – cross-spin nonlinear dissipation;
Δ – effect of the magnetic fied (in Hamiltonian $\sim \Delta(|\psi_r|^2 - |\psi_l|^2)$);
J – electric field, stress or due to asymmetry of quantum well interfaces;

Magnetic field, Δ, drives the transition from synchronized to desyncronized regimes for $\eta = \tau = 0$.

For nonzero η there is a second transition at Δ_{c2} back to synchronized state, $\Delta_{c2} \simeq (2\alpha/\eta)(\sigma - \tau + \eta u_a)/(\sigma + \tau + \eta(2 - u_a))$ (dashed line)



• –synchronized states (vortex-free states or synchronized vortices);
○– desynchronized states (vortices of opposite sign for l and r).
Conclude: homogeneous model gives good prediction of spatially varying system.

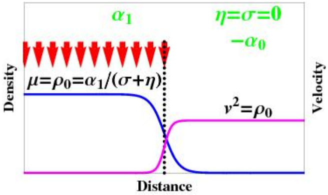
Vortex formation

Vortex formation in equilibrium condensates:

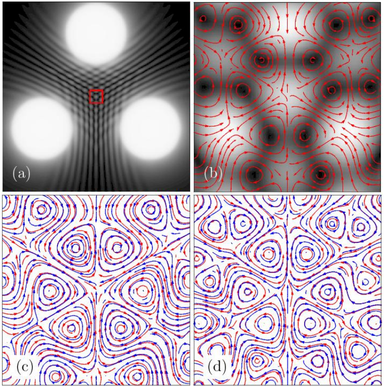
- interactions of finite amplitude sound waves;
- existence of critical velocities of the flow;
- modulational instabilities.

In addition in nonequilibrium condensates – pattern forming, interaction of fluxes with a disorder etc.

Vortex formation due to interference of supercurrents

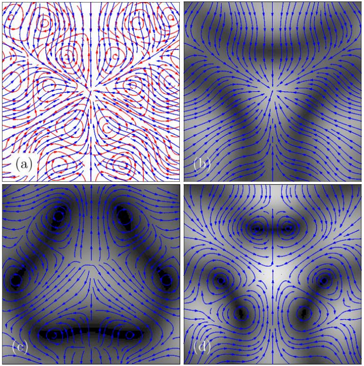


Analytical solution for the velocity $u(r)$ on $\infty < r < \infty$.



(a) $\Delta = 0$ showing geometry of pumping;
(b) Desynchronized $\Delta = 20$ steady majority density with streamlines;
(c) Lower synchronized $\Delta = 5$ steamlines of both polarizations;
(d) Upper synchronized $\Delta = 40$ steamlines of both polarizations.

"Half-vortices" have been seen in experiments:
[Lagoudakis et al Nature Phys. (2009)]
Are "half-vortices" pinned and stabilized by disorder?



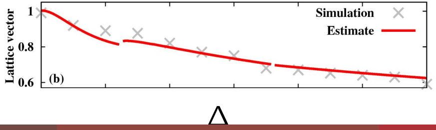
(a) Desyncronized $\Delta = 20$ half-vortex lattice;
(b) -(c) -(d) evolution of minority component in desyncronized regime $\Delta = 20$.

Majority component is stationary in both regimes;
Minority component is stationary in syncronized regime only.
In desyncronized regime averages to vortex-free state.

Currents are neglible at the pumping centre, $\mu(\rho_{l,r})$;
away from pumping spot – densities are neglible.

Synchronized regime: away from the pump
$\mu - |\vec{u}|^2 \mp \Delta/2 = J\left(\rho_l/\rho_r\right)^{\mp 1/2}\cos(\theta)$ and
$\nabla \cdot (\rho_{l,r}\vec{u}) + \alpha_1 \rho_{l,r} = \mp J\sqrt{\rho_l \rho_r}\sin(\theta)$.
These are solved by $\sin(\theta) = 0$ and $\nabla(\rho_l/\rho_r) = 0$,
so $|\vec{u}|^2 = \mu + \sqrt{J^2 + \Delta^2/4}$.

Desynchronized regime: θ and ρ_l/ρ_r are not time independent, so one calculates averages. If $\rho_r \gg \rho_l$, then for majority component
$\langle|\vec{u}_r|^2\rangle = \langle\mu_r\rangle + \Delta/2$.
Superposition of such currents results in hexagonal vortex lattice with spacing $l = (2\pi/|\vec{u}|) \times 2/3\sqrt{3}$.



Classical Turbulence

In 50th Batchelor wrote to his friend and close colleague, Alan Townsend, who remained in Australia:

You will come to Cambridge, study turbulence, and work with G. I. Taylor.

The answer came immediately: *I agree, but I have two questions:*

who is G. I. Taylor and ... **what is turbulence?**

Classical Turbulence

In 50th Batchelor wrote to his friend and close colleague, Alan Townsend, who remained in Australia:

You will come to Cambridge, study turbulence, and work with G. I. Taylor.

The answer came immediately: *I agree, but I have two questions: who is G. I. Taylor and ...* **what is turbulence?**

Classical Turbulence

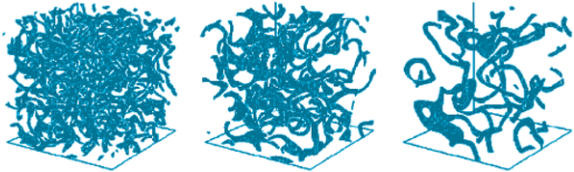
In 50th Batchelor wrote to his friend and close colleague, Alan Townsend, who remained in Australia:

You will come to Cambridge, study turbulence, and work with G. I. Taylor.

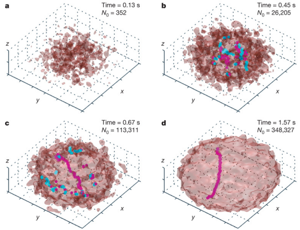
The answer came immediately: *I agree, but I have two questions: who is G. I. Taylor and ...* **what is turbulence?**

Classical turbulence – cascading vorticity;
Superfluid turbulence – quantisation of velocity circulation – differences with classical turbulence;
Strong turbulence– unstructured vortices (distance between vortices of the order of their core);
Weak turbulence regime – almost independent motion of weakly interacting dispersive waves.

Stages in condensate formation from a nonequilibrium state:
[Berloff & Svistunov Phys Rev A (2002)]
weak turbulence → strong turbulence → superfluid turbulence → condensate

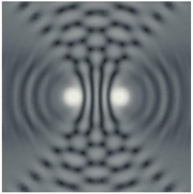


Vortex formed during nonequilibrium kinetics of BEC
[Weiler et al. Nature (2008)]

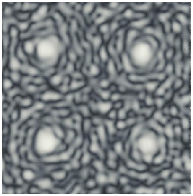


Reverse the process going from condensate to weak turbulent state?
[Henn at el PRL (2009)]: applied an external oscillatory perturbation to produce vortices.

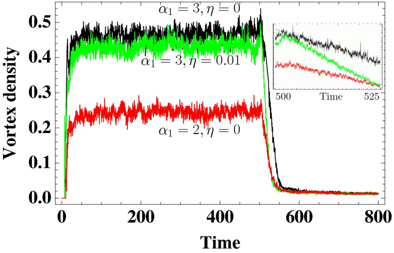
Regular emission of vortices



Many irregular spots: turbulence

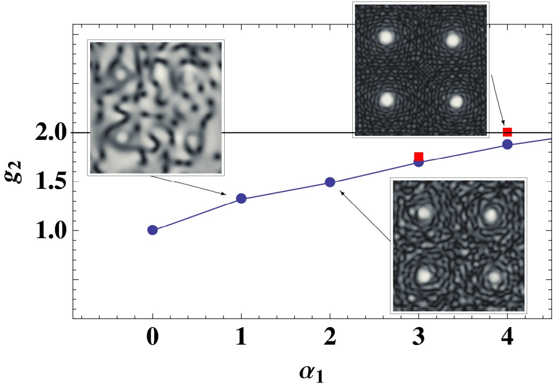


Two regimes: forced turbulence and turbulence decay.



Weak turbulence

In forced turbulence it is possible to reach a weak turbulence state: $g_2 = \langle|\psi|^4\rangle/\langle|\psi|^2\rangle^2$. Weak turbulence implies $g_2 \sim 2$.



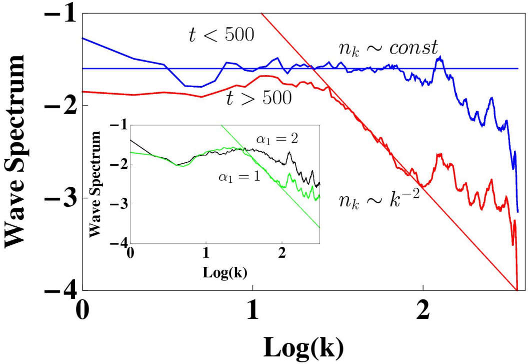
Red Squares – nonzero η facilitates the transition to weak turbulence.

Assume
(i) the existence of inertial range in the momentum space;
(ii) neglect pumping and dissipation there.

Weak turbulence theory
[Zhakharov et al (1992); Salman and Berloff, Physica D (2009)]:

Main idea:
Use random phase approximation to obtain evolution equation for the wave spectrum $\left\langle a_{\mathbf{k}_1} a^*_{\mathbf{k}_2} \right\rangle = n_{\mathbf{k}_1}\delta(\mathbf{k}_1 - \mathbf{k}_2)$,
$a_{\mathbf{k}}$ – the Fourier transform of ψ and $\mathbf{k}_i$ are discrete wave vectors.
$\partial_t n_{\mathbf{k}_1}(t) =$
$\int d^2k_2 d^2k_3 d^2k_4 W_{k_1,k_2;k_3,k_4}\left(n_{\mathbf{k}_3}n_{\mathbf{k}_4}n_{\mathbf{k}_1} + n_{\mathbf{k}_3}n_{\mathbf{k}_4}n_{\mathbf{k}_2} - n_{\mathbf{k}_1}n_{\mathbf{k}_2}n_{\mathbf{k}_3} - n_{\mathbf{k}_1}n_{\mathbf{k}_2}n_{\mathbf{k}_4}\right)$,
where $W_{k_1,k_2;k_3,k_4} = \frac{4\pi}{(2\pi)^2}\delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4)\delta(k_1^2 + k_2^2 - k_3^2 - k_4^2)$

Two solutions:
(i) Equipartition of the total kinetic energy $E = \int k^2 n_k \, d\mathbf{k}$, so that $n_k \sim k^{-2}$;
(ii) Equipartition of the total number of particles $N = \int n_k \, d\mathbf{k}$, so that $n_k \sim \mathrm{const}$.



Conclusions-1

- Nonequilibrium condensates: condensates made of light
    - Gross-Pitaevskii equation with loss and gain

$$\mathrm{i}\partial_t\psi = \left[-\nabla^2 + r^2 + |\psi|^2 + i\left(\alpha\Theta(r_0 - r) - \sigma|\psi|^2\right)\right]\psi.$$

    - Radially symmetric stationary states: narrowing of density profile
    - Spiral vortex states
    - Vortex lattices

- Nonequilibrium condensates: condensates made of light
  - Gross-Pitaevskii equation with loss and gain

$$\mathrm{i}\partial_t \psi = \left[-\nabla^2 + r^2 + |\psi|^2 + i\left(\alpha\Theta(r_0 - r) - \sigma|\psi|^2\right)\right]\psi.$$

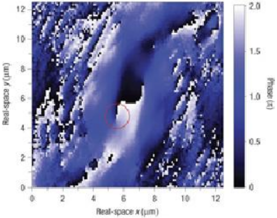
  - Radially symmetric stationary states: narrowing of density profile
  - Spiral vortex states
  - Vortex lattices

Conclusions-1

- Nonequilibrium condensates: condensates made of light
  - Gross-Pitaevskii equation with loss and gain

$$\mathrm{i}\partial_t \psi = \left[-\nabla^2 + r^2 + |\psi|^2 + i\left(\alpha\Theta(r_0 - r) - \sigma|\psi|^2\right)\right]\psi.$$

  - Radially symmetric stationary states: narrowing of density profile
  - Spiral vortex states

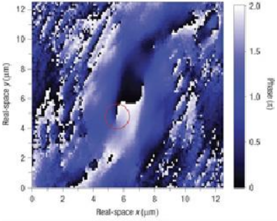


  - Vortex lattices

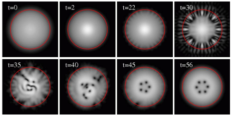
- Nonequilibrium condensates: condensates made of light
  - Gross-Pitaevskii equation with loss and gain

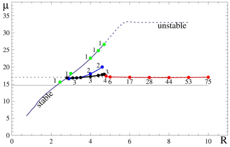
$$\mathrm{i}\partial_t\psi = \left[-\nabla^2 + r^2 + |\psi|^2 + i\left(\alpha\Theta(r_0 - r) - \sigma|\psi|^2\right)\right]\psi.$$

  - Radially symmetric stationary states: narrowing of density profile
  - Spiral vortex states



  - Vortex lattices





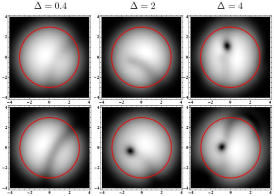
Conclusions-2

- Non-equilibrium spinor system

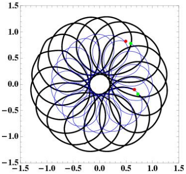
$$i\partial_t\psi_L = \Big[-\nabla^2 + V(r) + \frac{\Delta}{2} + |\psi_L|^2 + (1-u_a)|\psi_R|^2 + i\left(\alpha\Theta(r_0 - r) - \sigma|\psi_L|^2\right)\Big]\psi_L + J\psi_R$$

- Effect of Δ and J on vortices.

Densities of L and R components for $J = 1$



Trajectories for $\Delta = 4$



Spirographs (epitrochoids/hypotrochoid)

- Synchronization/desynchronization with the region of bistability.

Conclusions-3

- Turbulence in exciton-polariton condensates may lead to novel regimes of turbulence of classical matter field.
  - The regimes can be distinguished by finding second order correlation function; by looking at the wave spectrum.
  - What are the stages in transition from strong turbululence to weak turbulence and back?
- Spinor condensates: predictions of homogeneous model (syncronization/desynchronization) are not significantly modified by spatial inhomogeneity.
  - Observation of the experimental behaviour in an applied field can thus be used to distinguish the the loss nonlinearities σ, τ and η.
  - Vortices, vortex lattices and half-vortex latices in spinor condensates. Being stationary these textures can be studied experimentally.
- Turbulence in spinor condensates.

Scaling laws? Cross-overs of different regimes? Interplay between turbulent regimes and the effects of magnetic field?...