3P1d **Quantum Field Theory: Example Sheet 4** Michaelmas 2019

Corrections and suggestions should be emailed to B.C.Allanach@damtp.cam.ac.uk. *Starred questions may be handed in to your supervisor for feedback prior to the class if you wish.*

1*. The Lagrangian density for a fermionic Yukawa theory is given by

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 + \bar{\psi}(i\,\partial\!\!\!/ - m)\psi - \lambda\phi\bar{\psi}\psi. \tag{1}$$

(a) Consider $\psi\bar{\psi} \to \psi\bar{\psi}$ scattering, with initial and final states given by

$$|i\rangle = \sqrt{4E_{\vec{p}}E_{\vec{q}}}\, b_{\vec{p}}^{s\dagger} c_{\vec{q}}^{r\dagger}\,|0\rangle \tag{2}$$

$$|f\rangle = \sqrt{4E_{\vec{p}}E_{\vec{q}}}\, b_{\vec{p}'}^{s'\dagger} c_{\vec{q}'}^{r'\dagger}\,|0\rangle\,. \tag{3}$$

Show that the amplitude is given by

$$\mathcal{A} = -(-i\lambda)^2\left(\frac{[\bar{u}^{s'}(\vec{p'})\cdot u^s(\vec{p})][\bar{v}^r(\vec{q})\cdot v^{r'}(\vec{q'})]}{t-\mu^2} - \frac{[\bar{v}^r(\vec{q})\cdot u^s(\vec{p})][\bar{u}^{s'}(\vec{p'})\cdot v^{r'}(\vec{q'})]}{s-\mu^2}\right). \tag{4}$$

where $t = (p-p')^2$ and $s = (p+q)^2$ are the usual Mandelstam variables. Draw the two Feynman diagrams that correspond to these two terms.

(b) Now consider $\psi(p,s)\phi(q) \to \psi(p',s')\phi(q')$ scattering. Show that the amplitude is given by

$$\mathcal{A} = (-i\lambda)^2\left(\frac{[\bar{u}^{s'}(\vec{p'})\gamma^\mu\left((p_\mu - q'_\mu)+m\right)u^s(\vec{p})]}{u-m^2} + \frac{[\bar{u}^{s'}(\vec{p'})\gamma^\mu\left((p_\mu + q_\mu)+m\right)u^s(\vec{p})]}{s-m^2}\right)$$

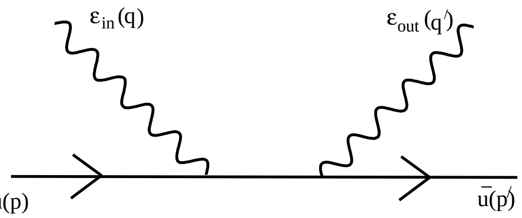
Draw any Feynman diagrams that contribute.

2. The Lagrangian density for a pseudoscalar Yukawa interaction is given by

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{2}\mu^2\phi^2 + \bar{\psi}(i\,\partial\!\!\!/ - m)\psi - \lambda\phi\bar{\psi}\gamma^5\psi\,. \tag{5}$$

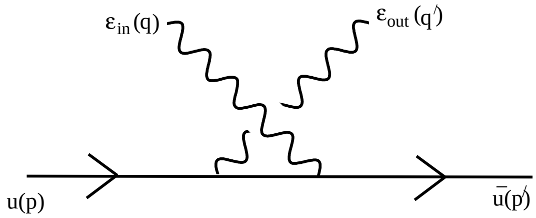
Write down the Feynman rules for this theory. Use these to write down the amplitude at order λ^2 for $\psi\psi \to \psi\psi$ scattering and $\psi\bar{\psi} \to \psi\bar{\psi}$ scattering.

3. Consider Compton scattering in which a photon and an electron scatter off each other. Let the incoming photon have polarisation vector ϵ^μ_{in} and the outgoing photon have polarisation $\epsilon^\mu_{\text{out}}$. Use the Feynman rules to derive the following amplitude associated to the lowest order diagram,



$$= i(-ie)^2\bar{u}^{r'}(\vec{p'})\,\epsilon\!\!/_{\text{out}}\frac{p\!\!\!/ + q\!\!\!/ + m}{s-m^2}\,\epsilon\!\!/_{\text{in}}u^s(\vec{p}). \tag{6}$$

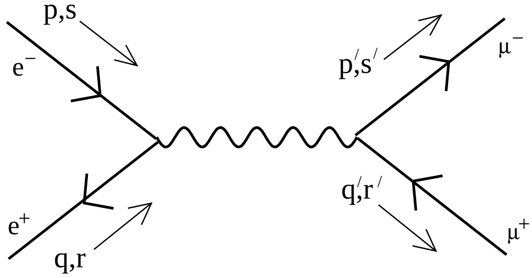
Also, compute the contribution from the diagram



The complete amplitude at order e^2 is the sum of these two contributions. Show that the total amplitude vanishes if ϵ_{in} is replaced by the incoming photon momentum q. Check that the same holds true if ϵ_{out} is replaced by q'. Finally, calculate $d\sigma/dt$ in the massless electron limit for Compton scattering in terms of Mandelstam variables. *Note: it will be helpful to recall the equation* $\left(\not{p} - m \right) u(\vec{p}) = 0$ *satisfied by the spinor.*

4. Take QED coupled to a charge 1 complex scalar ϕ. Write down the Lagrangian, and hence derive the Feynman rules for any interactions between ϕ, $\phi^\dagger$ and the photon A^μ.

5. Use the Feynman rules to show that the QED amplitude for $e^+e^- \to \mu^+\mu^-$ is given at lowest order in e by



$$= (-ie)^2 \frac{[\bar{v}_e^r(\vec{q})\gamma_\mu u_e^s(\vec{p})][\bar{u}_m^{s'}(\vec{p'})\gamma^\mu v_m^{r'}(\vec{q'})]}{s}, \quad (7)$$

where the subscripts e and m denote whether the spinors satisfy the Dirac equation for electrons or for muons, respectively and s is the usual Mandelstam variable.

6*. Calculate the total massless-fermion spin-averaged cross-section at leading order for the process $e^+e^- \to \mu^+\mu^-$,

$$\sigma_{QED} = \frac{4\pi\alpha^2}{3s}$$

where s is the usual Mandelstam variable, fermion masses have been neglected and $\alpha = e^2/(4\pi)$. This agrees with experimental data:

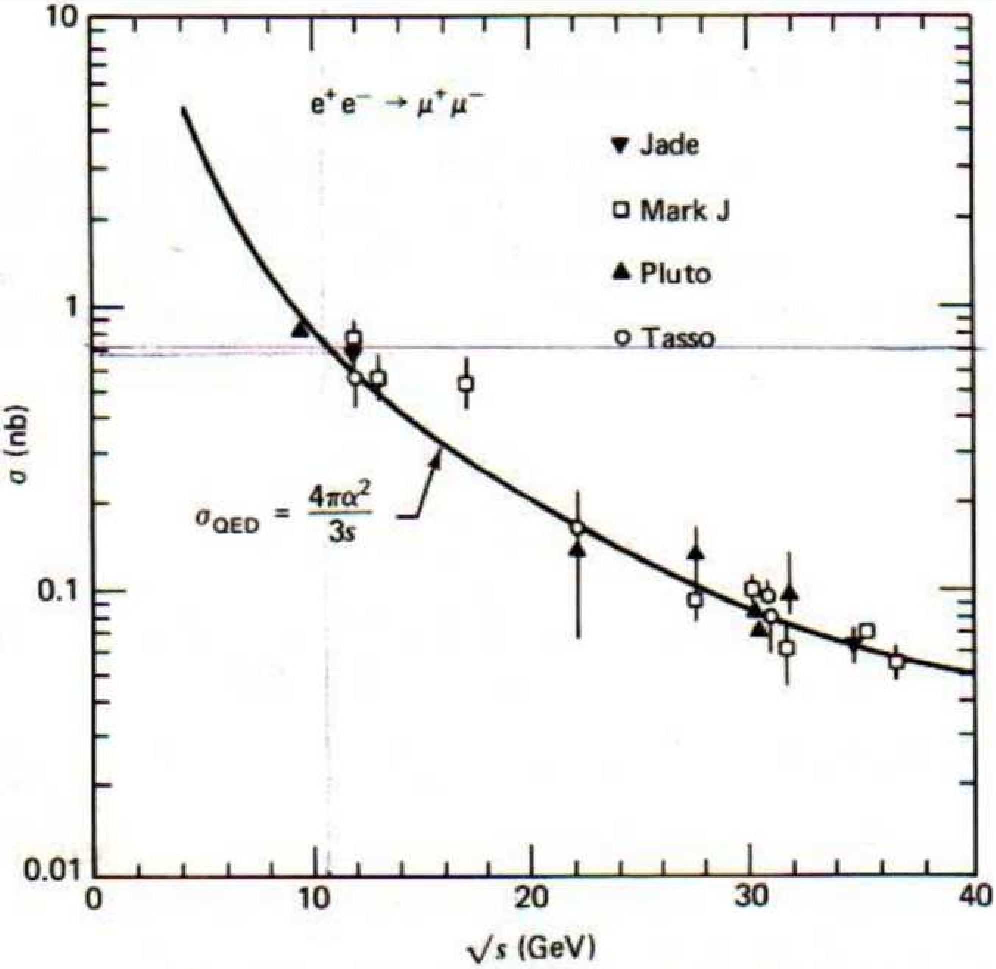


Fig. 6.6 The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

Draw a Feynman diagram at the next higher order than the one depicted in question 5. If this diagram were included in the calculation, the expression for σ would be corrected to which order in $\alpha \approx 1/137$? How much would this change the curve?