Brillouin zone pictures

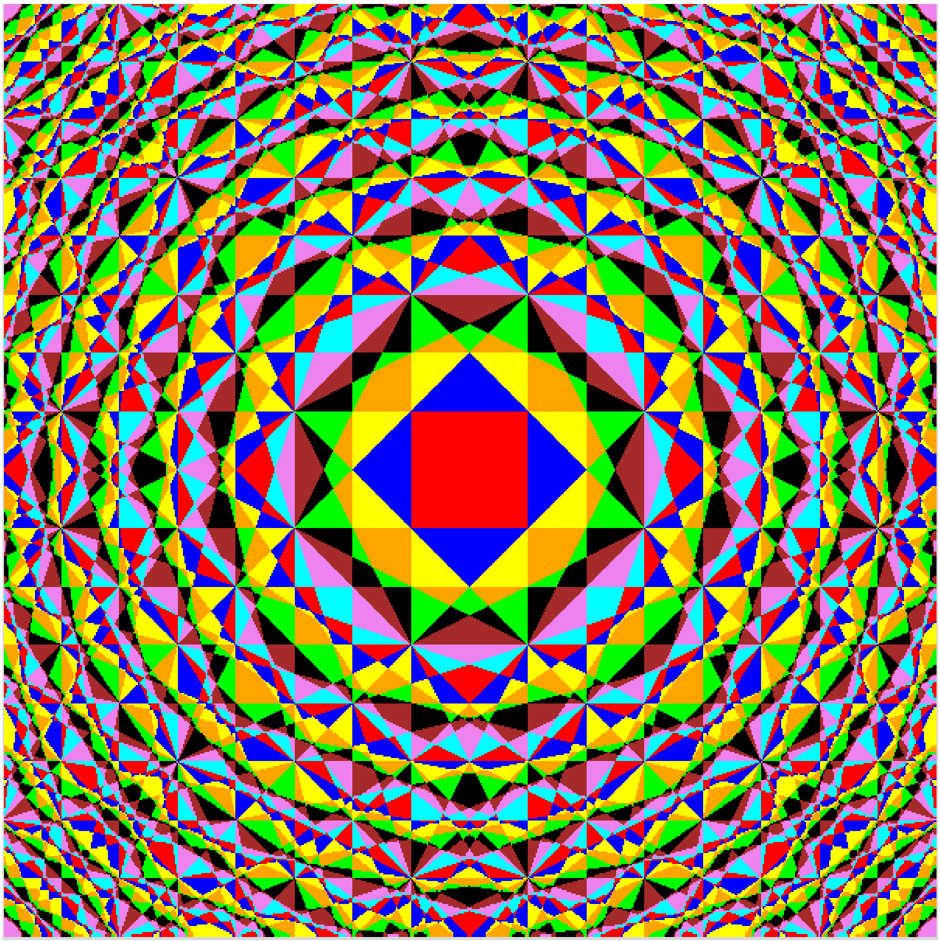

Figure 1: The Brillouin zones for a cubic 2D lattice. The red centre zone is B_1 and get to the others by counting out from B_1.

In Figure 1 I have constructed a large number of the Brillouin zones for the 2D cubic lattice.

In Figures 2 and 3 I show a few different 2D Brillouin zone pictures. To construct my pictures I used that for all $\boldsymbol{k} \in B_n$ the origin is the n-th nearest point in Λ^* to $\boldsymbol{k}$. This is the same as using the result that if $\boldsymbol{k}$ lies in a boundary of a Brillouin zone then $k^2 = |\boldsymbol{k} + \boldsymbol{q}|^2$, for some $\boldsymbol{q} \in \Lambda^*$. This is the same as defining the boundaries using the Bragg formula: $2\boldsymbol{k} \cdot \boldsymbol{q} - \boldsymbol{q} \cdot \boldsymbol{q} = 0$ with $\boldsymbol{q} \in \Lambda^*$. The bases I used for Λ^* are, respectively,

cubic	$\boldsymbol{b}_1 = (1, 0)$	$\boldsymbol{b}_2 = (0, 1)$
hexagonal	$\boldsymbol{b}_1 = (1, 0)$	$\boldsymbol{b}_2 = (1/2, \sqrt{3}/2)$
cubic	$\boldsymbol{b}_1 = (1.5, 0)$	$\boldsymbol{b}_2 = (0, 1)$

There is a very good website from Material Science which has a lot of stuff. The website is: http://www.doitpoms.ac.uk/tlplib. Go to the section on Brillouin zones. In particular, there are animations showing the construction of the Brillouin zones using the Bragg formula given above. Also, especially look at the **zone folding** section. This is animated and shows how to construct the reduced zone scheme from the extended scheme in 2D by translating $B_n,\ n > 1$ by $\boldsymbol{q} \in \Lambda^*$ to fit into the Voronoi cell (i.e. the region covered by B_1).

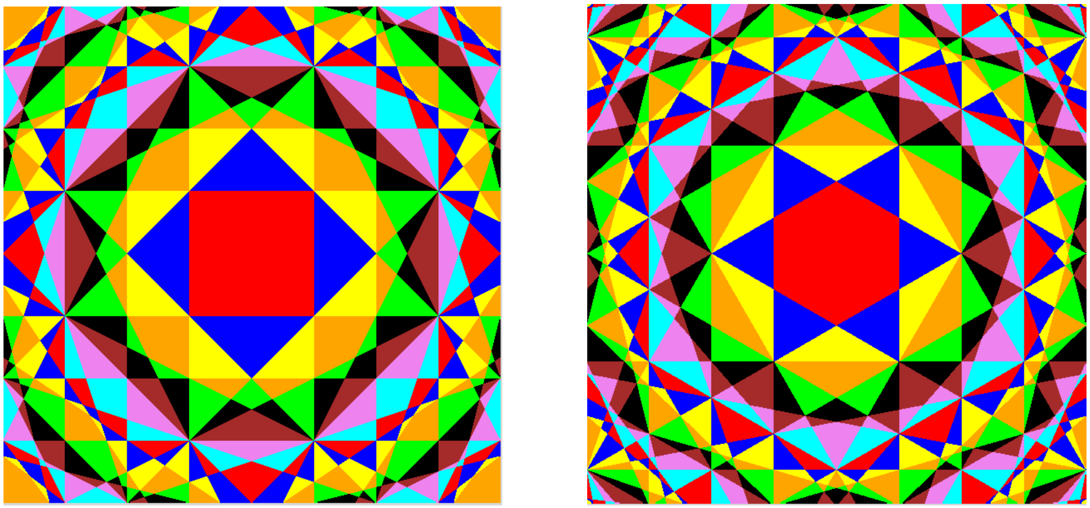

Figure 2: The Brillouin zones for a cubic 2D lattice (left) and the hexagonal lattice (right). The red centre zone is B_1 and get to the others by counting out from B_1.

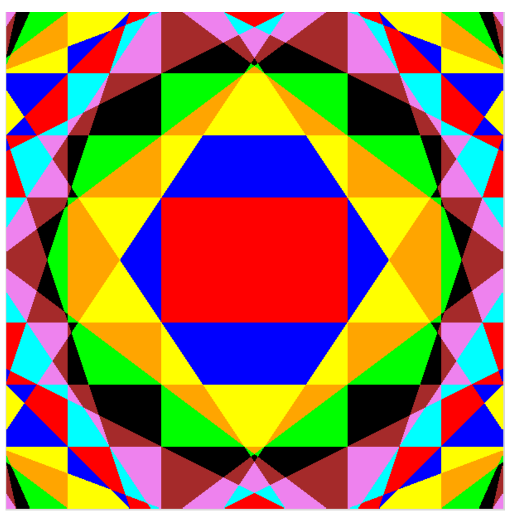

Figure 3: The Brillouin zones for a rectangular 2D lattice. The red centre zone is B_1 and get to the others by counting out from B_1