Inching Towards Strange Metallic Holography

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Based on “Towards Strange Metallic Holography”, 0912.1061
with Sean Hartnoll, Joe Polchinski and Eva Silverstein

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Plan of the Talk

- Motivation
  - Remedial Introduction to Conductivity
  - Anomalous Properties of Strange Metals

- Conductivity from Lifshitz Geometry
  - Lifshitz Geometry and D-Brane Probes
  - DC and Hall Conductivity
  - Optical Conductivity

Drude Model: DC Conductivity

- Ohm's Law: $\vec{E} = \rho\vec{j}$

Resistivity: ρ

Conductivity: $\sigma = 1/\rho$

$$\boxed{\rho = \frac{m}{ne^2\tau}}$$

Scattering Time

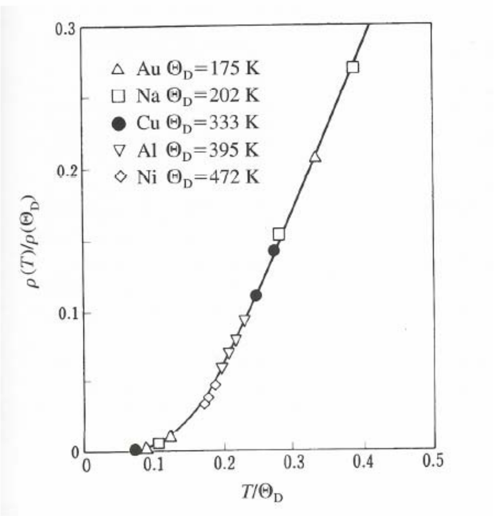
Scattering Mechanisms

- The temperature dependence of the resistivity sits in the scattering time:

Impurities: $\rho \sim T^0$

Phonons: $$\begin{cases} \rho \sim T & (T > \Theta_D) \\ \rho \sim T^5 & (T < \Theta_D) \end{cases}$$

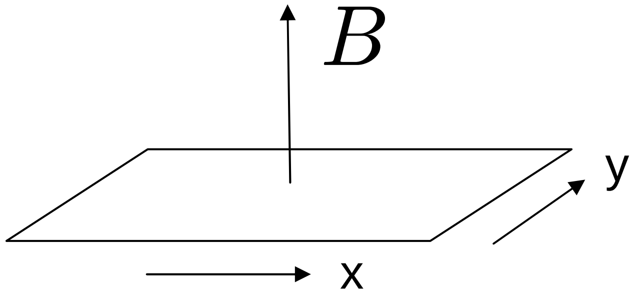
Electrons: $\rho \sim T^2$



Bardeen, 1940

- Resistivity in metals is typically due to phonons and impurities.

Drude Model: Hall Conductivity



$$\vec{j} = \sigma \vec{E}$$

Now a 2x2 matrix

$$\sigma_{xx} = \frac{1}{\rho}\frac{1}{1+\omega_c^2\tau^2} \qquad \sigma_{xy} = \frac{1}{\rho}\frac{\omega_c\tau}{1+\omega_c^2\tau^2} \qquad \left(\omega_c = \frac{eB}{mc}\right)$$

$$\boxed{\frac{\sigma_{xx}}{\sigma_{xy}} = \frac{1}{\omega_c\tau}}$$

Drude Model: AC Conductivity

- Fourier Transform: $$\vec{j}(\omega) = \sigma(\omega)\vec{E}(\omega)$$

$$\vec{E} = \vec{E}(\omega)e^{-i\omega t}$$

$$\vec{j} = \vec{j}(\omega)e^{-i\omega t}$$

$$\boxed{\sigma(\omega) = \frac{1}{\rho}\frac{1}{1-i\omega\tau}}$$

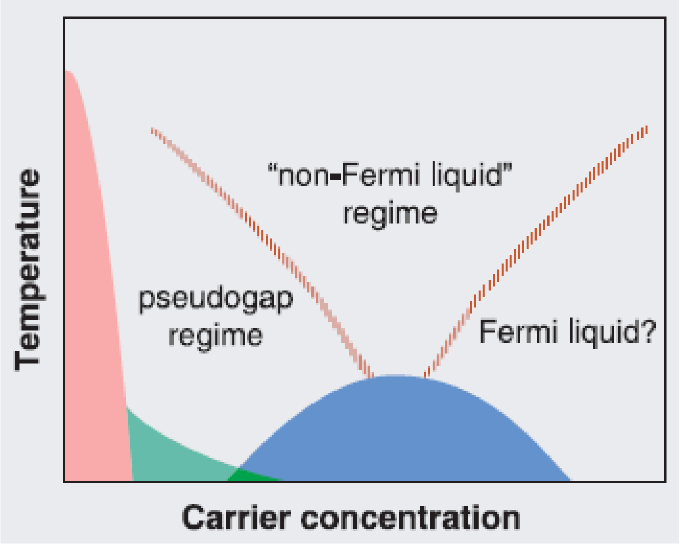
- Note: $\sigma(\omega) \to i/\omega$ as $\omega \to \infty$

Strange Metals

- High Tc Superconductors at optimal doping
  - e.g, cuprates

- Suggestions that behaviour is governed by quantum critical point.

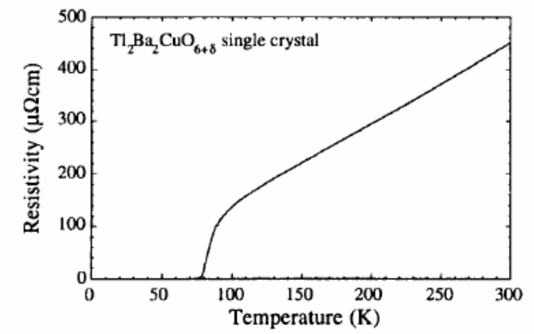
- Heavy Fermion materials have similar properties



Anomalous Properties

- Resistivity:

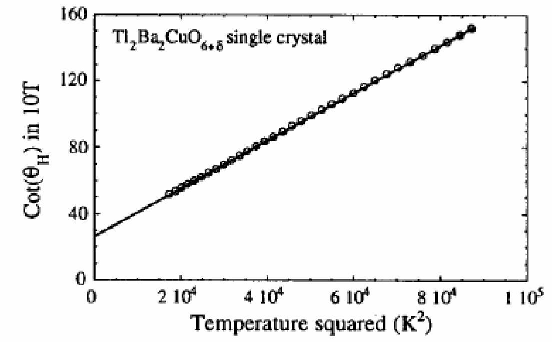
$$\rho \sim T$$



Mackenzie, 1997

- Hall Conductivity

$$\sigma_{xx}/\sigma_{xy} \sim T^2$$

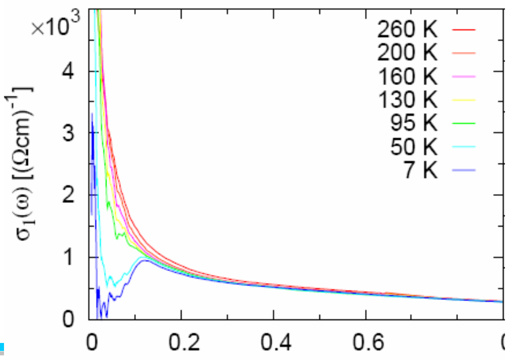


Mackenzie, 1997

- AC Conductivity

$$\sigma(\omega) \to (i/\omega)^\nu$$

$$\nu \approx 0.65$$



Van der Marel et al., 2003

Challenge

- Reproduce this anomalous behaviour in conductivity from a (presumably strongly coupled) field theory.

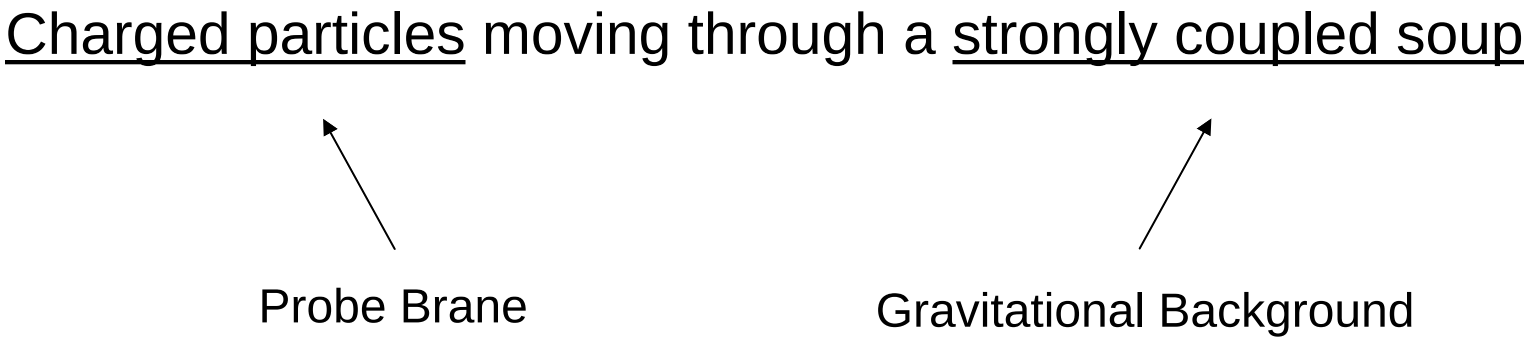
Strategy

- Use AdS/CFT
  - Build up ingredients we want by writing an effective theory in the bulk. e.g.

- Pros:
  - Very simple to compute transport properties in *a* strongly interacting theory

- Cons:
  - Can’t build microscopic theories to order
  - But even worse…don’t know microscopic degrees of freedom!

The Set-Up



- Charged particles moving through a strongly coupled soup

Probe Brane

Gravitational Background

- Our boundary theory will live in d=2+1 dimensions
- We want a current with *massive* charge carriers
- We will take the strongly coupled soup to obey *Lifshitz Scaling*

Lifshitz Scaling

- Non-Relativistic Conformal Invariance

z: dynamical critical exponent

$$\vec{x} \to \lambda \vec{x} \qquad\qquad t \to \lambda^z t$$

$\vec{x} = (x, y)$: d=2 spatial dimensions

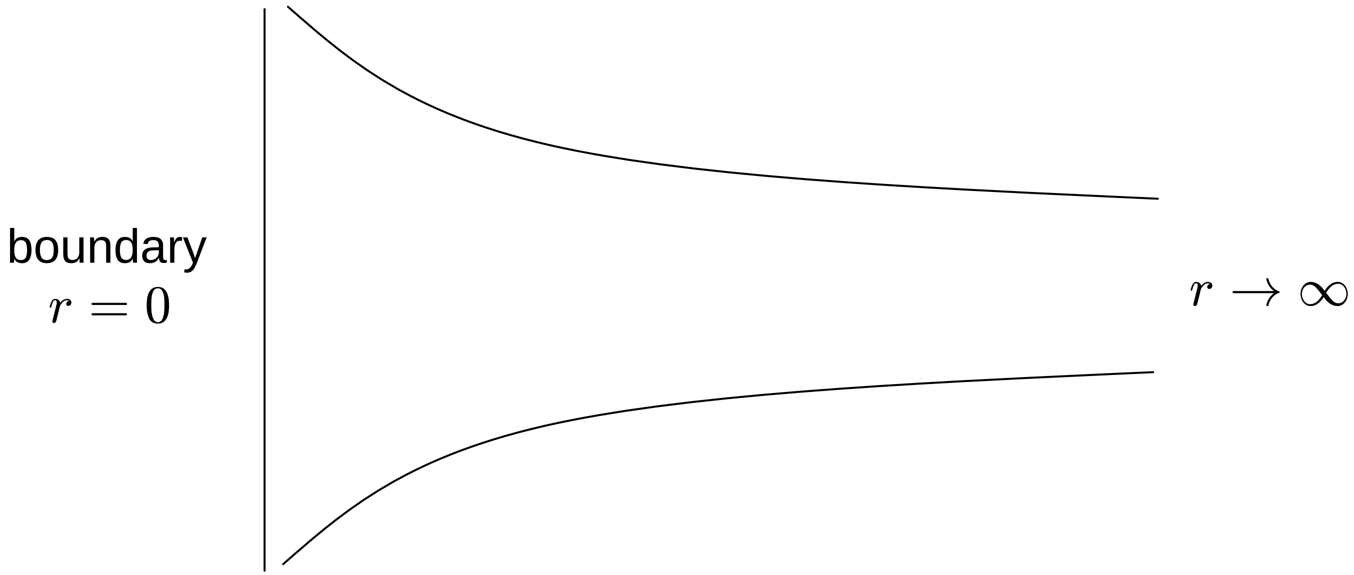
- A weakly coupled example with z=2:

$$S = \int dt d^d x \ \left[\dot{\phi}^2 - (\nabla^2 \phi)^2 \right]$$

Lifshitz Geometry

Kachru, Liu, Mulligan '08

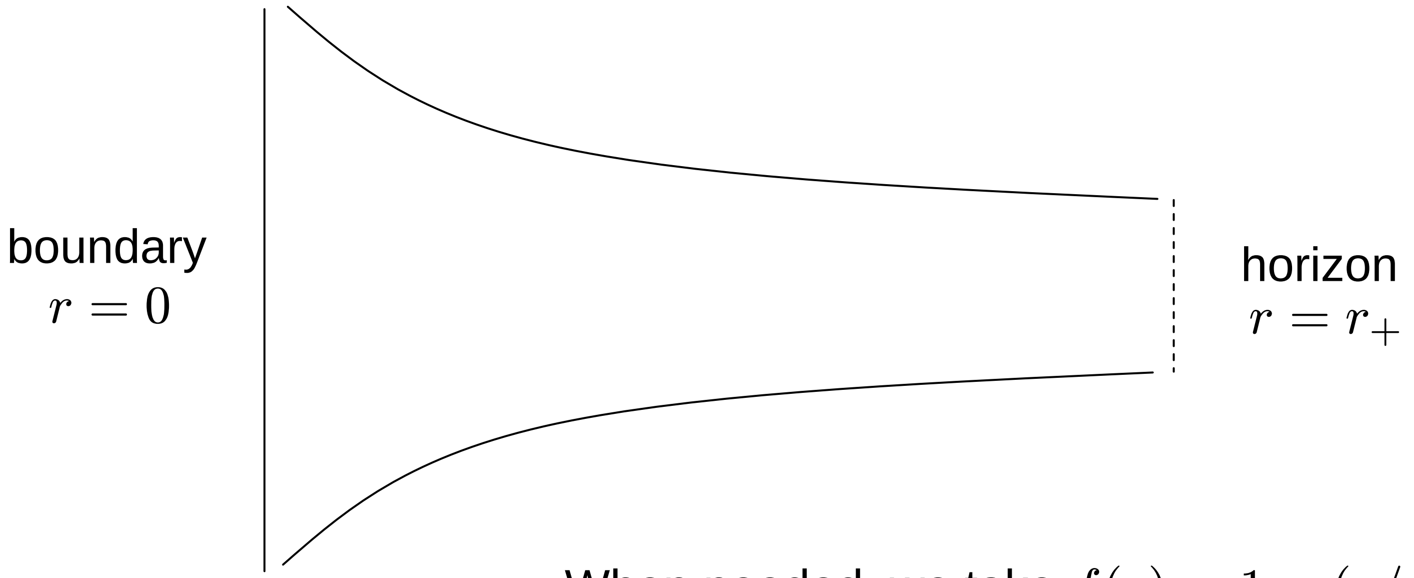
$$ds^2 = L^2 \left(-\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx^2+dy^2}{r^2} \right)$$



Hot Lifshitz Geometry

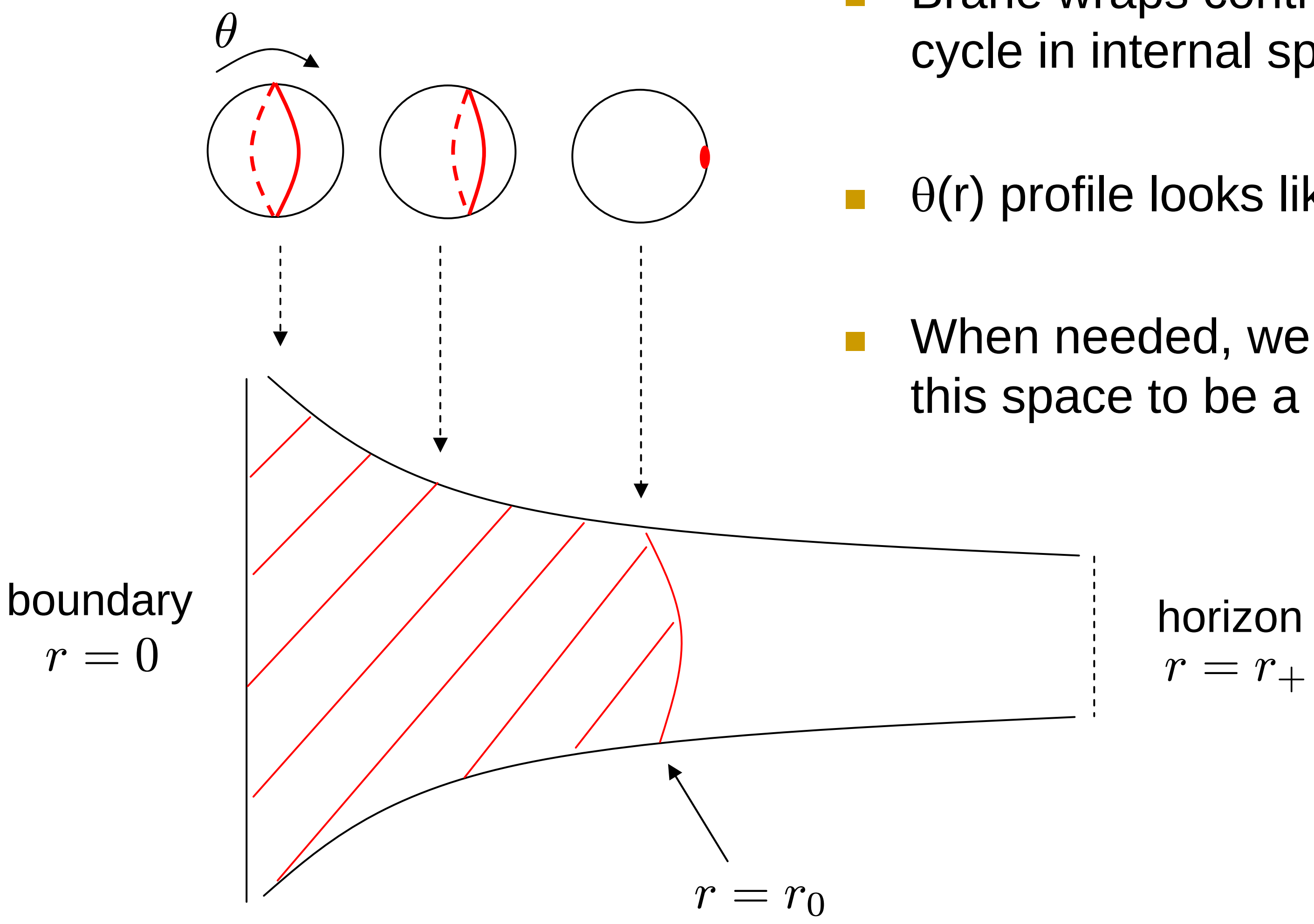
$$ds^2 = L^2 \left(-\frac{f(r)}{r^{2z}} dt^2 + \frac{dr^2}{f(r) r^2} + \frac{dx^2 + dy^2}{r^2} \right)$$

$$f(r_+) = 0 \quad \Longrightarrow \quad T \sim f'(r_+)/r_+^{z-1} \sim 1/r_+^z$$



When needed, we take $f(r) = 1 - (r/r_+)^{z+2}$

Adding Probe Branes



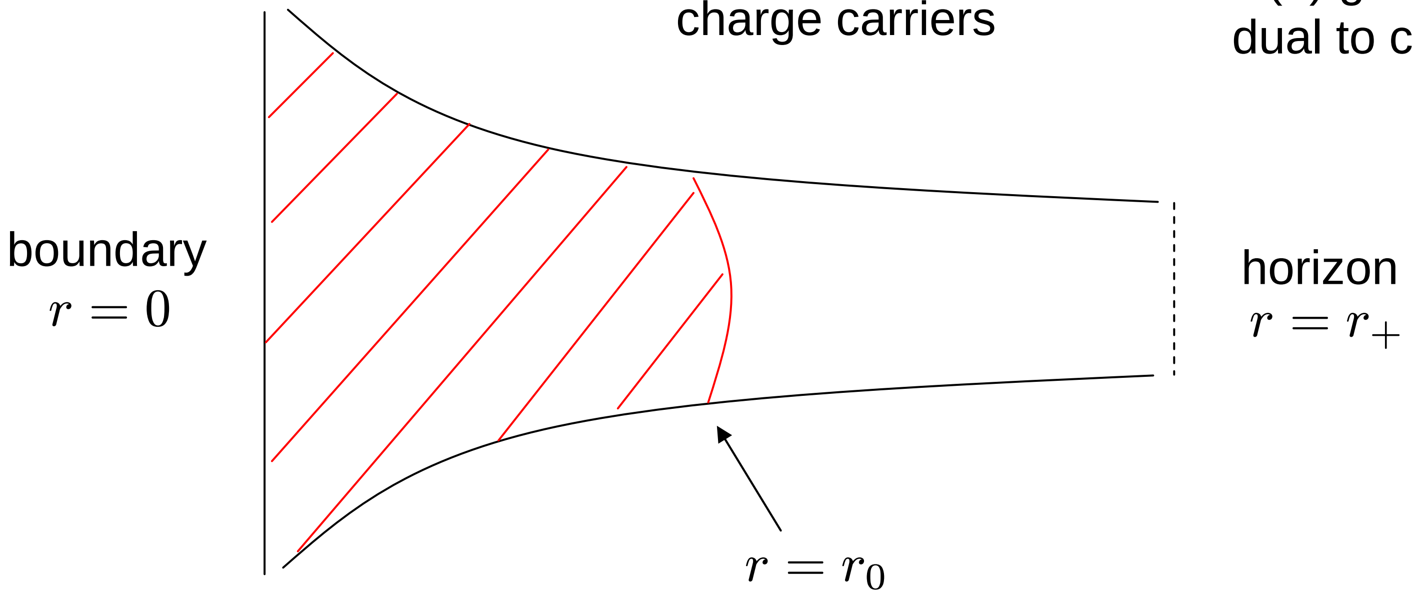
- Brane wraps contractible cycle in internal space
- θ(r) profile looks like cigar
- When needed, we take this space to be a sphere

Adding Probe Branes

$$S_{\mathrm{DBI}} = -\tau \int dt d^3\sigma \sqrt{-det\left[g_{ab} + \partial_a\theta\partial_b\theta + F_{ab}\right]}$$

Dual to mass of charge carriers

U(1) gauge field, dual to current, J


boundary
$r = 0$

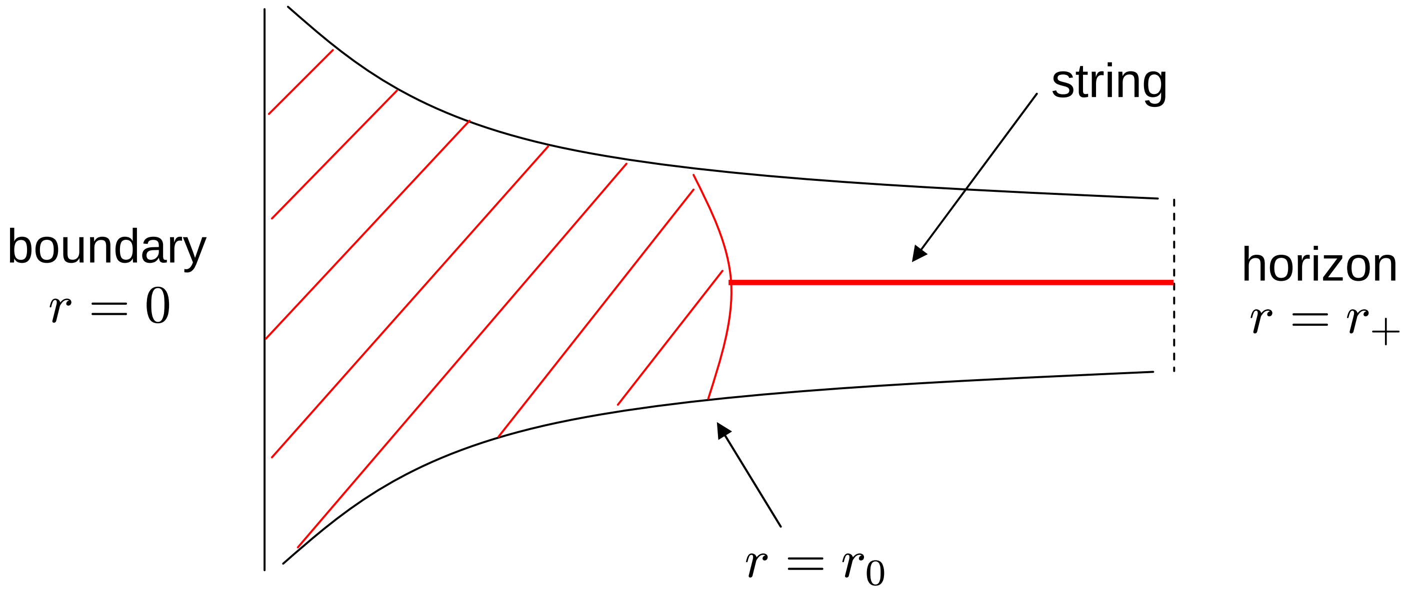
horizon
$r = r_+$

$r = r_0$

In this talk, we'll drop powers of τ, L^2, α'

Charge Carriers

- Charge carrier = string: $m = E_{\rm gap} \sim 1/r_0^z$
- Note: $m \gg T$ ($E_{\rm gap} \sim 1eV$, $T \sim 10 \to 10^3 K$)

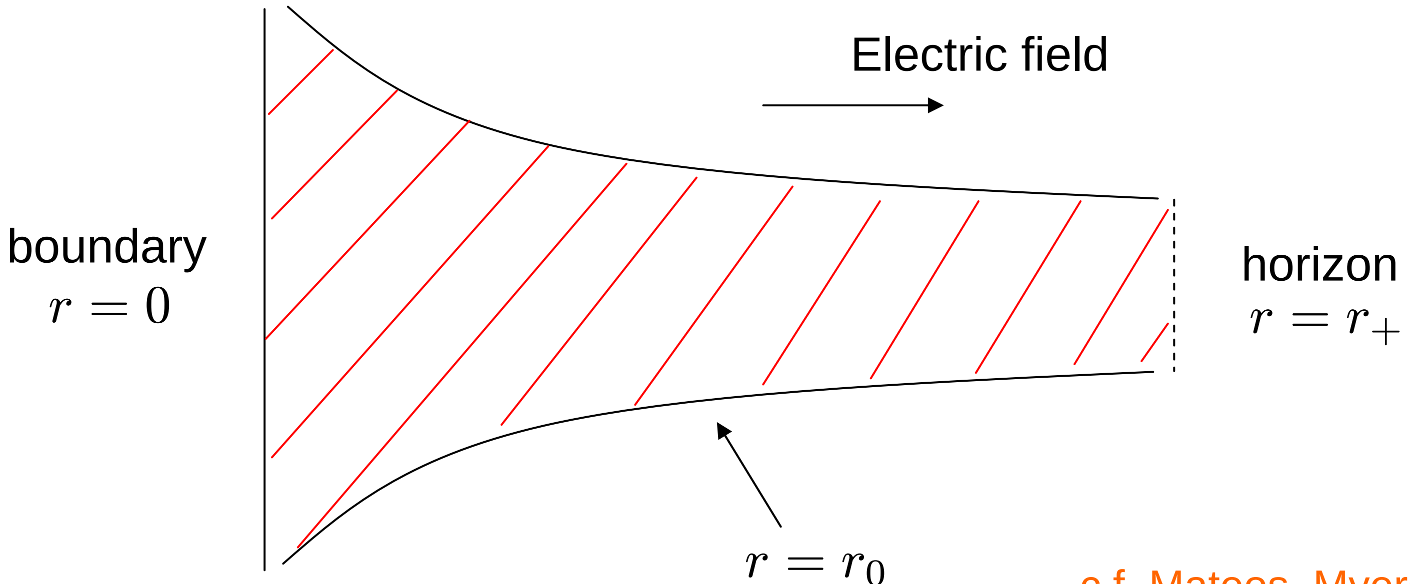


Charge Carriers

chemical potential

charge density

- Finite charge density: $A_0 \to \mu - J^t r^{2-z} + \ldots$
- Brane now stretches to horizon. θ(r) profile looks like tube.



c.f. Mateos, Myers et. al. ‘06

DC Conductivity

Karch, O’Bannon ‘07

$$A_0 \to \mu - J^t r^{2-z} + \ldots$$

$$A_x \to Et + J^x r^z + \ldots$$

electric field current

- Compute full non-linear conductivity

$$J^x = \sigma(E,T)\,E$$

- Without doing any work….!

DC Conductivity

$$\sigma(E,T) = \sqrt{\text{const.} + r_\star^4 (J^t)^2}$$

due to pair creation

due to background charge

- where $f(r_\star) = E^2\, r_\star^{2z+2}$
- when $E_{\text{gap}},\ (J^t)^{z/2} \gg T$

$$\boxed{\rho = \frac{1}{\sigma} \sim \frac{T^{2/z}}{J^t}}$$

- Comments:
  - ❑ Finite DC conductivity only because we’re ignoring backreaction
  - ❑ Linear for z=2

Hall Conductivity

O'Bannon '07

- Similar technique: Find temperature dependence

$$\boxed{\frac{\sigma_{xx}}{\sigma_{xy}} \sim T^{2/Z}}$$

- This is like the Drude result: no sign of anomalous behaviour.
  - Caveat: The term from pair creation actually scales as

$$\frac{\sigma_{xx}}{\sigma_{xy}} \sim \frac{T^{4/z}}{J^t B}$$

  - Good for z=2, but unclear why this term would dominate

AC Conductivity

$$A_x(\omega) = \frac{E_x(\omega)}{i\omega} + J^x(\omega)r^z + \dots$$

- Boundary conditions:
  - $E_x(\omega)$ at r=0
  - Ingoing boundary conditions at black hole horizon
- Compute
  - Current $J^x(\omega)$
  - Numerical results for all ω
  - Analytic results for $T \ll \omega \ll E_{\text{gap}}$

$\omega \sim 0.1 \rightarrow 1 \text{ eV}$

AC Conductivity

$$\sigma(\omega) \sim \begin{cases} (J^t)^{z/2}\,\omega^{-1} & z < 2 \\ J^t\,(\omega \log \omega)^{-1} & z = 2 \\ J^t\,\omega^{-2/z} & z > 2 \end{cases}$$

- Note: z=3 matches data!
- Linearity in J^t is now dynamical

Comment on the z=2 Crossover

- Dimensional Analysis: $[x] = -1 \quad [t] = -z$
- Operators are relevant if $[\mathcal{O}] \leq d + z$

$$\int dt d^d x \quad \mathcal{O}$$

- Examples:
  - Charge density: $[J^t] = d \Longrightarrow (J^t)^2$ relevant for z>d
  - Probe Inertia: $\int dt\ \dot{x}^2 \Longrightarrow$ irrelevant for z>2
    - For z>2, all inertia due to stuff that particle drags around with it

Summary

- Charge carriers moving in strongly coupled Lifshitz backgrounds yields:
  - DC conductivity: $\rho \sim T^{2/z}$
  - Hall conductivity: $\sigma_{xx}/\sigma_{xy} \sim \rho$
  - AC conductivity $\sigma(\omega) \sim 1/\omega^{2/z}$ for z>2
- Starting point for model building?