D-Branes in Field Theory

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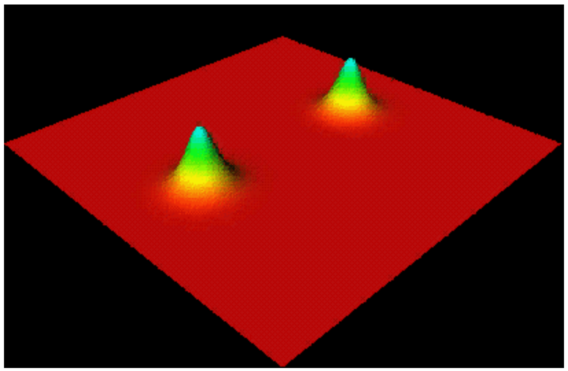
hep-th/0512192, JHEP 02 (2006) 030

The Plan

- Objective
  - To study D-brane like objects in field theories, removed from the complexities of gravity
- Motivations
  - To understand the relationship between string and gauge theories for semi-classical, magnetic strings.
  - To develop a new perspective on soliton scattering in field theory.

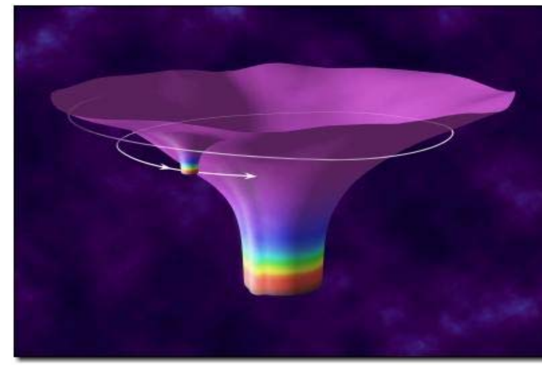
Soliton Scattering

Field Theory | **String Theory**



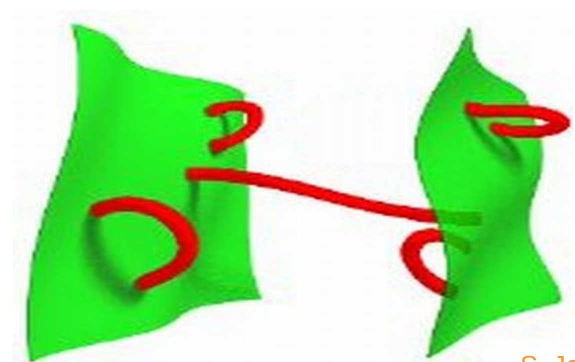
J. Moore and E.P. Shellard

Closed String Description



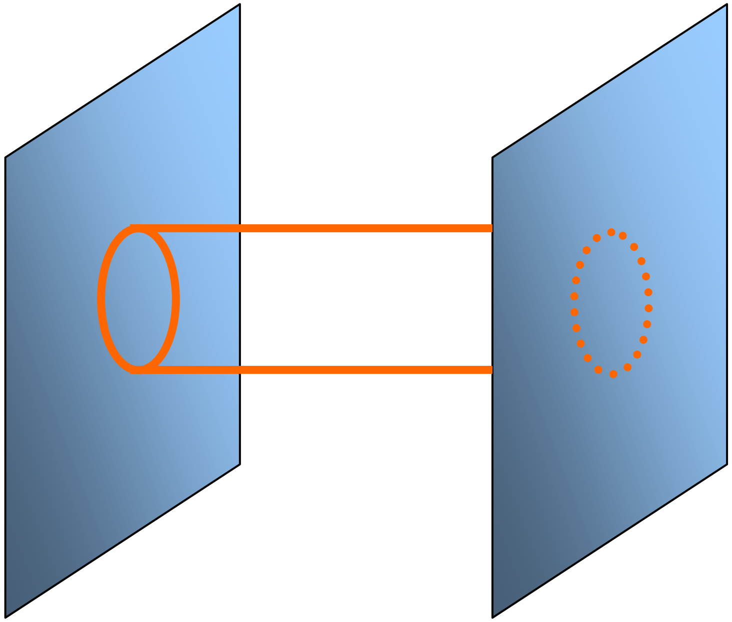
D. Davies and K. Thorne

Open String Description



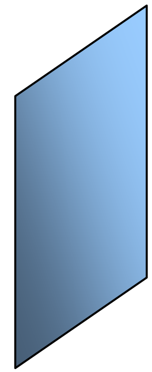
S. Jensen

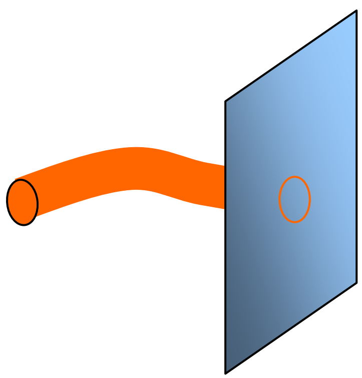
Open-Closed String Duality



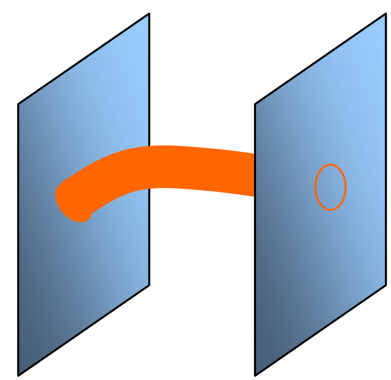
- Tree level closed string = one-loop open string
- Sum over all modes to see equivalence
- In certain regimes, can restrict attention to lowest mode

D-Branes in Field Theory

- Both strings  and branes  exist as solitons



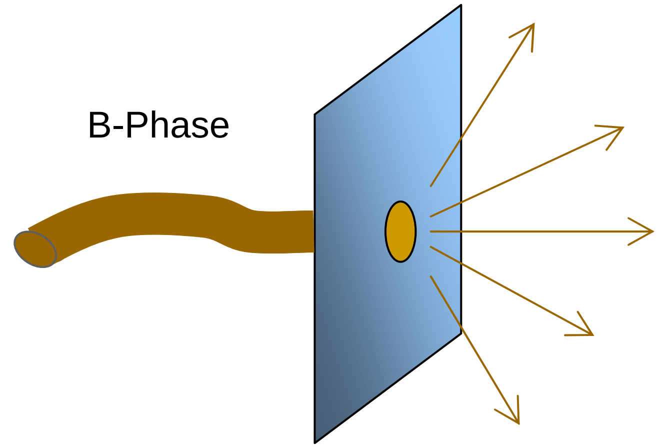
- The string can end on the brane



- When branes approach, the dynamics is governed by light, stretched strings

D-Branes in Nature

- D-Branes in ^{3}He



- D-Branes in Fluid Dynamics

- D-Branes in Wyoming

But they only satisfy the first 2 criteria

D-Branes in Field Theory

There are 3 field theories that admit D-branes with all properties

- $\mathcal{N}=1$ super Yang-Mills in d=3+1: The domain wall is a D-brane for the QCD flux tube. Witten; Acharya and Vafa

- $\mathcal{N}=2$ super QCD in d=3+1: The domain wall is a D-brane for the magnetic vortex string.

- $\mathcal{N}=2$ super Yang-Mills in d=5+1: The monopole 2-brane is a D-brane for the instanton string. Hanany and Witten

The Lagrangian: $\mathcal{N}=2$ Supersymmetric U(1) gauge theory

gauge coupling e^2

neutral, real scalar

N complex, charged scalars

$$L = \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2e^2}\partial\phi^2 + \sum_{i=1}^{N}|\mathcal{D}q_i|^2$$
$$-\sum_i(\phi - m_i)^2|q_i|^2 - \frac{e^2}{2}\Big(\sum_i|q_i|^2 - v^2\Big)^2$$

real masses m_i

Fayet-Iliopoulos parameter v^2

N isolated vacua: $\phi = m_i$ and $|q_j|^2 = v^2\delta_{ij}$

The Spectra

There are three perturbative energy scales:

E

Landau Pole

$$\Lambda \sim ev\,\exp(+1/e^2 N)$$

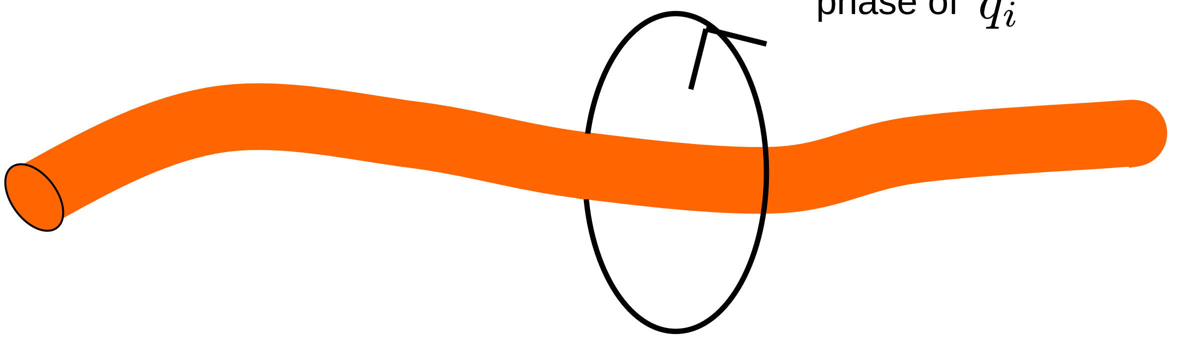
$$M_\gamma \sim ev$$

$$M_q \sim |m_i - m_j|$$

- Classically, in the strong coupling limit $e^2 \to \infty$ the theory reduces to a massive sigma model (GLSM).
- We assume a UV completion.

Vortices

Broken U(1) gauge symmetry → vortices



$$B_3 = e^2\left(\sum_i |q_i|^2 - v^2\right)$$
$$\mathcal{D}_z q_i = 0$$

$z = x_1 + ix_2$

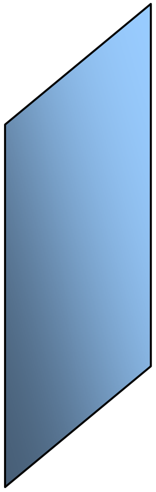
$$T_{\rm vortex} = 2\pi v^2$$

Nielsen and Olesen, '73

Domain Walls

- Isolated vacua ⟹ domain walls

$\phi = m_i$



$\phi = m_j$

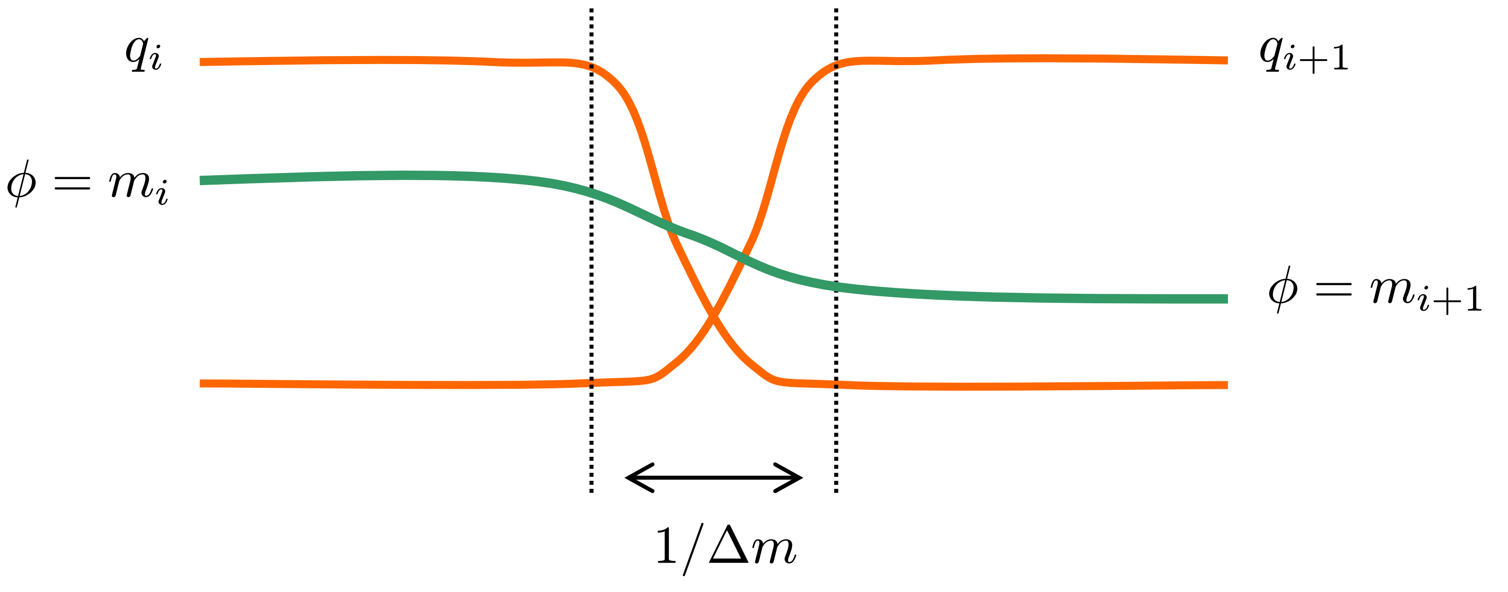
$$\partial_3 \phi = e^2 \left(\sum_i |q_i|^2 - v^2\right)$$
$$\mathcal{D}_3 q_i = (\phi - m_i) q_i$$

$$\boxed{T_{\text{wall}} = |m_i - m_j| v^2}$$

Abraham and Townsend, ‘91

Wall Profile

choose $m_1 > m_2 > \ldots > m_N$



Two Collective Coordinates

- Center of Mass X
- Phase from U(1) flavour action: $q_i \to e^{i\sigma} q_i$

$$q_{i+1} \to e^{-i\sigma} q_{i+1}$$

⇒ Moduli Space is $\mathbf{R} \times \mathbf{S}^1$

Domain Wall Dynamics

The collective coordinates are promoted to dynamical degrees of freedom on the domain wall worldvolume:

$$\mathcal{L}_{\rm wall} = T_{\rm wall} \int d^3x \; (\partial X)^2 + (\partial\sigma)^2 + \text{fermions}$$

But in d=2+1, σ is dual to a U(1) gauge field: $\partial_\alpha \sigma \sim \epsilon_{\alpha\beta\gamma} F^{\beta\gamma}$

low energy dynamics is $\mathcal{N}=2$ U(1) gauge theory

$$\mathcal{L}_{\rm wall} = \int d^3x \; \frac{1}{4g^2} F_{\alpha\beta}F^{\alpha\beta} + \frac{1}{2g^2}(\partial\psi)^2 + \text{fermions}$$

$\frac{1}{g^2} = \frac{\Delta m}{4\pi^2 v^2}$ and $\psi = 2\pi v^2 X = T_{\rm vortex} X$

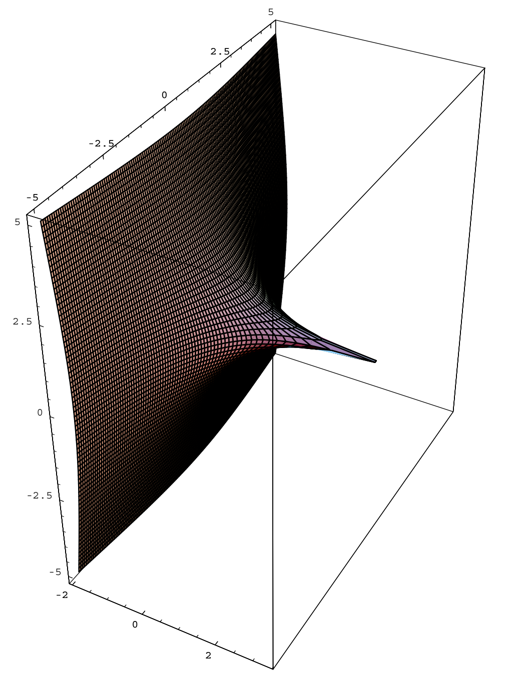
The Domain Wall as a D-Brane

Gauntlett, Portugues, Tong and Townsend, '00
Shifman and Yung,'03

Simplest to see from domain wall worldvolume. String appears as a "BIon"

$$X + i\sigma = -\log z$$

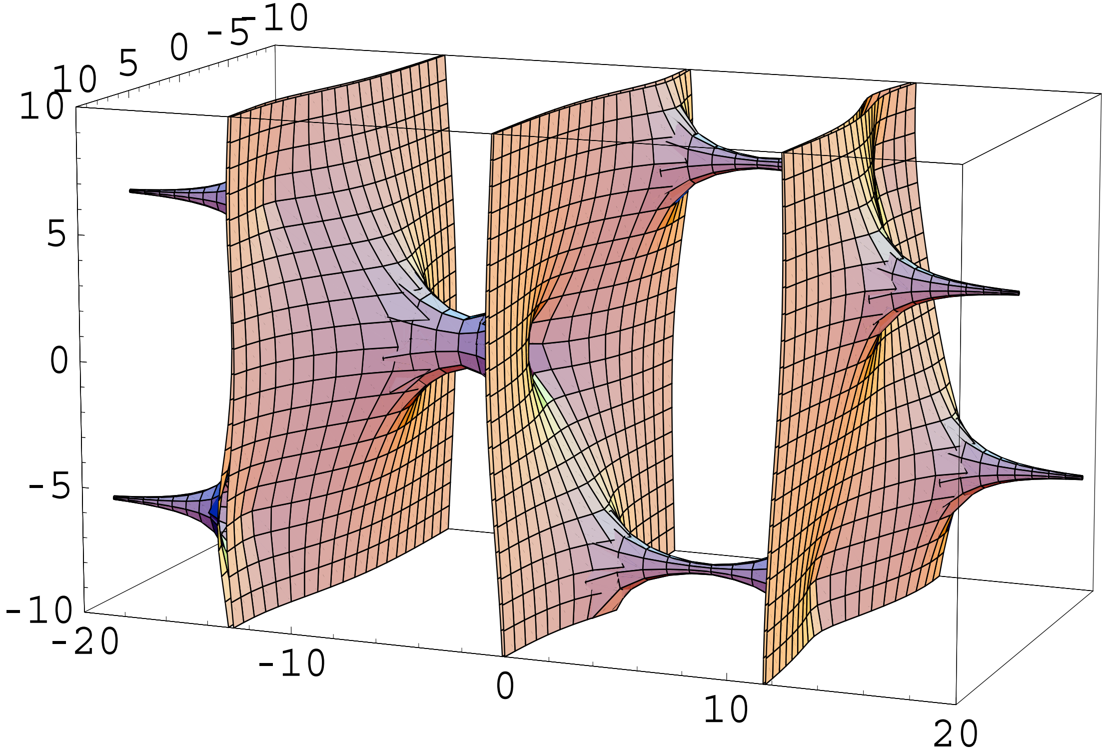
with $z = x^1 + ix^2 = re^{i\theta}$



$\sigma \sim \theta$ ⇒ $F_{0r} \neq 0$ ⇒ end of string electrically charged

Bulk Solutions

Analytical solutions to bulk equations known in simplest case; numerical solutions found in others



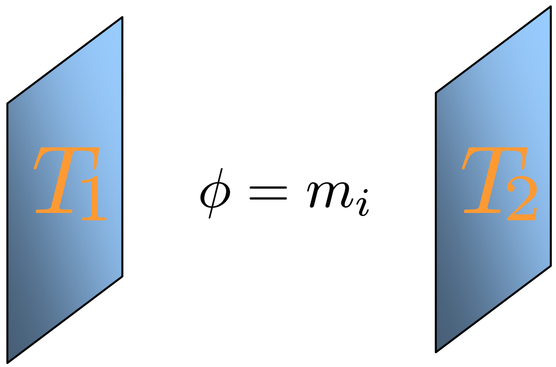
Isozumi, Nitta, Ohashi and Sakai ‘04

Scattering of Two Walls

Solutions exist for all separations

Gauntlett, Tong, Townsend '00

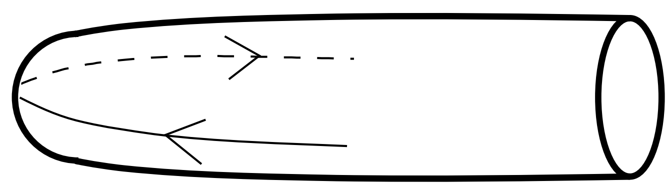
$\phi = m_{i-1}$



$\phi = m_i$

$\phi = m_{i+1}$

Moduli Space is: $\mathcal{M}_{2-\mathrm{wall}} \cong \mathbf{R} \times \mathbf{S}^1 \times \mathcal{M}_{\mathrm{cigar}}$

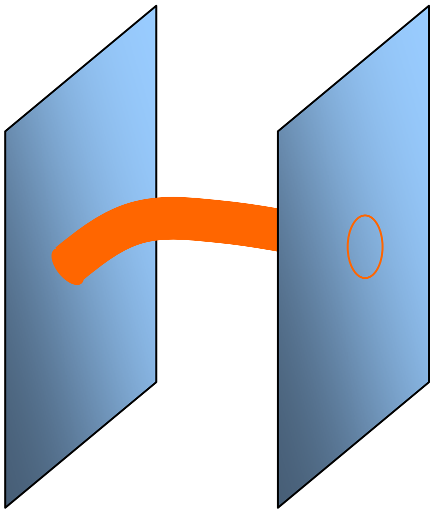


Metric is

- known, but messy
- smooth at the tip
- exponentially close to the flat cylinder

Tong '01

Open String Description



Low-energy dynamics is $U(1)^2$ gauge theory, coupled by the vortex string

vortex charge = (+1,-1)

vortex mass = $T_{\rm vortex}\Delta X = (\psi_2 - \psi_1)$

What is BPS state?

- Vector multiplet
- Chiral multiplet 

→ Add charged chiral multiplet, with fields q and λ

Chern-Simons-Higgs Theory

The relative dynamics of the two walls is governed by U(1) gauge theory coupled to chiral multiplet

But: Integrate in Dirac fermion in d=2+1 ⇒ Induce a Chern-Simons term

$$\mathcal{L}_{\rm wall} = \frac{1}{4g^2}F_{\alpha\beta}^2 + \frac{1}{2g^2}(\partial\psi)^2 + |\mathcal{D}q|^2 + \psi^2|q|^2 + \kappa A\wedge F + \frac{g^2}{2}(|q|^2 - \kappa\psi)^2 + \text{fermions}$$

$\psi = \psi_2 - \psi_1$

where $\kappa = -\frac{1}{2}$ from integrating charged fermion λ

The Vacuum Moduli Space

- Chern-Simons theory captures domain wall dynamics
- Vacuum manifold $\cong$ domain wall moduli space

$$V = \psi^2 |q|^2 + \frac{g^2}{2}(|q|^2 - \kappa\psi)^2$$

Set $\psi \neq 0$ ⇨ $V = \frac{1}{2}\kappa^2 g^2 \psi^2$. Integrate out q and λ

$$\kappa_{\text{eff}} = -\frac{1}{2} + \frac{1}{2}\,\text{sign}(\,\text{Mass}[\lambda]) = -\frac{1}{2} + \frac{1}{2}\,\text{sign}\,\psi = \begin{cases} 0 & \psi > 0 \\ -1 & \psi < 0 \end{cases}$$

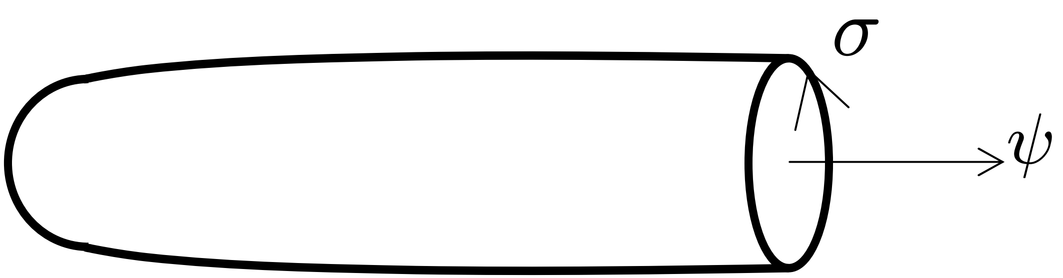
↑ Yukawa coupling $\psi\bar{\lambda}\lambda$

⇨ Massless $F_{\alpha\beta}$ and ψ when $\psi > 0$ ⇨ Walls cannot pass!

Vacuum Moduli Space

The low energy dynamics of the light fields on the wall is
a sigma model involving the separation ψ and dual photon σ

$$ds^2 = \frac{1}{2g_{\rm eff}^2} d\psi^2 + \frac{1}{2} g_{\rm eff}^2 \, d\sigma^2 \qquad \text{with} \qquad \frac{1}{g_{\rm eff}^2} = \frac{1}{g^2} + \frac{1}{2\psi} + \ldots$$

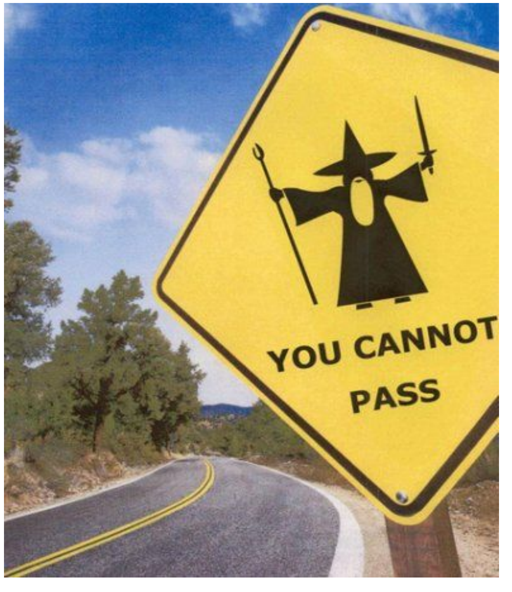


Metric is
- smooth at the tip
- different from flat cylinder by power-law corrections

Summary

- We can study either
  - Classical solutions of bulk field equations
  - Quantum dynamics of open strings

- Both lead to the same qualitative physics



Regimes of Validity

The bulk and open string calculation are valid in different regimes

Bulk:

- Bulk theory is non-renormalizable.
- Expect higher derivative corrections of order $(\partial/v)^2$
- These are negligible for domain walls when:

$$\frac{\Delta m}{v} \ll 1$$

Open String:

- The energy of the open string ground state $Xv^2 \ll \Delta m$, the excitation of the string.
- Wall separation $X \gg 1/\Delta m$, the width of the wall

$$\frac{\Delta m}{v} \gg 1$$

String Coupling?

- Question: Does the D-brane tension scale as 1/g_s ?
- Problem: It's not clear how to compute g_s.

$$T_{\text{wall}} = \left(\frac{T_{\text{vortex}}}{2\pi}\right)^{3/2} \frac{\Delta m}{v} \quad \Rightarrow \quad g_s \stackrel{?}{=} \frac{v}{\Delta m}$$

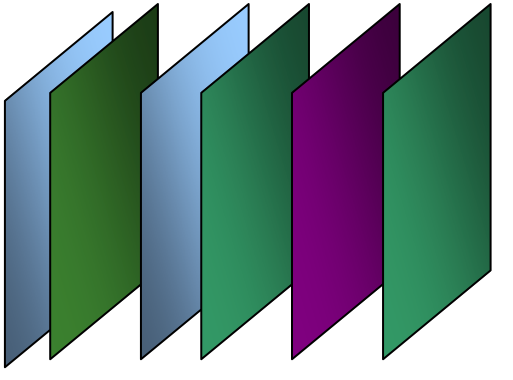
- For $e^2 \ll 1$ the vortices always reconnect and $g_s \sim 1$
- Our massive sigma model arises from the $e^2 \to \infty$ limit and the relationship $g_s = v/\Delta m$ seems plausible.
- If true, interesting for cosmic strings
- With this definition, open strings require $g_s \ll 1$

More General Domain Walls

Can consider $U(N_c)$ with $N_f > N_c$ fundamental flavors

$$\Rightarrow \begin{pmatrix} N_f \\ N_c \end{pmatrix} \text{ vacua}$$

- Many interpolating domain walls
- Interesting ordering properties (penetrable and impenetrable walls)
- Vortices end on only certain domain walls



Isozumi, Nitta, Ohashi and Sakai '04
Sakai and Tong '05

Open string description: Abelian Chern-Simons-Higgs theory.

(No non-abelian symmetry enhancement)

Other D-Branes

- N=1 d=3+1 SU(N) Super Yang-Mills:
  - k domain wall theory is d=2+1, N=1 U(k) + adjoint multiplet + CS term at level N — Acharya and Vafa
  - Two-loop computation in 1/N reproduces qualitative force between domain walls — Armoni and Hollowood

- N=2 d=5+1 SU(2) Super Yang-Mills: (LST)
  - k monopole theory is d=2+1, N=4 SU(k) super Yang-Mills — Hanany and Witten
  - Quantum Coulomb branch reproduces monopole moduli space
  - Monopole has $1/g_s$ tension with $g_s = 1/e\phi$ — Seiberg and Witten

Chalmers and Hanany

Conclusion

- We have presented an open string description for semi-classical D-branes in d=3+1 field theory
- Agrees qualitatively with classical bulk description

- Questions:
  - Can we quantize the semi-classical string with Dirichlet boundary conditions?
  - Is the bulk calculation truly a “closed string” description?
    - ‘t Hooft limit underlying d=3+1 confining theory
    - Little string underlying d=5+1 SYM
  - Is there a little string theory underlying the d=3+1 massive sigma model?