

# Contents

|                                                             |               |
|-------------------------------------------------------------|---------------|
| <i>Preface</i>                                              | <i>page</i> 1 |
| <b>1 Newtonian Mechanics</b>                                | 6             |
| 1.1 Newton's Laws of Motion                                 | 7             |
| 1.1.1 Newton's First Law                                    | 8             |
| 1.1.2 Galilean Relativity                                   | 9             |
| 1.1.3 Newton's Second Law                                   | 12            |
| 1.1.4 Looking Forwards: The Validity of Newtonian Mechanics | 14            |
| <b>2 Forces</b>                                             | 16            |
| 2.1 Potentials in One Dimension                             | 16            |
| 2.1.1 Moving in a Potential                                 | 19            |
| 2.1.2 Why (Almost) Everything is a Harmonic Oscillator      | 22            |
| 2.2 Potentials in Three Dimensions                          | 25            |
| 2.2.1 Conservative Forces                                   | 25            |
| 2.2.2 Work Done                                             | 26            |
| 2.2.3 Central Forces and Angular Momentum Conservation      | 28            |
| 2.3 Gravity                                                 | 29            |
| 2.3.1 The Gravitational Field                               | 31            |
| 2.3.2 Escape Velocity                                       | 34            |
| 2.3.3 Inertial vs Gravitational Mass                        | 36            |
| 2.4 Electromagnetism                                        | 36            |
| 2.4.1 Motion in a Constant Magnetic Field                   | 38            |
| 2.4.2 The Electric Field of a Point Charge                  | 42            |
| 2.5 Friction                                                | 44            |
| 2.5.1 The Damped Harmonic Oscillator                        | 48            |
| 2.5.2 The Damped, Driven Harmonic Oscillator                | 50            |
| 2.5.3 Terminal Velocity                                     | 52            |
| 2.5.4 Ohm's Law                                             | 55            |
| 2.5.5 Three-Dimensional Motion with Linear Drag             | 56            |
| 2.6 Appendix: Two Digressions                               | 58            |
| 2.6.1 Line Integrals and Conservative Forces                | 58            |
| 2.6.2 The Poisson Equation                                  | 60            |
| <b>3 Interlude: Dimensional Analysis</b>                    | 65            |

|          |                                                   |     |
|----------|---------------------------------------------------|-----|
| <b>4</b> | <b>Systems of Particles</b>                       | 76  |
| 4.1      | Interactions Between Particles                    | 76  |
| 4.1.1    | Newton's Third Law                                | 77  |
| 4.1.2    | Centre of Mass Motion                             | 78  |
| 4.1.3    | Angular Momentum                                  | 79  |
| 4.1.4    | Energy                                            | 80  |
| 4.2      | Collisions                                        | 84  |
| 4.2.1    | Bouncing Balls                                    | 85  |
| 4.2.2    | More Bouncing Balls and the Digits of $\pi$       | 86  |
| 4.3      | Variable Mass Problems                            | 88  |
| 4.3.1    | Rockets: Things Fall Apart                        | 89  |
| 4.3.2    | Avalanches: Stuff Gathering Other Stuff           | 93  |
| <b>5</b> | <b>The Two-Body Problem</b>                       | 95  |
| 5.1      | Setting the Scene                                 | 95  |
| 5.1.1    | The Two-Body Problem is Really a One-Body Problem | 96  |
| 5.2      | Conservation of Angular Momentum                  | 97  |
| 5.2.1    | Polar Coordinates in the Plane                    | 98  |
| 5.3      | The Effective Potential                           | 99  |
| 5.3.1    | Getting a Feel for Orbits                         | 102 |
| 5.3.2    | The Stability of Circular Orbits                  | 104 |
| 5.4      | Orbits                                            | 106 |
| 5.4.1    | The Kepler Problem                                | 107 |
| 5.4.2    | Kepler's Laws of Planetary Motion                 | 112 |
| 5.4.3    | The Runge-Lenz Vector                             | 114 |
| 5.4.4    | Perihelion Precession in General Relativity       | 116 |
| 5.5      | A Brief History of Newton's Time                  | 118 |
| 5.5.1    | What Would Newton Do?                             | 120 |
| 5.6      | Scattering: Throwing Stuff at Other Stuff         | 129 |
| 5.6.1    | The Impact Parameter                              | 129 |
| 5.6.2    | Rutherford Scattering                             | 131 |
| 5.6.3    | A First Look at the Cross-Section                 | 132 |
| 5.6.4    | A Bit More History: The Discovery of the Nucleus  | 134 |
| <b>6</b> | <b>Rotating Reference Frames</b>                  | 136 |
| 6.1      | Rotating Frames                                   | 136 |
| 6.1.1    | Velocity and Acceleration in a Rotating Frame     | 138 |
| 6.2      | Newton's Equation of Motion in a Rotating Frame   | 139 |
| 6.3      | The Centrifugal Force                             | 141 |
| 6.3.1    | An Example: Apparent Gravity                      | 141 |
| 6.3.2    | The Roche Limit                                   | 144 |
| 6.4      | The Coriolis Force                                | 145 |
| 6.4.1    | Particles, Baths, and Hurricanes                  | 146 |
| 6.4.2    | Balls and Towers                                  | 148 |

|          |                                                  |            |
|----------|--------------------------------------------------|------------|
| 6.4.3    | Foucault's Pendulum                              | 150        |
| 6.4.4    | Orbital Precession in a Magnetic Field           | 152        |
| <b>7</b> | <b>The Lagrangian Formalism</b>                  | <b>154</b> |
| 7.1      | Calculus of Variations                           | 155        |
| 7.1.1    | The Euler–Lagrange Equations                     | 156        |
| 7.1.2    | Fermat's Principle                               | 160        |
| 7.1.3    | The Second Variation                             | 162        |
| 7.2      | The Principle of Least Action                    | 165        |
| 7.2.1    | Systems of Particles                             | 168        |
| 7.2.2    | Galilean Relativity Revisited                    | 171        |
| 7.2.3    | Looking Forwards: Quantum Mechanics and Beyond   | 172        |
| 7.3      | Generalised Coordinates                          | 173        |
| 7.3.1    | Rotating Coordinate Systems                      | 175        |
| 7.3.2    | Joseph-Louis Lagrange (1736–1813)                | 176        |
| 7.4      | Constraints                                      | 177        |
| 7.4.1    | Holonomic Constraints                            | 178        |
| 7.4.2    | A Bead on a Rotating Hoop                        | 182        |
| 7.4.3    | The Double Pendulum                              | 184        |
| 7.4.4    | The Spherical Pendulum                           | 185        |
| 7.5      | Symmetries and Conservation Laws                 | 188        |
| 7.5.1    | Noether's Theorem                                | 190        |
| 7.5.2    | Emmy Noether (1882–1935)                         | 195        |
| 7.6      | Lagrangians for Fundamental Forces               | 196        |
| 7.6.1    | Gravity                                          | 196        |
| 7.6.2    | Electromagnetism                                 | 202        |
| 7.6.3    | A Brief Look at Curved Geometry                  | 208        |
| <b>8</b> | <b>Small Oscillations</b>                        | <b>210</b> |
| 8.1      | Examples                                         | 214        |
| 8.1.1    | The Double Pendulum Revisited                    | 214        |
| 8.1.2    | The Linear Triatomic Molecule                    | 215        |
| 8.1.3    | The Stability of Lagrange Points                 | 217        |
| 8.1.4    | A Lattice of Atoms                               | 221        |
| 8.2      | A Brief Look at Perturbation Theory              | 224        |
| 8.2.1    | The Anharmonic Oscillator                        | 225        |
| <b>9</b> | <b>Rigid Bodies</b>                              | <b>229</b> |
| 9.1      | Kinematics                                       | 229        |
| 9.1.1    | Angular Velocity                                 | 231        |
| 9.1.2    | An Aside: Path-Ordered Exponentials              | 233        |
| 9.2      | The Inertia Tensor                               | 236        |
| 9.2.1    | Angular Momentum                                 | 237        |
| 9.2.2    | Computing the Inertia Tensor for Simple Examples | 238        |

|           |                                                       |            |
|-----------|-------------------------------------------------------|------------|
| 9.2.3     | Parallel Axis Theorem                                 | 240        |
| 9.3       | Motion of a Rigid Body: Preliminaries                 | 241        |
| 9.3.1     | Roll, Don't Slip                                      | 243        |
| 9.3.2     | A Swinging Rod                                        | 244        |
| 9.3.3     | A Rolling Disc                                        | 245        |
| 9.4       | Euler's Equations                                     | 246        |
| 9.4.1     | The Symmetric Top                                     | 247        |
| 9.4.2     | The Asymmetric Top: Stability                         | 250        |
| 9.4.3     | The Poinsot Construction                              | 251        |
| 9.5       | Euler Angles                                          | 254        |
| 9.5.1     | Angular Velocity                                      | 257        |
| 9.5.2     | The Free Symmetric Top Revisited                      | 258        |
| 9.6       | The Heavy Symmetric Top                               | 260        |
| 9.6.1     | Uniform Precession                                    | 263        |
| 9.6.2     | The Sleeping Top                                      | 264        |
| 9.6.3     | The Precession of the Equinox                         | 265        |
| 9.7       | The Motion of Deformable Bodies                       | 267        |
| 9.7.1     | Kinematics                                            | 267        |
| 9.7.2     | Dynamics                                              | 270        |
| <b>10</b> | <b>The Hamiltonian Formalism</b>                      | <b>274</b> |
| 10.1      | Hamilton's Equations                                  | 274        |
| 10.1.1    | The Hamiltonian                                       | 277        |
| 10.1.2    | The Principle of Least Action                         | 281        |
| 10.1.3    | A Particle in a Potential                             | 282        |
| 10.1.4    | A Particle in an Electromagnetic Field                | 283        |
| 10.1.5    | William Rowan Hamilton (1805–1865)                    | 284        |
| 10.2      | Liouville's Theorem                                   | 285        |
| 10.2.1    | Liouville's Equation                                  | 289        |
| 10.2.2    | Poincaré Recurrence Theorem                           | 291        |
| 10.3      | Poisson Brackets                                      | 294        |
| 10.3.1    | Angular Momentum                                      | 296        |
| 10.3.2    | A Particle in a Magnetic Field, Revisited             | 297        |
| 10.3.3    | First-Order Vortex Dynamics in the Plane              | 299        |
| 10.3.4    | Spin and a Compact Phase Space                        | 302        |
| 10.4      | Canonical Transformations                             | 306        |
| 10.4.1    | Infinitesimal Canonical Transformations               | 309        |
| 10.4.2    | The Inverse Noether Theorem                           | 310        |
| 10.4.3    | Generating Functions                                  | 313        |
| 10.5      | Action-Angle Variables                                | 315        |
| 10.5.1    | The Harmonic Oscillator                               | 315        |
| 10.5.2    | Briefly, Integrable Systems                           | 317        |
| 10.5.3    | Action-Angle Variables for One-Dimensional Systems    | 318        |
| 10.5.4    | Action-Angle Variables for Higher-Dimensional Systems | 321        |

|           |                                                     |            |
|-----------|-----------------------------------------------------|------------|
| 10.6      | Adiabatic Invariants                                | 324        |
| 10.6.1    | A Particle in a Magnetic Field                      | 329        |
| 10.7      | The Hamilton–Jacobi Equation                        | 331        |
| 10.7.1    | Action and Angles from Hamilton–Jacobi              | 335        |
| 10.8      | Quantum Mechanics                                   | 337        |
| 10.8.1    | Hamilton, Jacobi, Schrödinger, and Feynman          | 339        |
| <b>11</b> | <b>Special Relativity</b>                           | <b>343</b> |
| 11.1      | Lorentz Transformations                             | 344        |
| 11.1.1    | Lorentz Transformations in Three Spatial Dimensions | 347        |
| 11.1.2    | Spacetime Diagrams                                  | 348        |
| 11.1.3    | A History of Light Speed                            | 349        |
| 11.2      | Relativistic Physics                                | 352        |
| 11.2.1    | Simultaneity                                        | 352        |
| 11.2.2    | Causality                                           | 354        |
| 11.2.3    | Time Dilation                                       | 356        |
| 11.2.4    | Length Contraction                                  | 360        |
| 11.2.5    | Addition of Velocities                              | 362        |
| 11.3      | The Geometry of Spacetime                           | 363        |
| 11.3.1    | The Invariant Interval                              | 363        |
| 11.3.2    | The Lorentz Group                                   | 367        |
| 11.3.3    | A Rant: Why $c = 1$                                 | 371        |
| 11.4      | Relativistic Kinematics                             | 372        |
| 11.4.1    | Proper Time                                         | 372        |
| 11.4.2    | 4-Velocity                                          | 373        |
| 11.4.3    | 4-Momentum                                          | 375        |
| 11.4.4    | Massless Particles                                  | 378        |
| 11.4.5    | Newton’s Laws of Motion                             | 380        |
| 11.4.6    | Acceleration                                        | 382        |
| 11.4.7    | Indices Up, Indices Down                            | 384        |
| 11.5      | Particle Physics                                    | 386        |
| 11.5.1    | Particle Decay                                      | 387        |
| 11.5.2    | Particle Collisions                                 | 388        |
| 11.6      | Lagrangians and Hamiltonians                        | 393        |
| 11.6.1    | The Covariant Action                                | 394        |
| 11.6.2    | The Hamiltonian for a Relativistic Particle         | 399        |
| 11.7      | The Lorentz Group and $SL(2, \mathbb{C})$           | 400        |
| 11.7.1    | A New Way of Looking at Spacetime                   | 400        |
| 11.7.2    | What the Observer Actually Observes                 | 404        |
| 11.7.3    | Spinors                                             | 409        |
|           | <i>Further Reading</i>                              | 411        |

|                   |     |
|-------------------|-----|
| <i>References</i> | 417 |
|-------------------|-----|

|              |     |
|--------------|-----|
| <i>Index</i> | 419 |
|--------------|-----|