The dawn of a new era: Exploring the Universe with Gravitational Waves

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DAMTP, University of Cambridge
LIGO Scientific Collaboration

The Archimedeans Mathematical Society
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Gravitational Waves: Ripples in spacetime

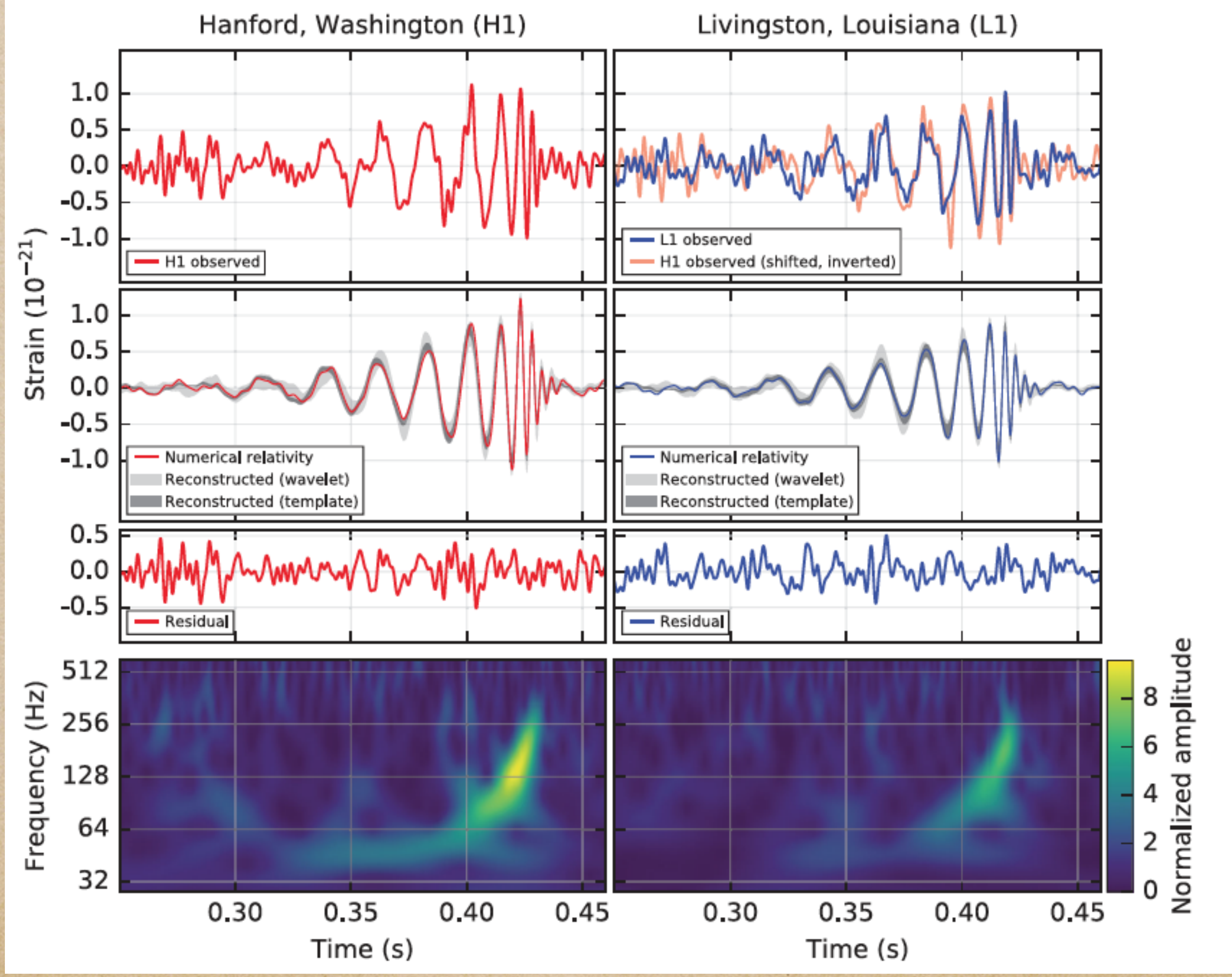
- Unusual news headlines on 11/12 February 2016
- First direct detection of gravitational waves: GW150914

So, what happened?

- Sep 14, 2015 at 09:50:45 UTC: SNR ~ 24

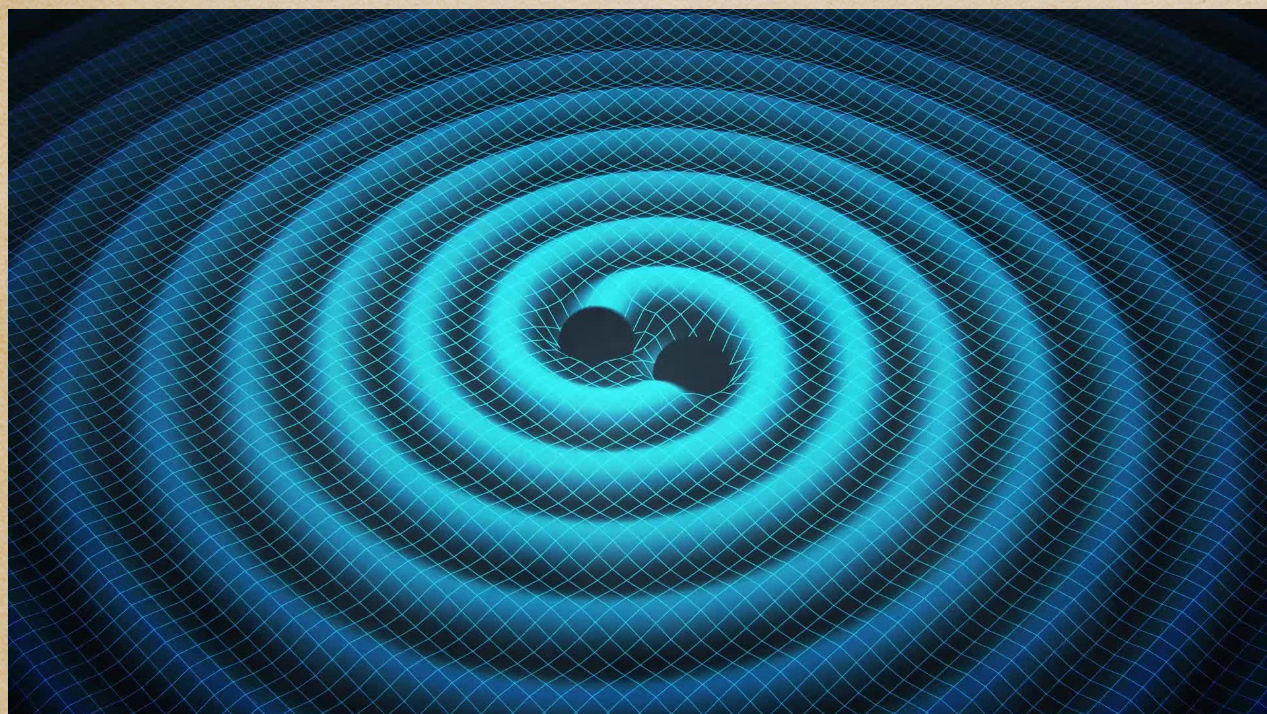
  Abbott et al. PRL 2016, Abbott et al. PRX 2016

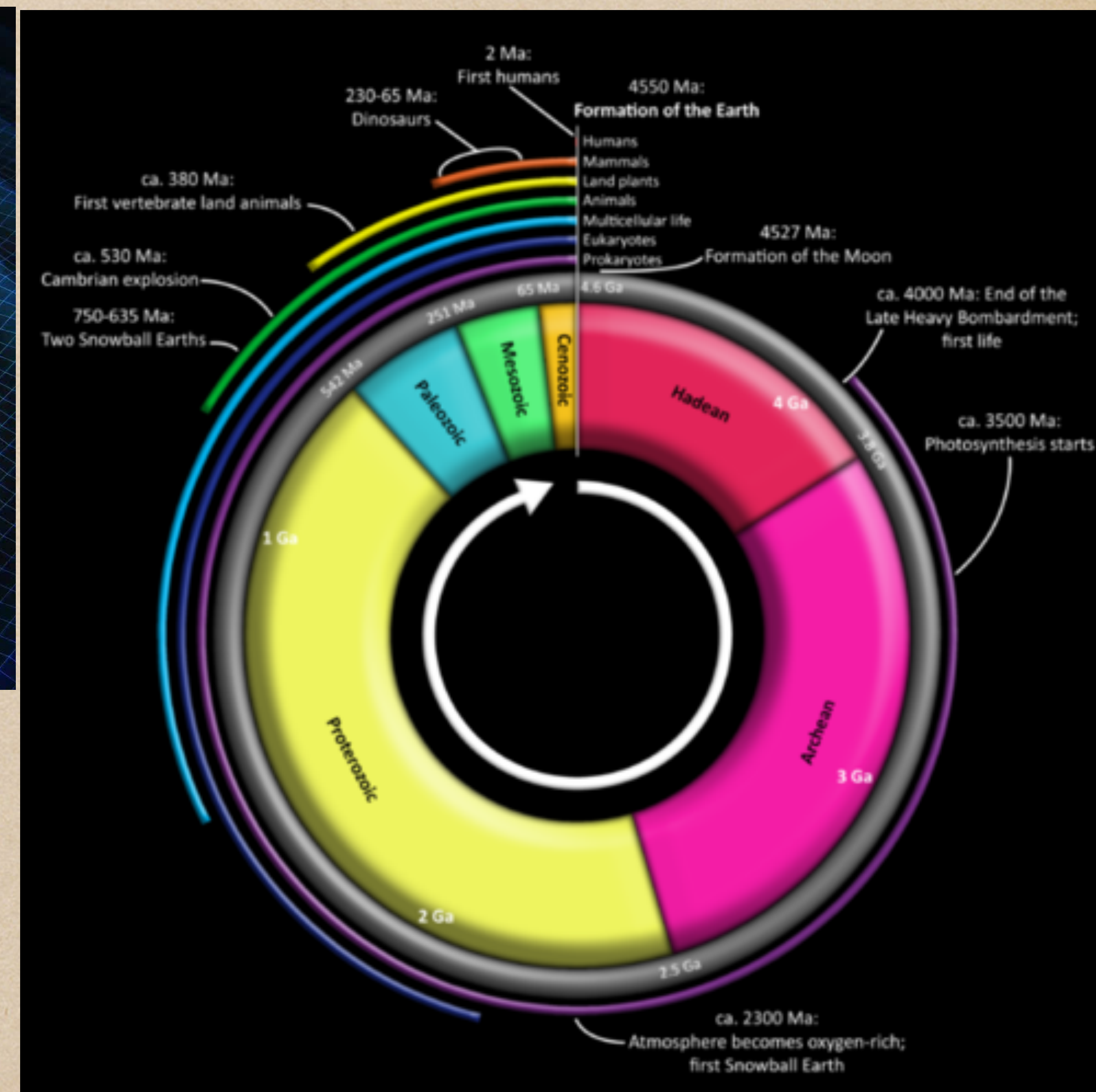
- BBH inspiral, merger and ringdown: $m_1 = 35^{+5}_{-3}\, m_\odot\,, \quad m_2 = 30^{+3}_{-4}\, M_\odot$



What really happened...

- Once upon a time: $1.34^{+0.52}_{-0.59}$ Gyr ago, somewhere in the universe





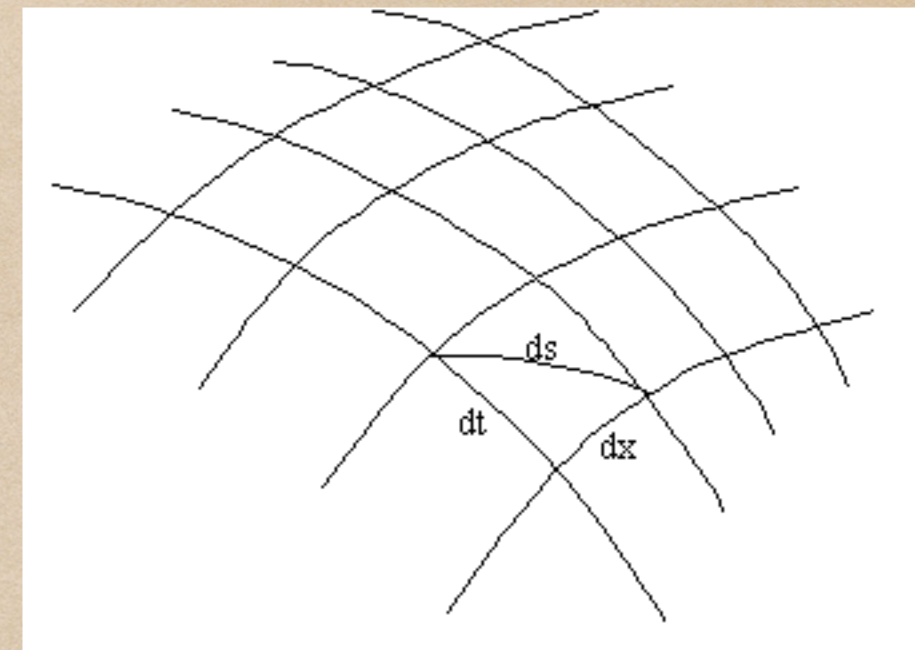
- Deep Precambrian

Overview

- A brief theory of gravitational waves
- Frequency windows, sources and detectors
- Parameter estimation and source modeling
- The GW events
- Some future applications
- Conclusions

Gravitational waves

General relativity in 30 seconds



- Spacetime as a curved manifold
- Key quantity: spacetime metric $g_{\alpha\beta}$
- Curvature, geodesics etc. all follow
- Einstein equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R + \Lambda g_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

- 10 non-linear PDEs for $g_{\alpha\beta}$
- $T_{\alpha\beta} =$ Matter fields
- Conceptually simple,
- hard in practice
- E.g. Schwarzschild

$$g_{\mu\nu} = \begin{pmatrix} \left(1 - \frac{2GM}{rc^2}\right) & 0 & 0 & 0 \\ 0 & -\left(1 - \frac{2GM}{rc^2}\right)^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2\theta \end{pmatrix}$$

$$ds^2 = c^2 dt^2 \left(1 - \frac{2GM}{rc^2}\right) - \frac{dr^2}{1 - 2GM/rc^2} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

General relativity in 30 seconds

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Gravitational waves: weak-field solutions

- Consider small deviations from Minkowski in Cartesian coordinates
- "Background": Manifold $\mathcal{M} = \mathbb{R}^4, \quad \eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$
- "Perturbation": $h_{\mu\nu} = \mathcal{O}(\epsilon) \ll 1 \quad \Rightarrow \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Coordinate freedom: "Transverse-traceless (TT)" gauge

$$h^{\mu}{}_{\mu} = 0, \quad \partial^{\nu} h_{\mu\nu} = 0$$

- Vacuum, no cosmological constant: $T_{\mu\nu} = 0, \quad \Lambda = 0$
- Einstein's eqs.: $\Box h_{\mu\nu} = 0$
- Plane wave solution in z direction: $h_{\mu\nu} = H_{\mu\nu} e^{ik_\sigma x^\sigma}$

$$k^{\mu} = \omega(1, 0, 0, 1) \qquad H_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & H_+ & H_\times & 0 \\ 0 & H_\times & -H_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Effect on particles

- Geodesic eq.
- Particle at rest at x^μ stays at $x^\mu = \mathrm{const}$ in TT gauge
- Proper separation:

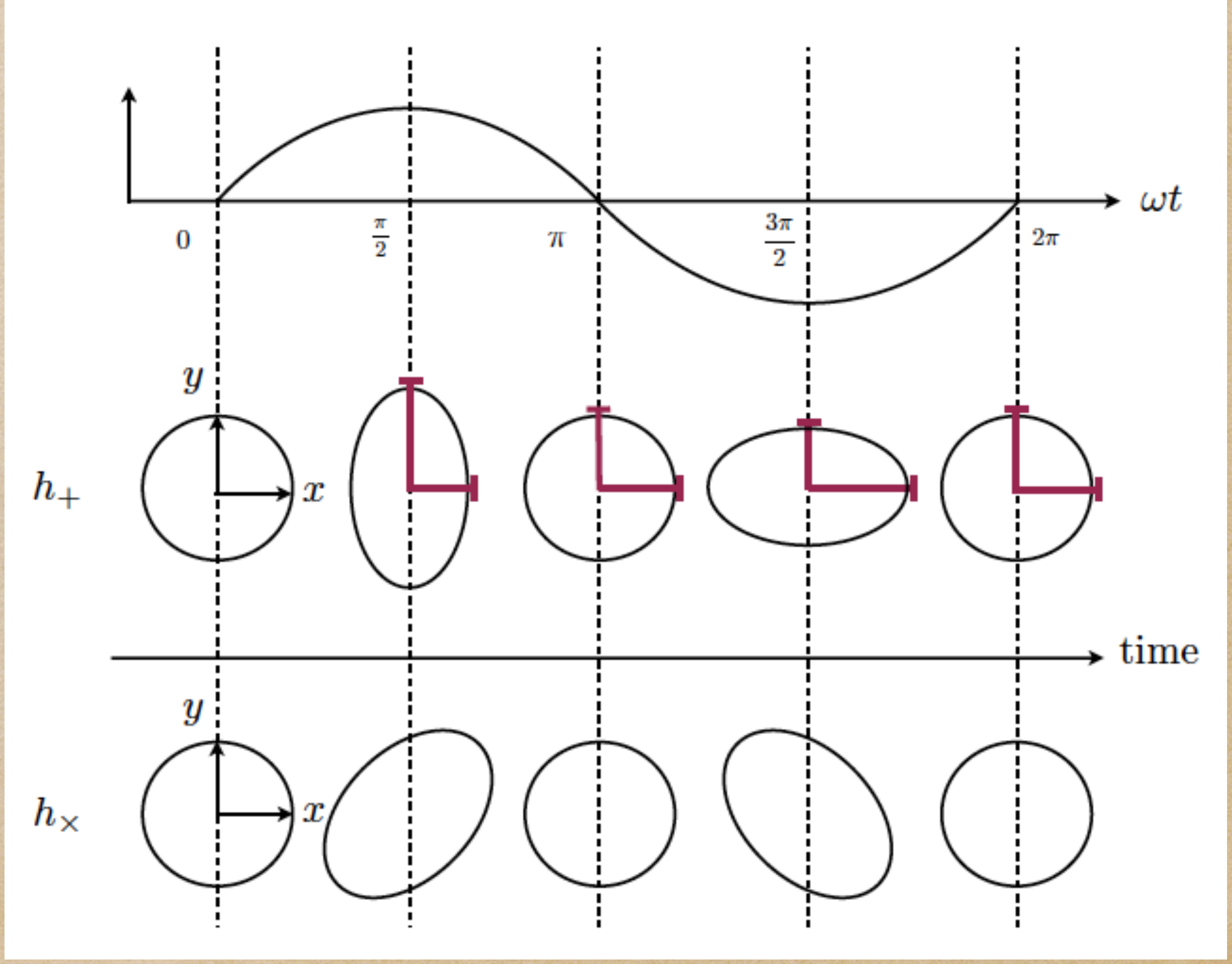
$$ds^2 = -dt^2 + (1+h_+)\,dx^2 + (1-h_+)\,dy^2 + 2h_\times\,dx\,dy + dz^2$$

- Effect on test particles:

  Mirshekari 1308.5240

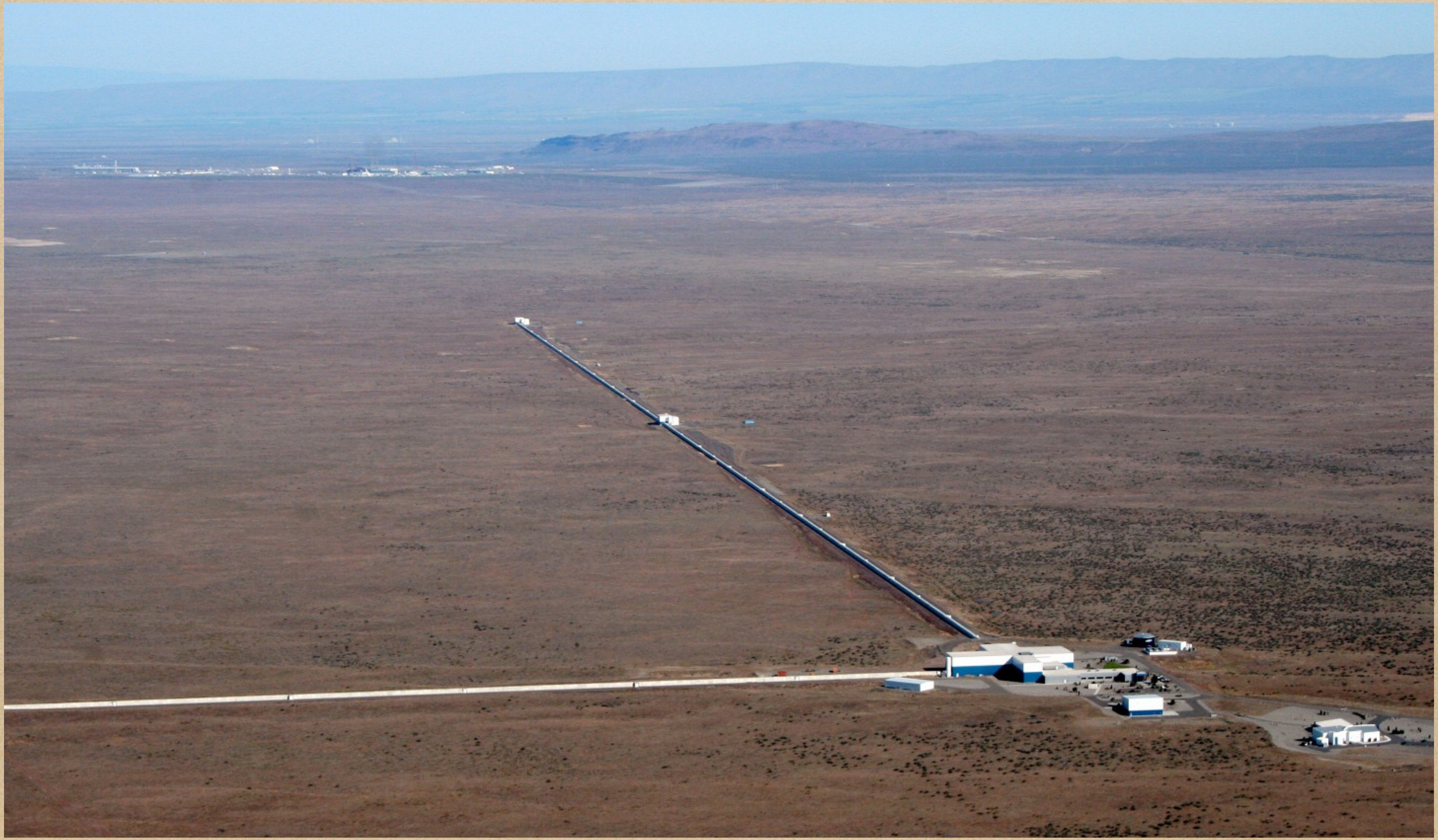
- Debate on physical reality until late 1950s

  e.g.Saulson GRG (2011)



Effect on particles

- Measure this effect; Michelson-Morley type interferometer



The GW spectrum, sources and detectors

The gravitational wave spectrum

- Source types and detection strategies $\Rightarrow$ 4 regimes

  Ultra low $f \sim 10^{-18} \ldots 10^{-15}$ Hz

  Very low $f \sim 10^{-9} \ldots 10^{-6}$ Hz

  Low $f \sim 10^{-4} \ldots 10^{-1}$ Hz

  High $f \sim 10^{1} \ldots 10^{3}$ Hz

- Major sources

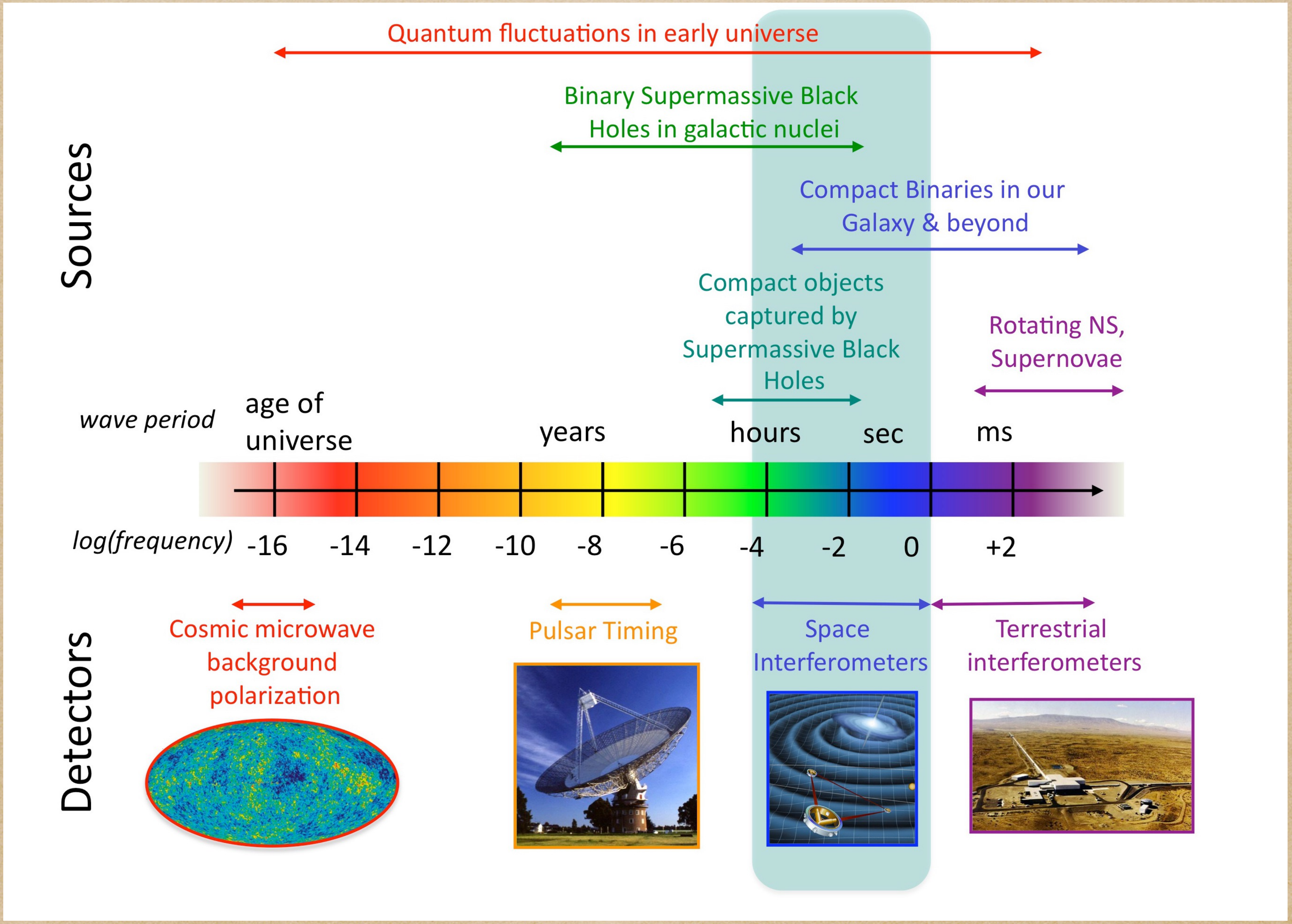
  Ultra low: Fluctuations in the early universe

  Very low: Supermassive BH binaries (high M, z)

  Low: SMBHs, EMRIs, Compact binaries,...

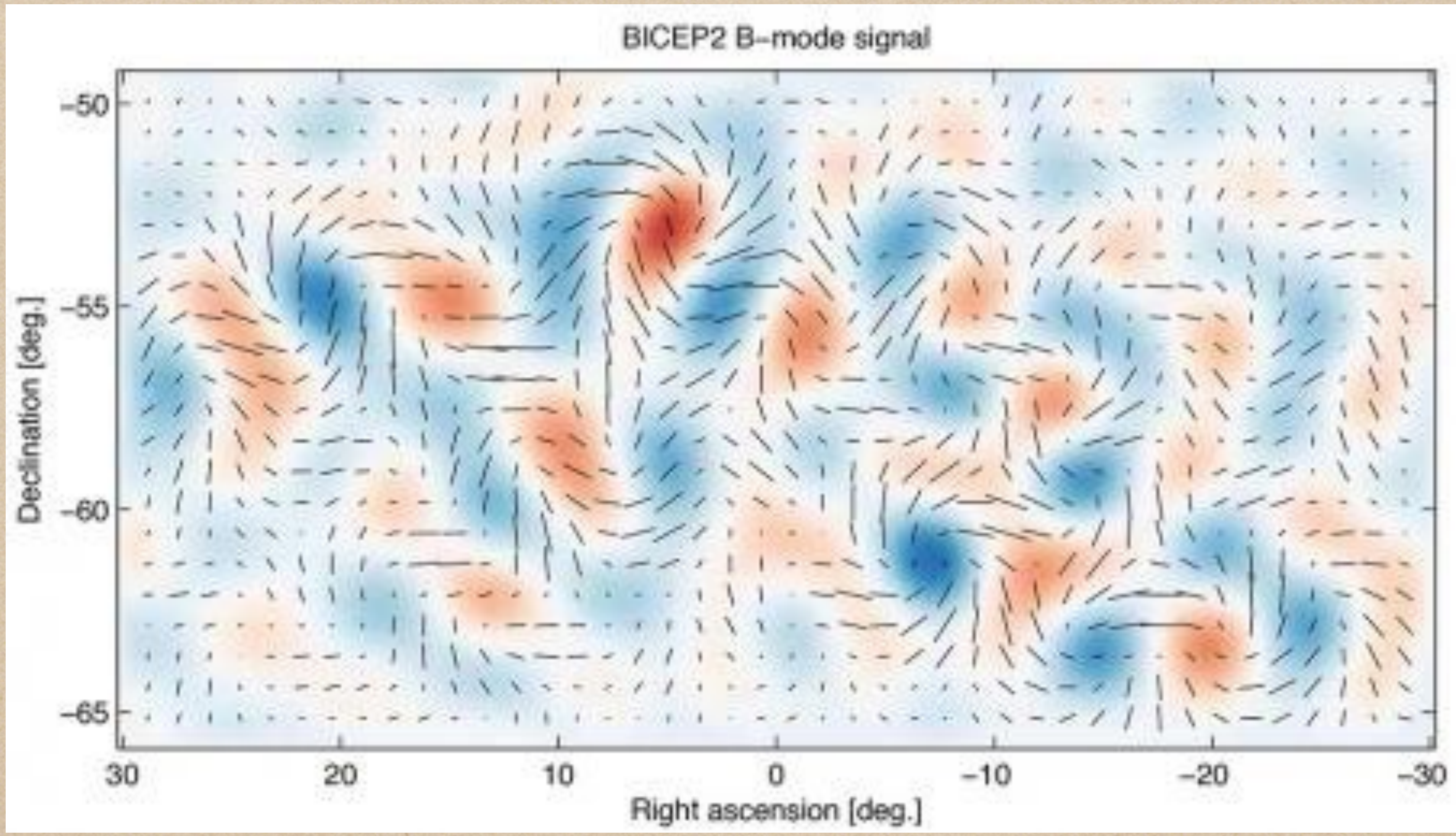
  High: Neutron star / BH binaries, supernovae,...

The gravitational wave spectrum



The ultra low frequency regime

- Wave periods $\sim$ Hubble time
- Primordial GWs $\longrightarrow$ Signature in polarization of CMB
- E.g. BICEP2

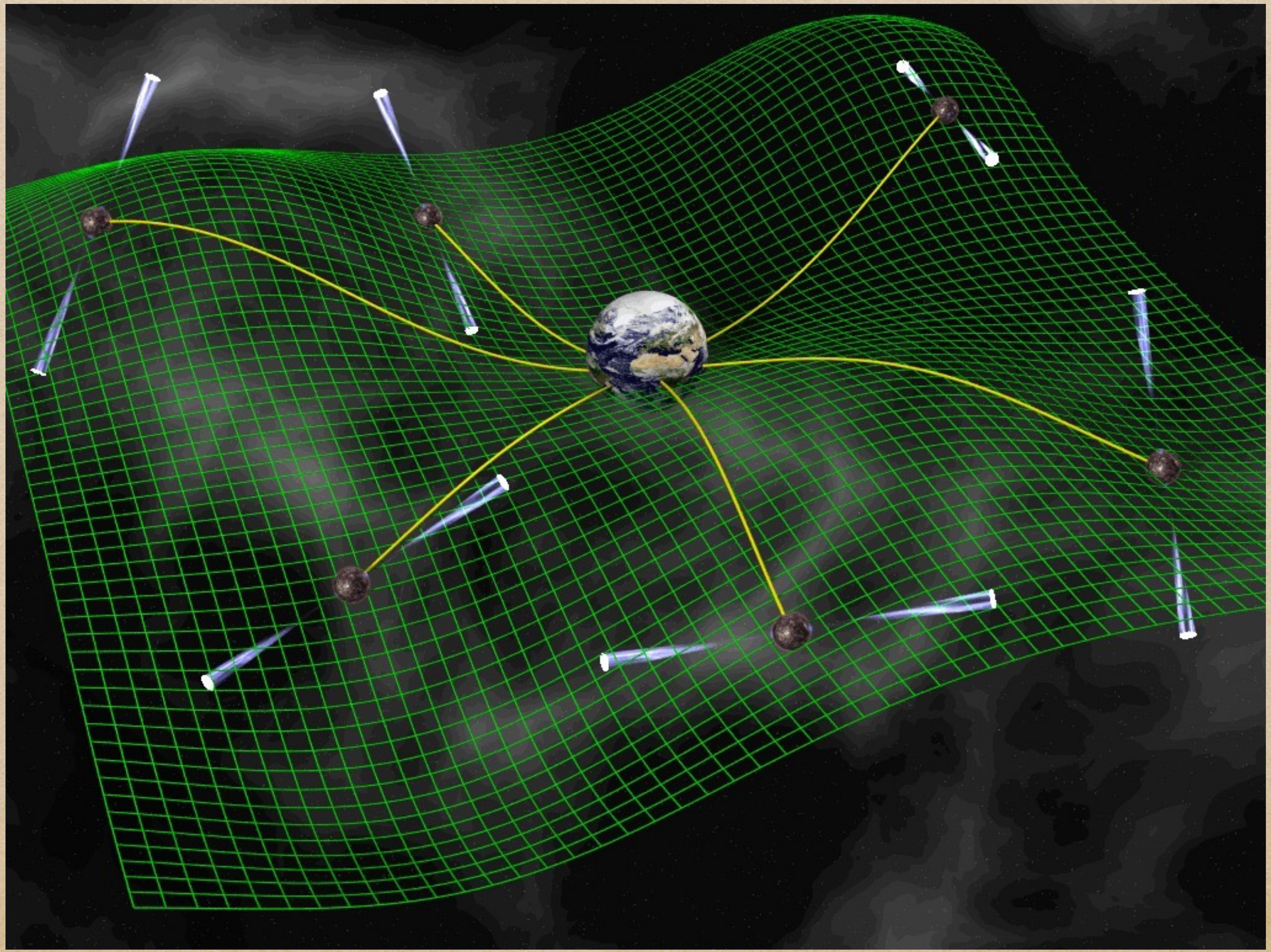


- Problem: Pattern can be attributed to galactic dust (BICEP2, Planck)

  See e.g. Flauger, Hill, Spergel 1405.7351

The very low frequency regime

- Pulsar timing arrays   PPTA, EPTA, NANOGrav
- Search for correlated arrival time delays of pulses



The very low frequency regime

- Exotic sources: Topological defects, cosmic strings (early Universe)
- SMBH binaries $\gtrsim 10^8 \ M_\odot$

  Most/all galaxies host BHs hole-halo correlation: $M_{\rm bh} \propto \sigma^{4.8 \pm 0.5}$

  Ferrarese & Merrit ApJ (2000), Gültekin et al, ApJ (2009)
- Galaxies merge $\Rightarrow$ SMBH merger

  But "Final parsec problem"
- Few individually observed systems possible.

  But mostly stochastic background.

  Model as power law

$$h_c = A \left(\frac{f}{\mathrm{yr}^{-1}} \right)^{\alpha}$$

The low frequency regime

The low frequency regime

- Interferometry with $\sim 10^6\ \mathrm{km}$ arms
- Realm of space missions
- LISA: L3 mission of ESA's "Cosmic Vision" Launch: ~ 2034
- Configuration still uncertain:

  2 arms vs. 3 arms

  $10^6\ \mathrm{km}$ vs. $5 \times 10^6\ \mathrm{km}$

  2 yr vs. 5 yr life span
- Verification binaries (WDs)
- Outstanding SNR
- LISA Pathfinder: Test mission

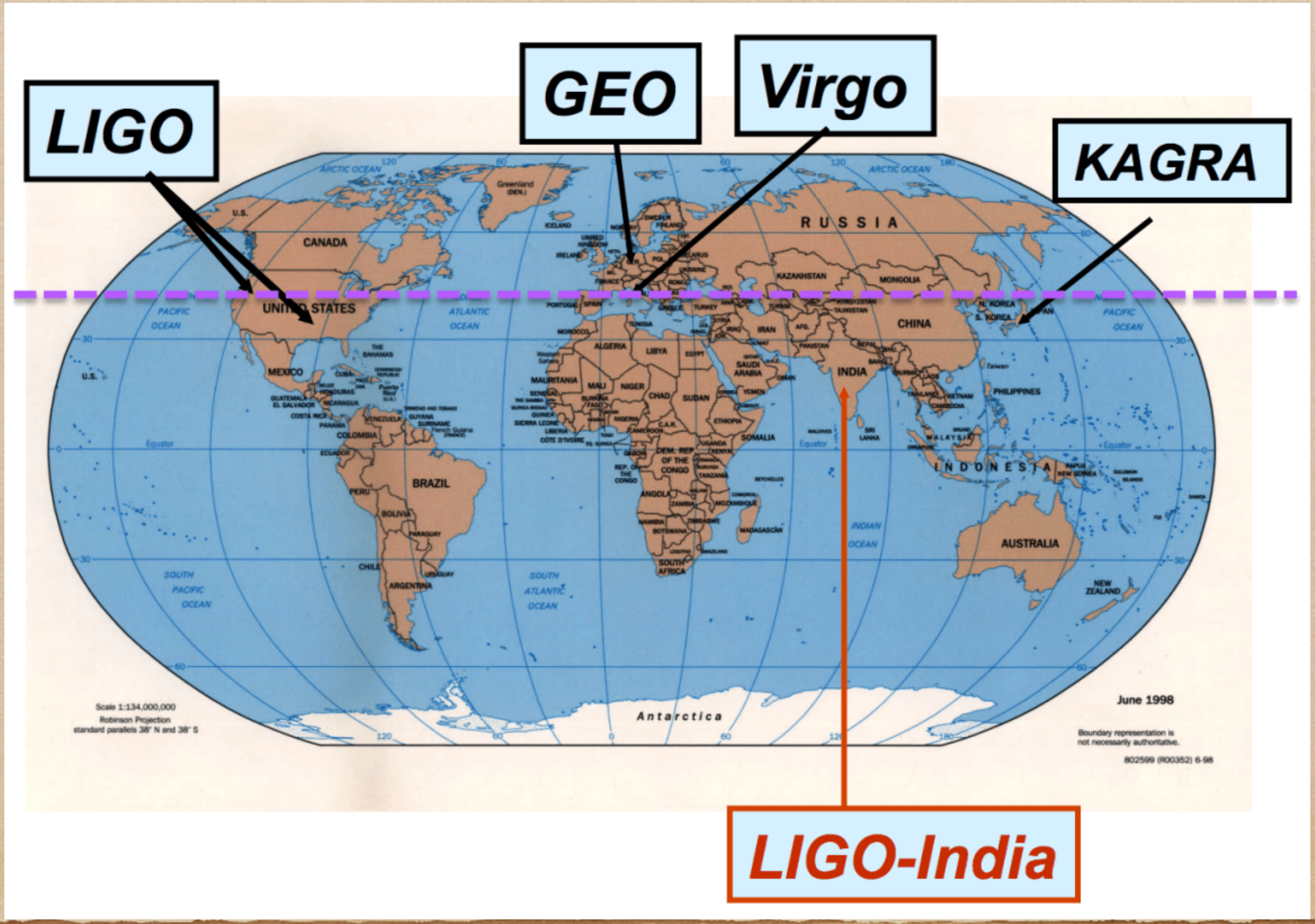
  Launched 3 Dec 2015

  Major success!!!

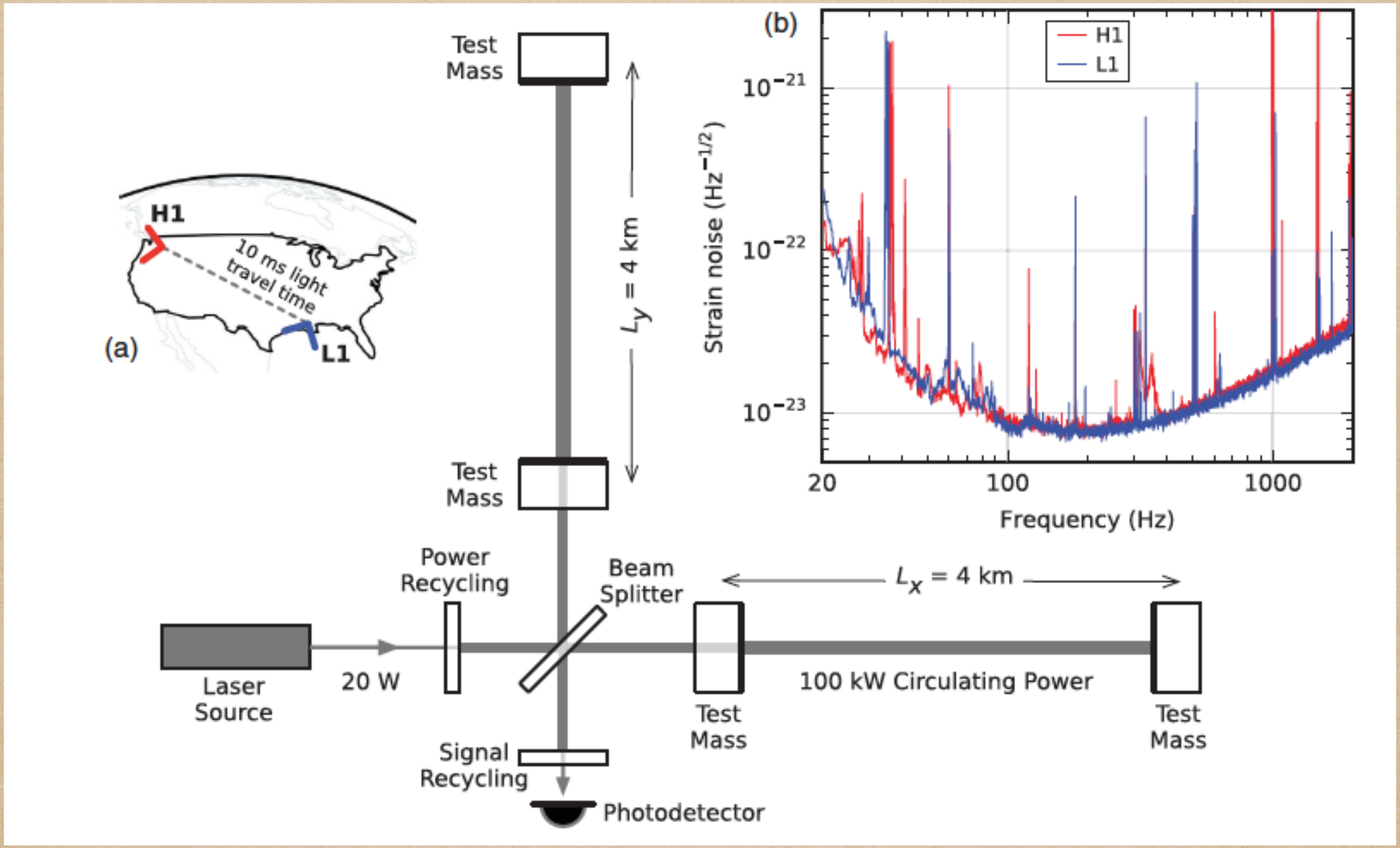
The high frequency regime

- Interferometry with $\sim \mathrm{km}$ arms
- Detector nework:

2 LIGO (2015), Virgo (2017), GEO600, KAGRA (2020), LIGO-India



The interferometer diagram: LIGO

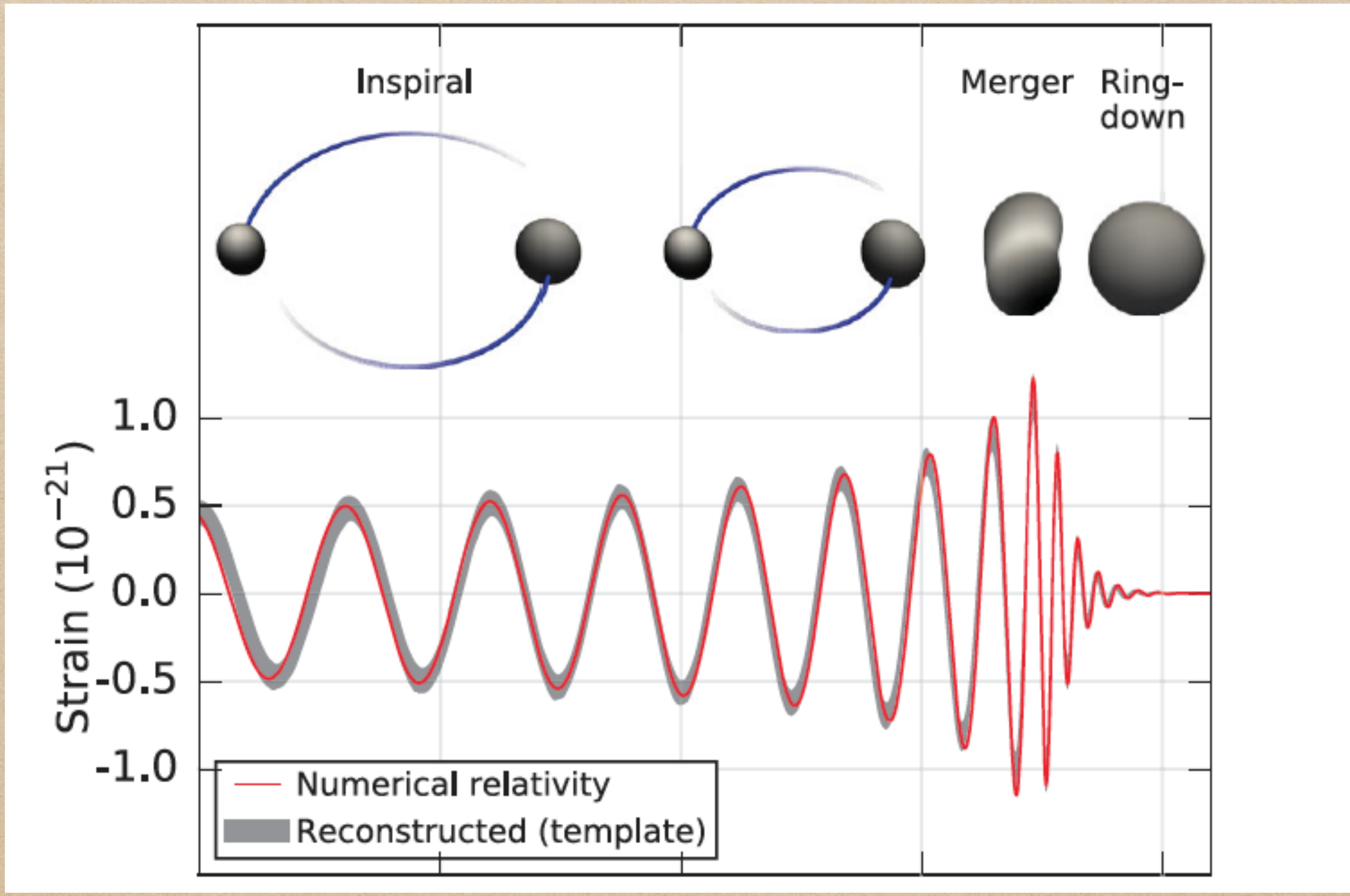


Abbott et al, PRL 116 (2016) 061102

Seismic, thermal, shot noise

The high frequency regime

- Supernovae
- Neutron star oscillations
- Neutron star / stellar-mass black hole binaries



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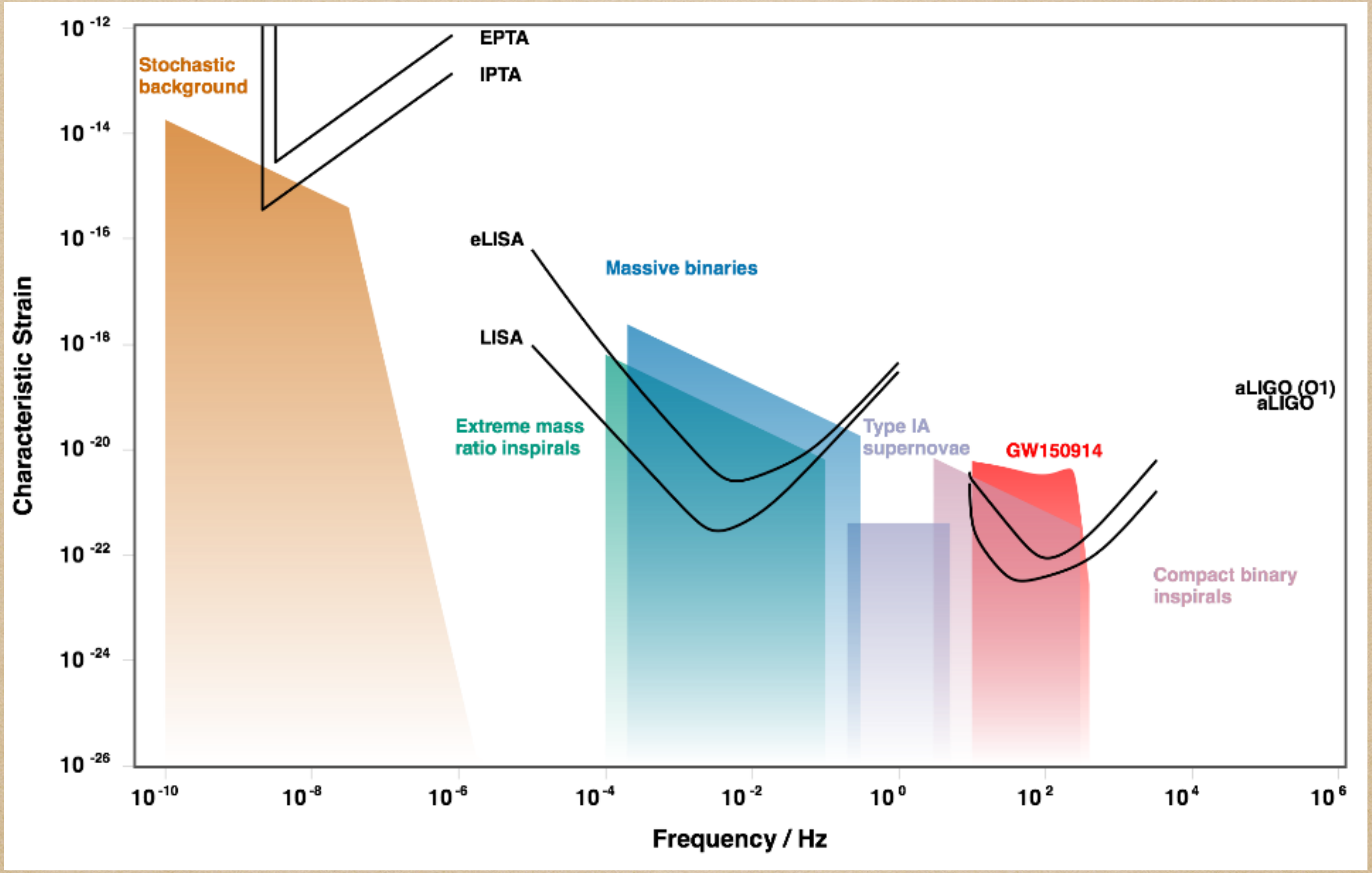
Summary: sensitivity curves



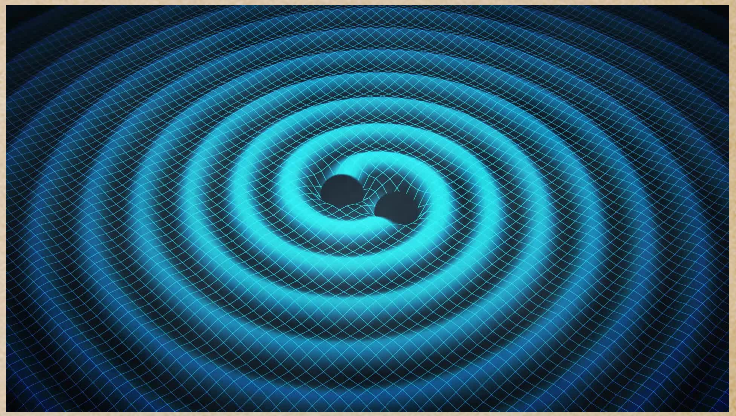
http://gwplotter.com

Parameter estimation and source modeling

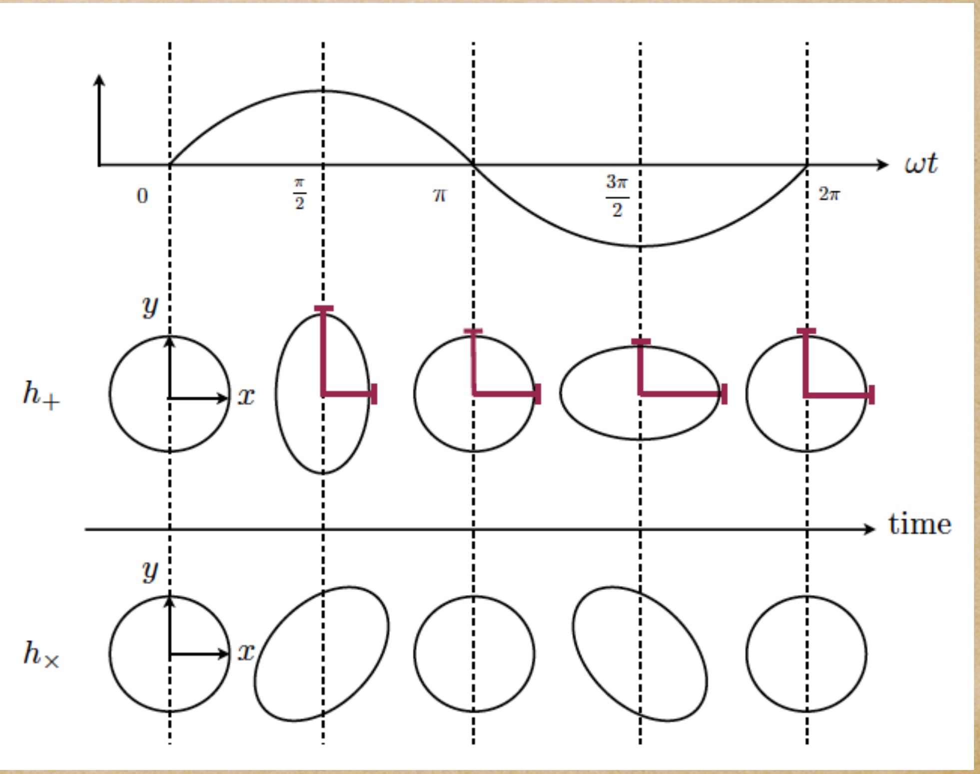
The search for GWs in the data stream

- Einstein's field equations

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}\,; \qquad \frac{8\pi G}{c^4} = 2.07 \times 10^{-43} \frac{\mathrm{s}^2}{\mathrm{m\,kg}}$$



  - Weak effect of matter on geometry
  - GWs carry huge energy but barely interact with anything
  - Induced changes in length: < atomic nucleus / km



Detection and parameter estimation

Generic transient search

- No specific waveform model
- Identify excess power in detector strain data
- Use multi detector maximum likelihood   Klimenko et al. 1511.05999

Binary coalescence search

- "Matched Filtering"
- Compare data stream with GW templates ("Finger print search")
- Bayesian analysis: Prior $\rightarrow$ Posterior

Black-hole binaries: parameters

- 8+1 Intrinsic parameters

  Masses $m_1\,,\ m_2$

  Spins $\boldsymbol{S}_1\,,\ \boldsymbol{S}_2$

  Eccentricity (often ignored; GW emission circularizes orbit)

- 7 Extrinsic parameters

  Location: Luminosity distance D_L, Right ascension α, Declination δ

  Orientation: Inclination ι, Polarization ψ

  Time t_c and Phase ϕ_c of coalescence

- Other parameters

  Anything beyond vacuum GR.... (ignored for starters...)

GW source modeling

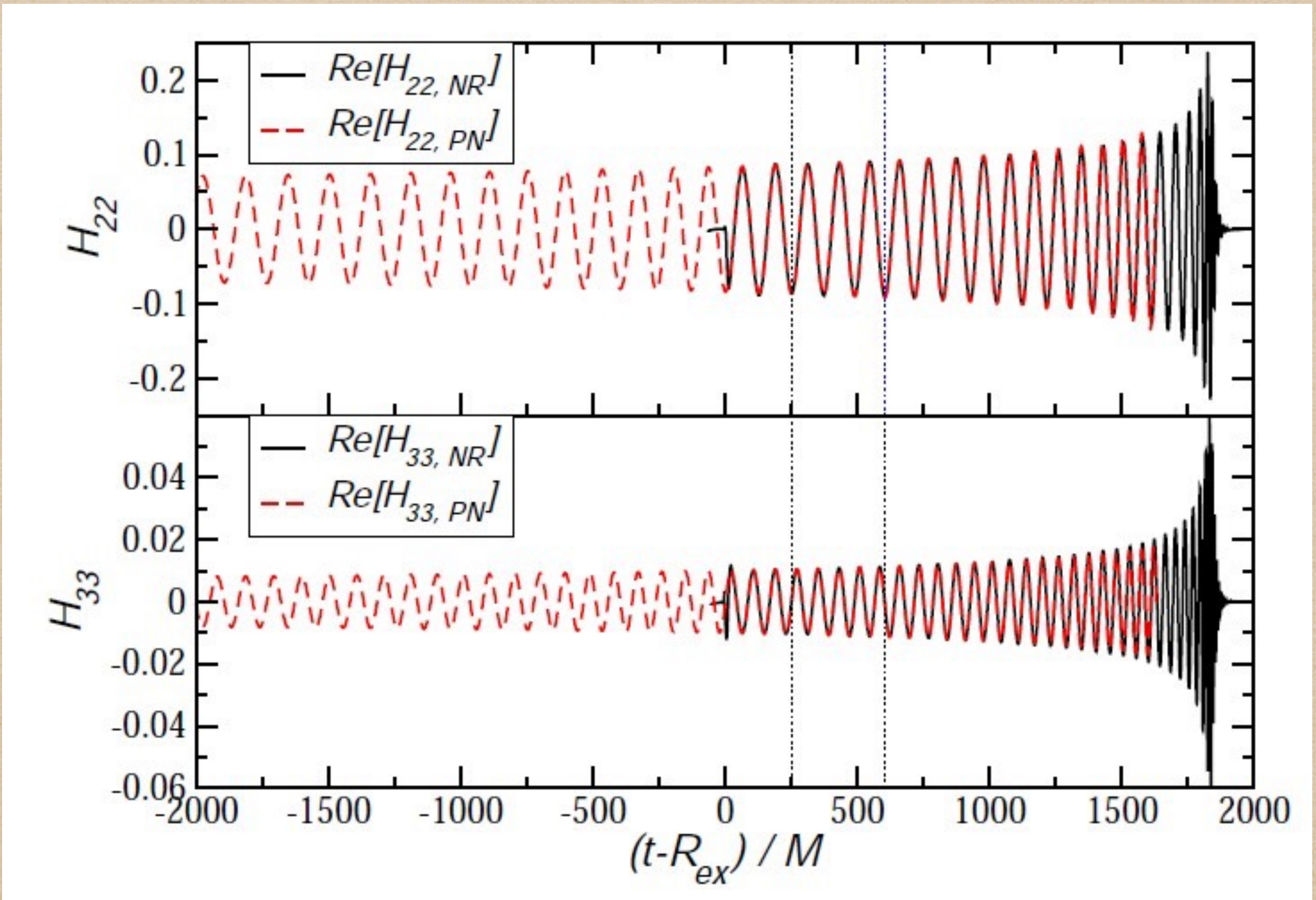
- Key requirement for matched filtering: GW template catalog
- Model black holes in general relativity
  - Post Newtonian theory $\rightarrow$ Inspiral   Blanchet Liv.Rev.Rel. 2006
  - Numerical relativity $\rightarrow$ final orbits, merger

    Pretorius PRL 2005, Baker et al PRL 2006, Campanelli et al PRL 2006
  - Perturbation theory $\rightarrow$ Ringdown
- Combine “NR” with “Post-Newtonian”, “Effective one body” methods
- 2 families in use: Phenomenological, Effective one body
- Use reduced bases or similar to cover parameter space
- Multipolar decomposition

$$h_+ - ih_\times = \sum_{\ell m} {}_{-2}Y_{\ell m}(\theta,\phi)h_{\ell m}(t)$$

Hybrid waveforms and catalogs

- Stitch together PN and NR waveforms

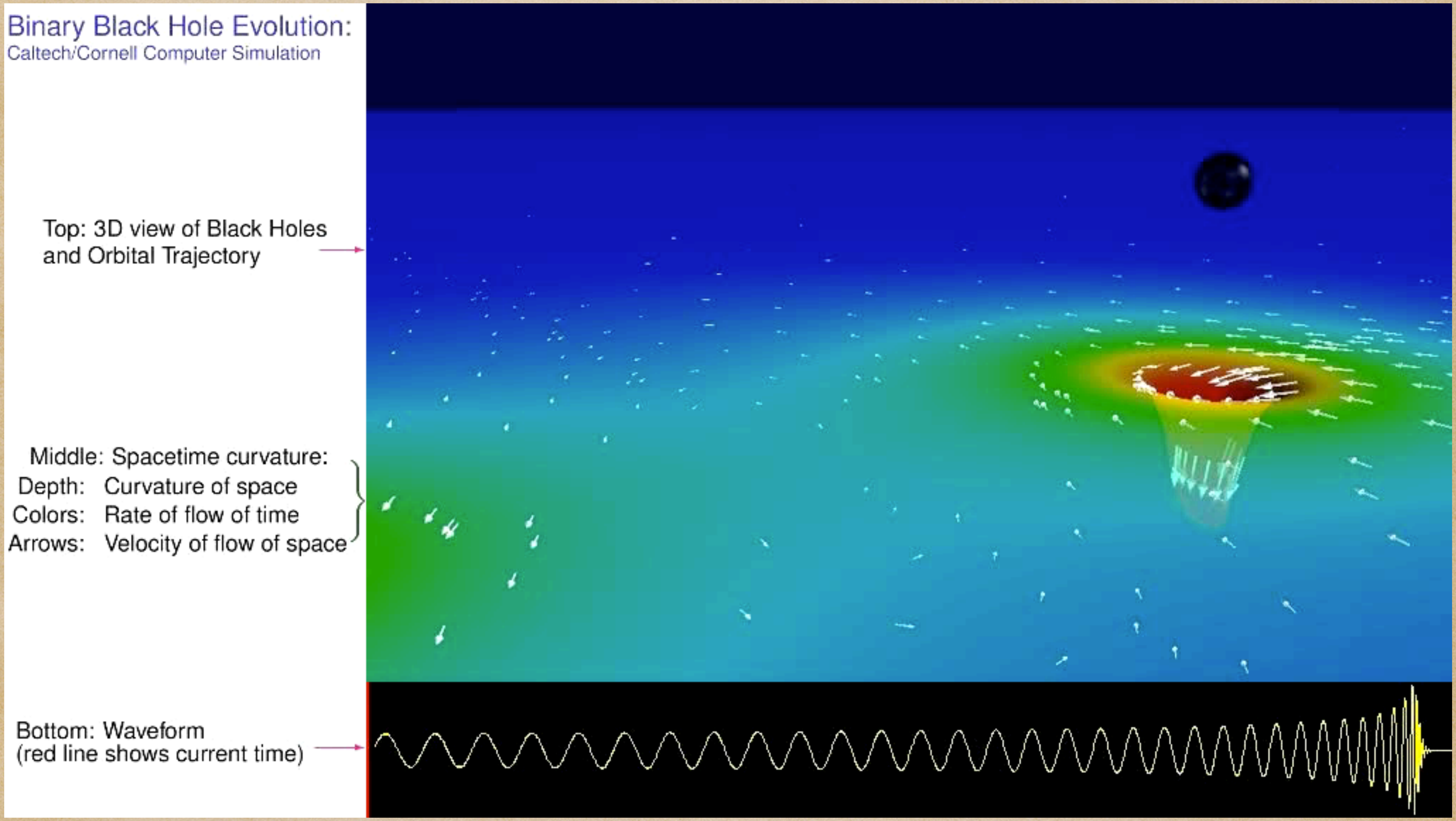


US et al CQG 2011

- Mass produce waveforms;

Hinder et al CQG 2013, Mroué et al PRL 2013, Boyle et al CQG 2019

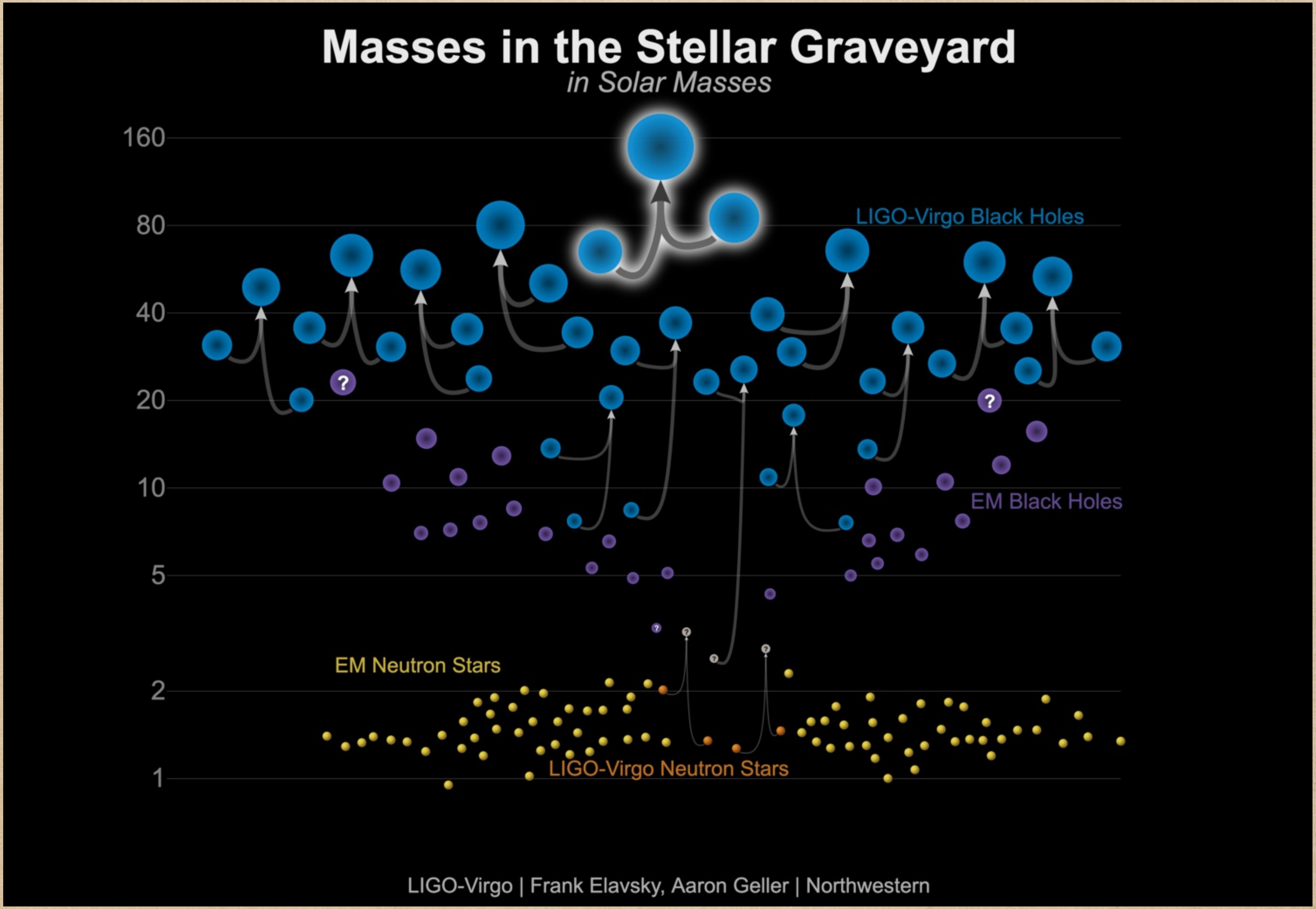
Anatomy of a BHB coalescence



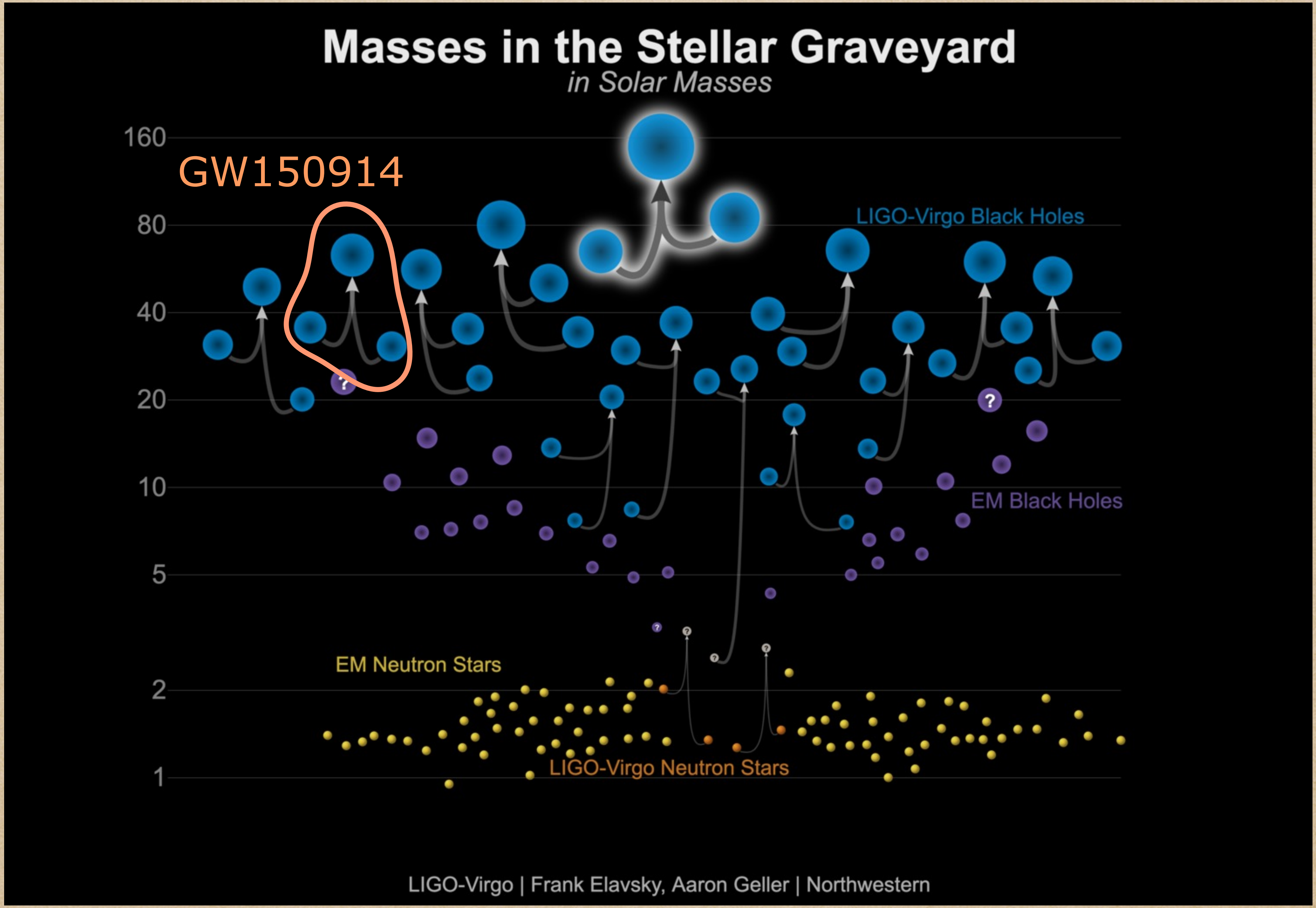
Thanks to Caltech-Cornell groups

The GW events

Subset of GW detections up to 2024



GW150914



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GW150914
LIGO-Virgo Black Holes
EM Black Holes
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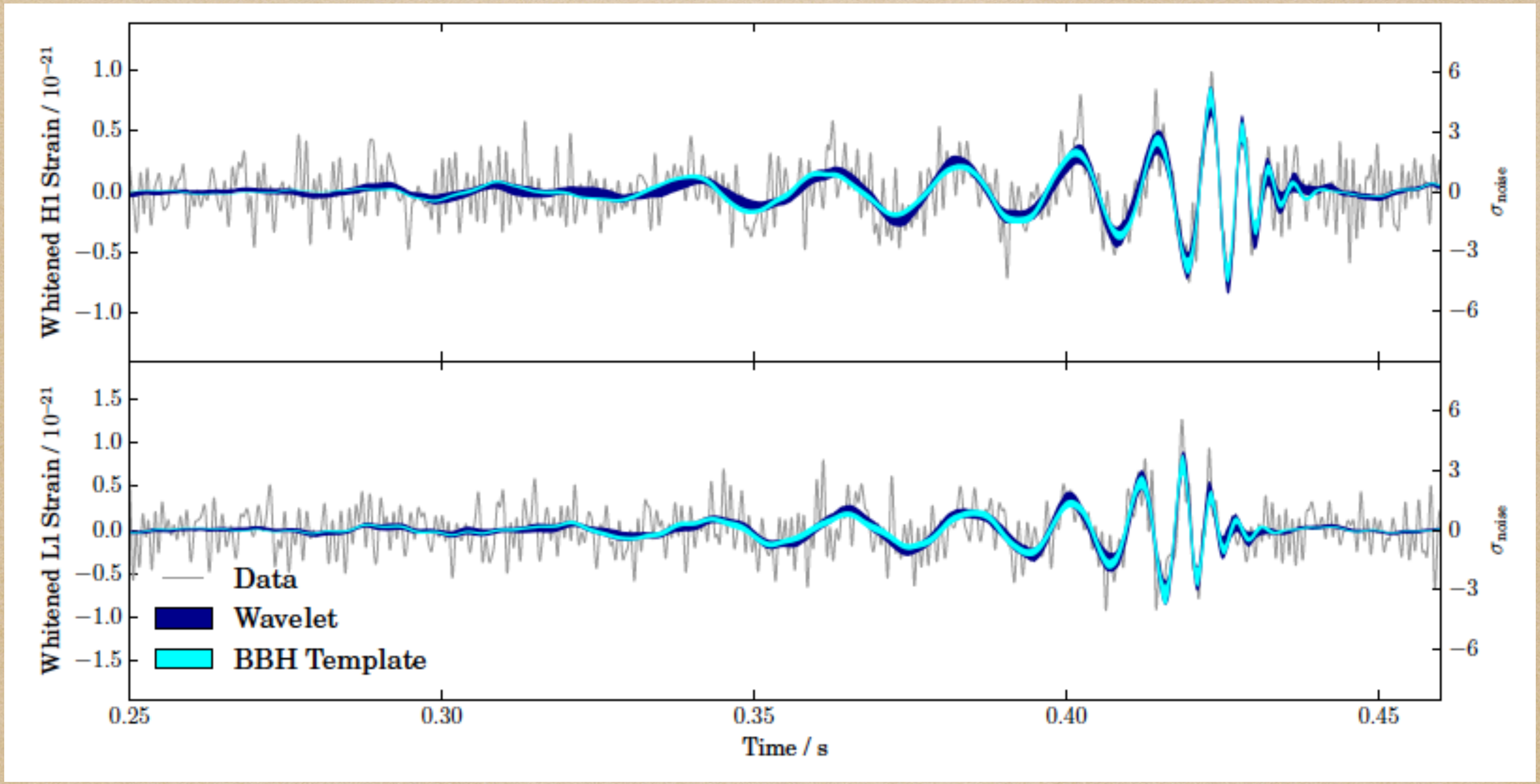
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

GW150914: The signal



Abbott et al 1602.03840

- Whitened by power spectral density
- Wavelet = Linear combination of sine-Gaussian pieces

GW150914: BH masses

- Source frame
- 2 Waveform models

Abbott et al. 1602.03840

$$m_1 = 36^{+5}_{-4}\ M_\odot$$

$$m_2 = 29^{+4}_{-4}\ M_\odot$$

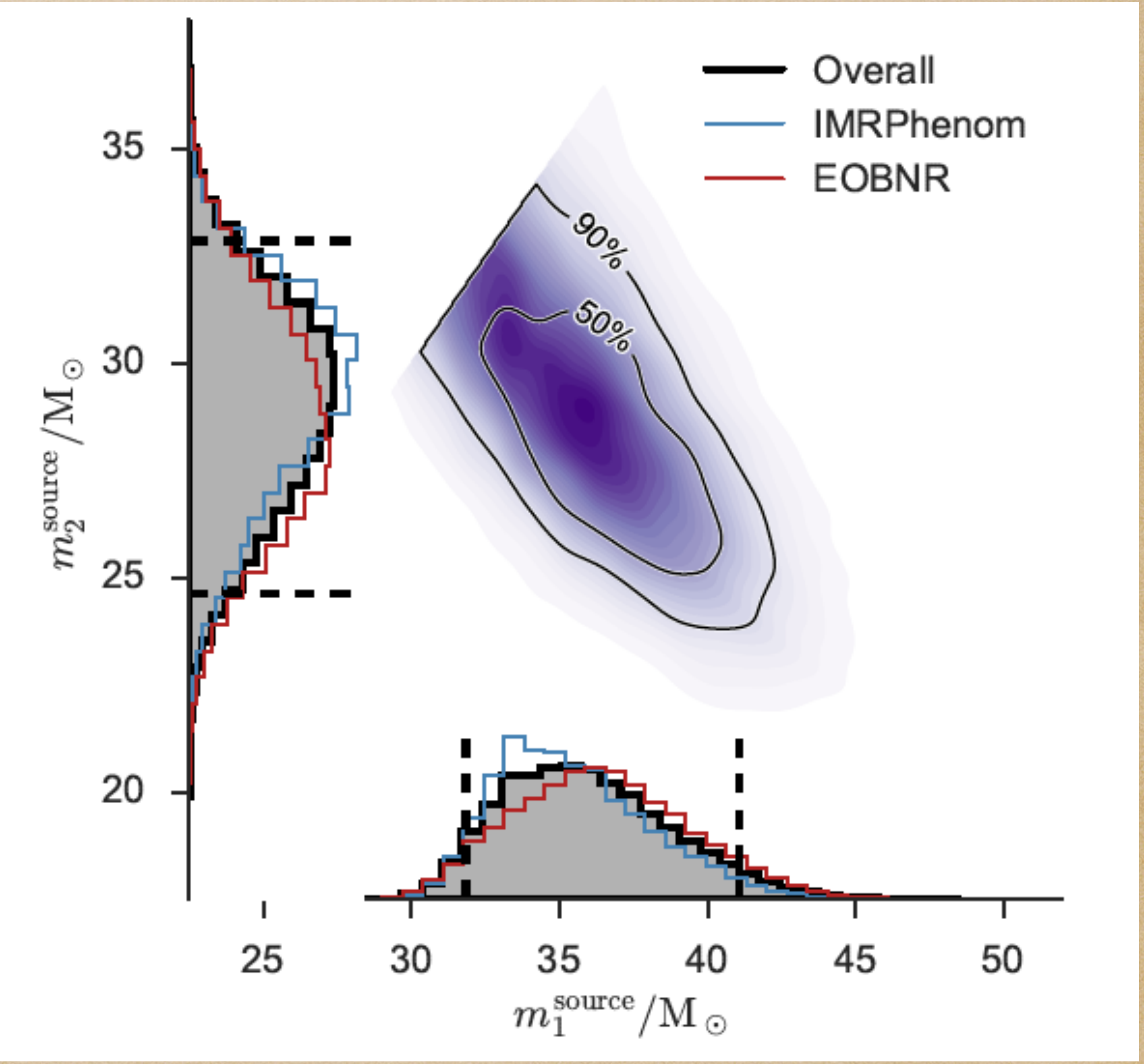
$$M_{\rm fin} = 62^{+4}_{-4}\ M_\odot$$

- Deficit in GWs!

$$\Delta M \approx 3\ M_\odot$$

$$\approx 5.4 \times 10^{54}\ {\rm erg}$$

$$L_{\rm max} \approx 3.6 \times 10^{56}\ {\rm erg/s}$$



Abbott et al 1602.03840

GW150914: BH parameters

- Mass ratio $q \equiv \frac{m_2}{m_1} = 0.65 \pm 0.03$
- Spins harder to measure: few cycles, no full-precession catalog

  $$\chi_1 = \frac{|\boldsymbol{S}_1|}{m_1^2} < 0.7\,, \quad \chi_2 = \frac{|\boldsymbol{S}_2|}{m_2^2} < 0.9$$

  $$\chi_{\rm fin} = 0.67^{+0.05}_{-0.07}$$

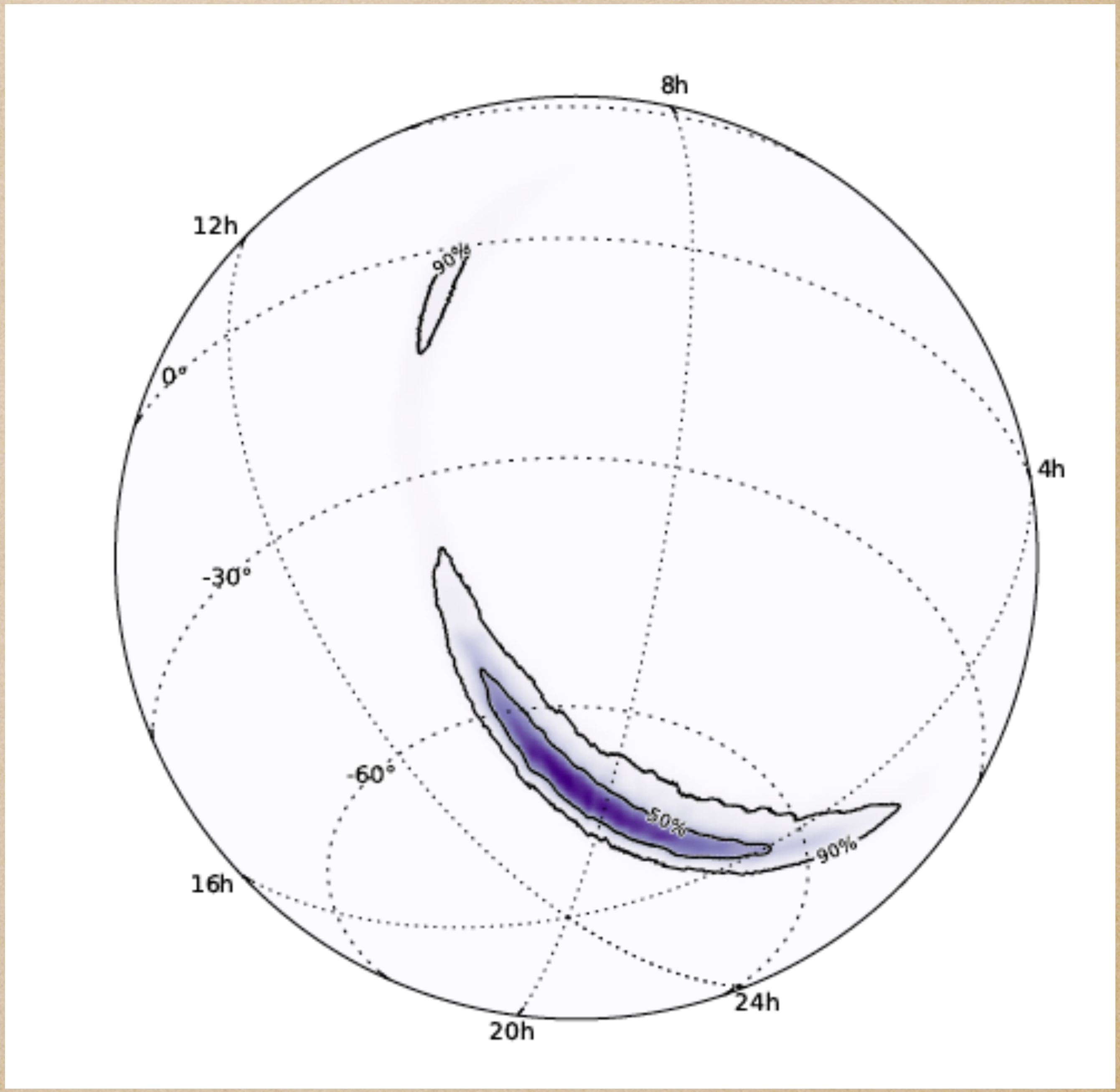
- Luminosity distance $D_L = 410^{+160}_{-180}\ \mathrm{Mpc}$
- Source redshift $z = 0.088^{+0.031}_{-0.038}$

GW150914: Sky location

- Important for EM follow-up
- GW detectors are all-sky
- Via triangulation
- 2 detectors

  $\sim 590 \text{ deg}^2$
- Southern hemisphere
- To be improved with

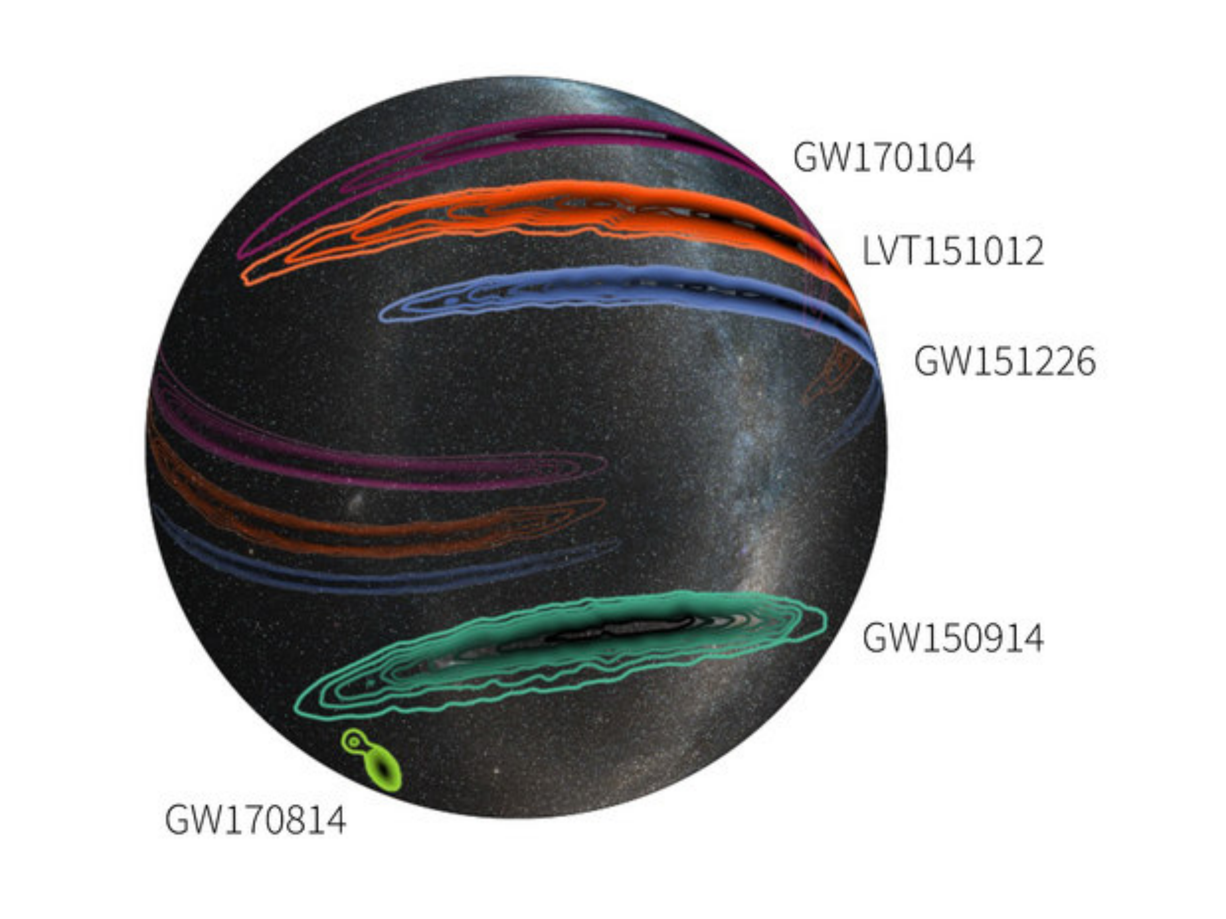
  Virgo, KAGRA, LIGO India



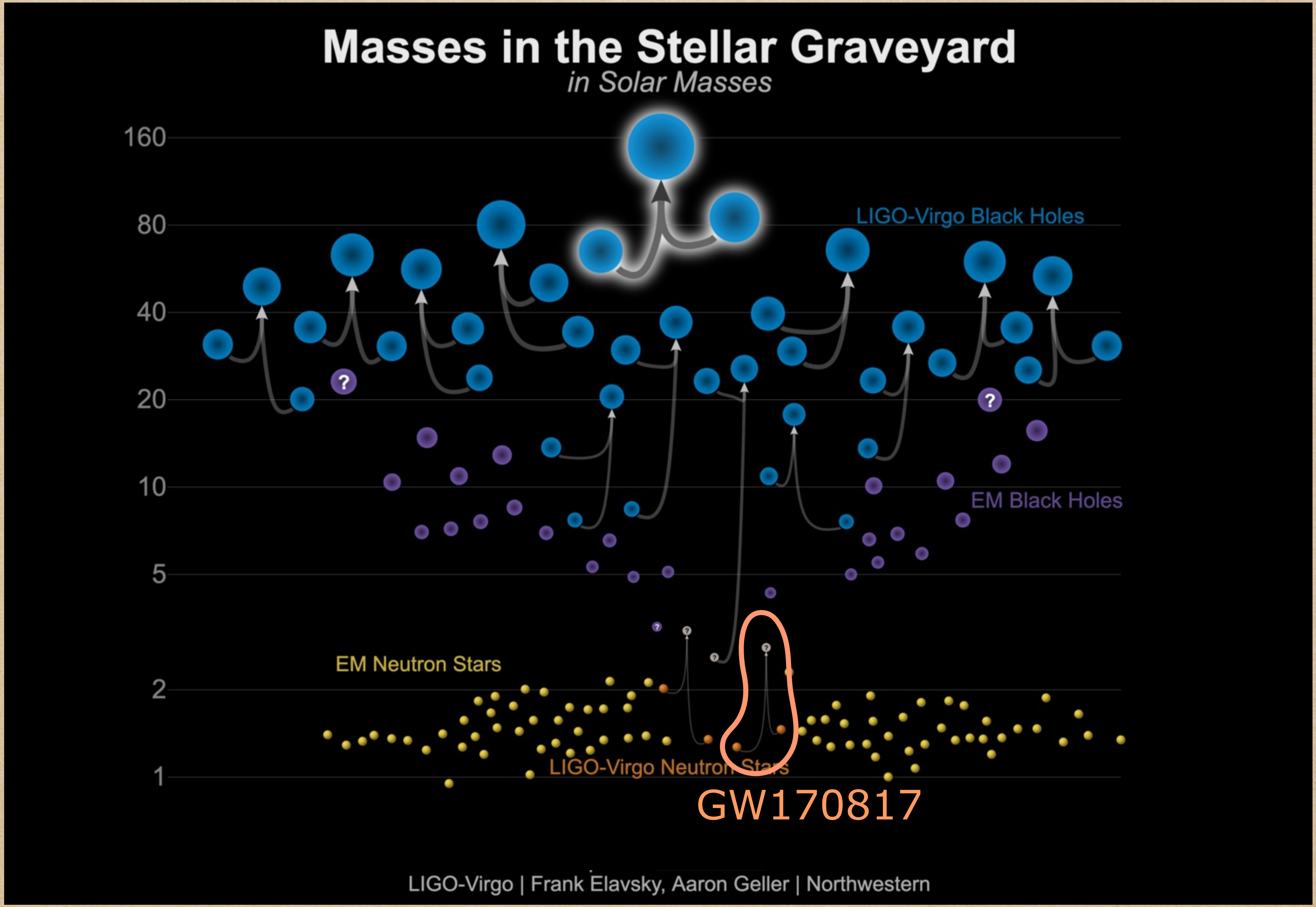
Abbott et al 1602.03840

2017: Virgo joins O2

- 3 detectors
- much improved sky location!
- First triple coincidence detection GW170814



GW170817



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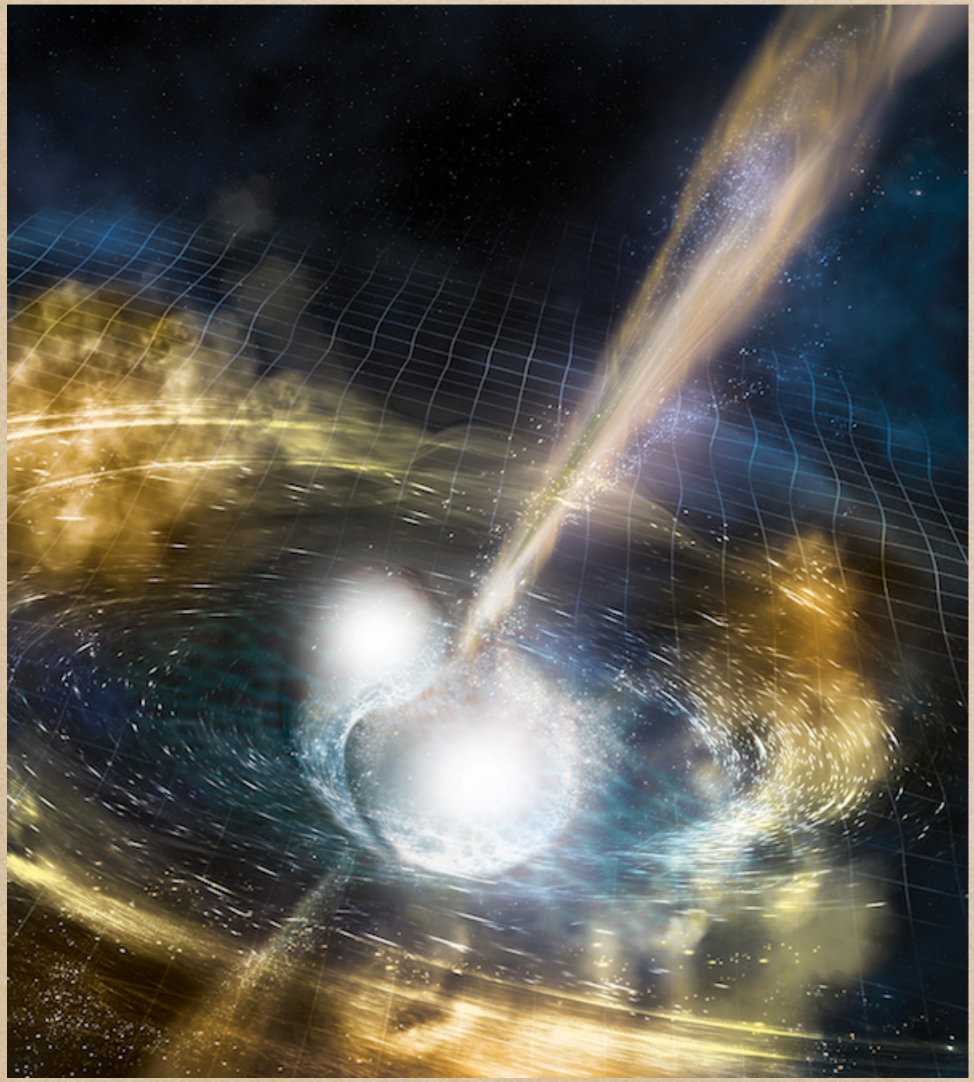
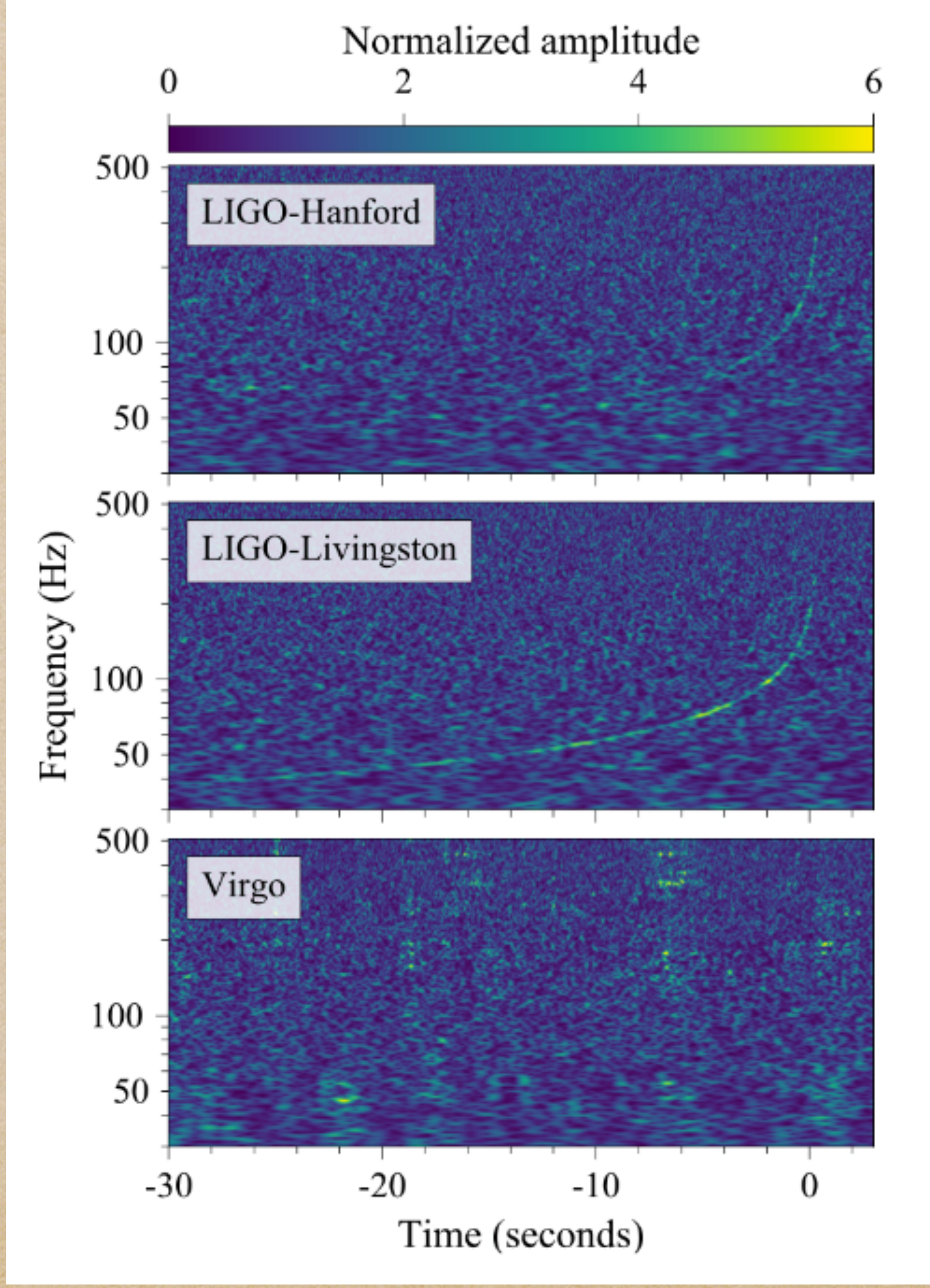
GW170817: Binary Neutron Star Merger

- Fermi and Gamma-Ray Burst Monitor send GRB170817A alert
- Within 6 minutes, LIGO, Virgo find GW170817 in the data stream





GW170817: Binary Neutron Star Merger

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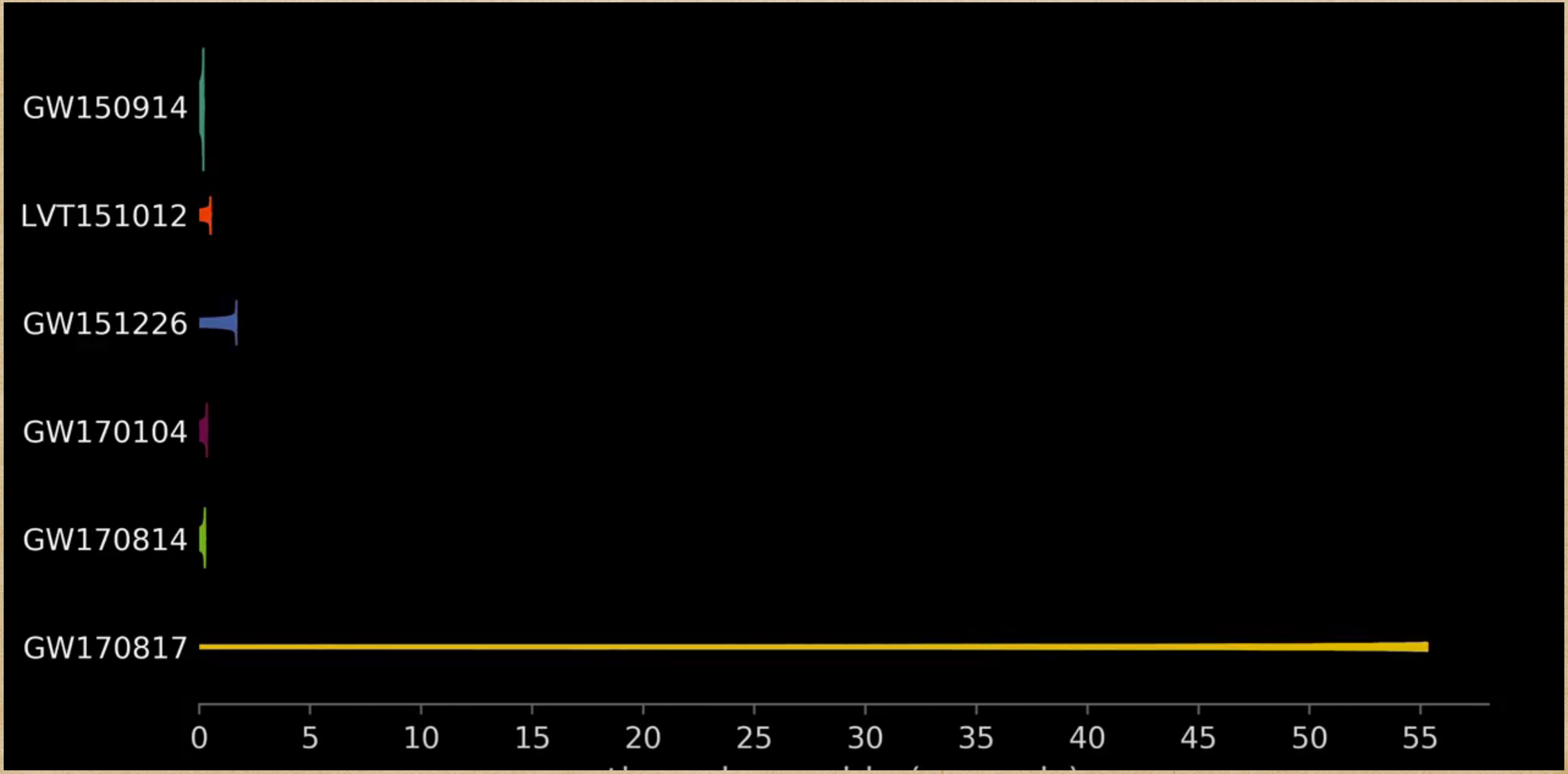
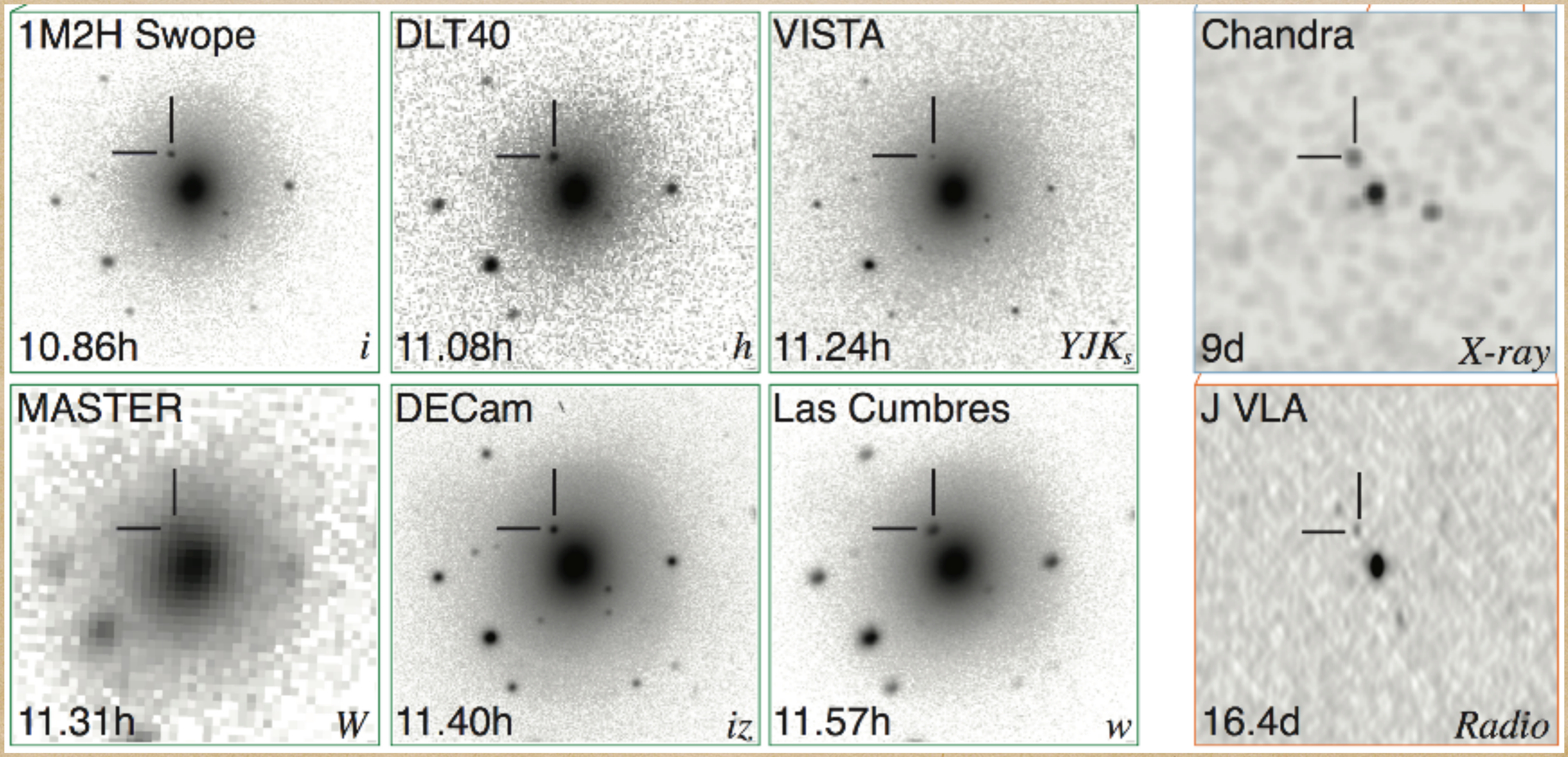
- Within 6 minutes, LIGO, Virgo find GW170817 in the data stream
- Note the duration!!!



GW170817: Multi messenger astronomy

- Gravitational Waves, Gamma rays
- Optical: 6 left panels
- X-ray, Radio: Chandra, Jansky VLA



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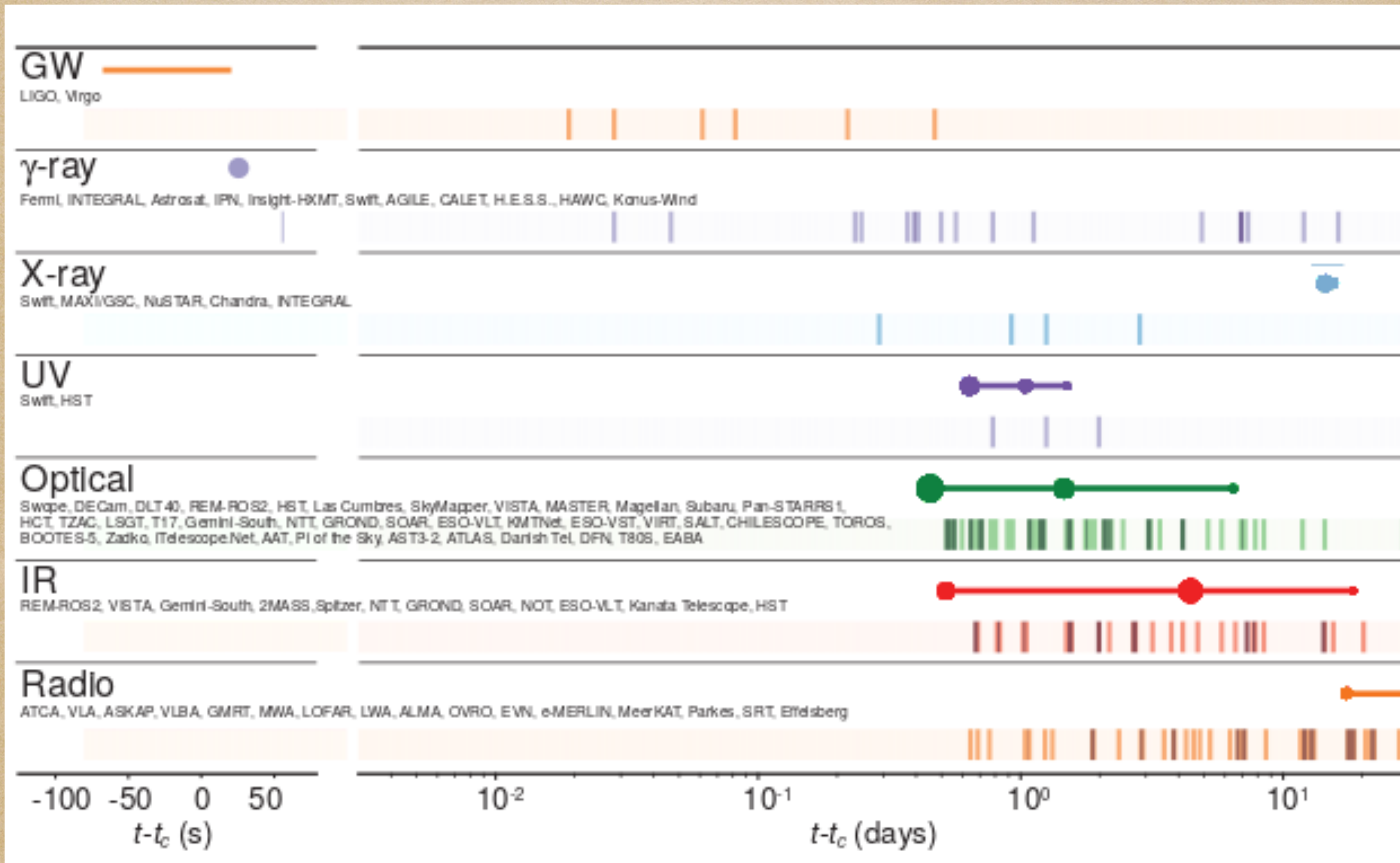
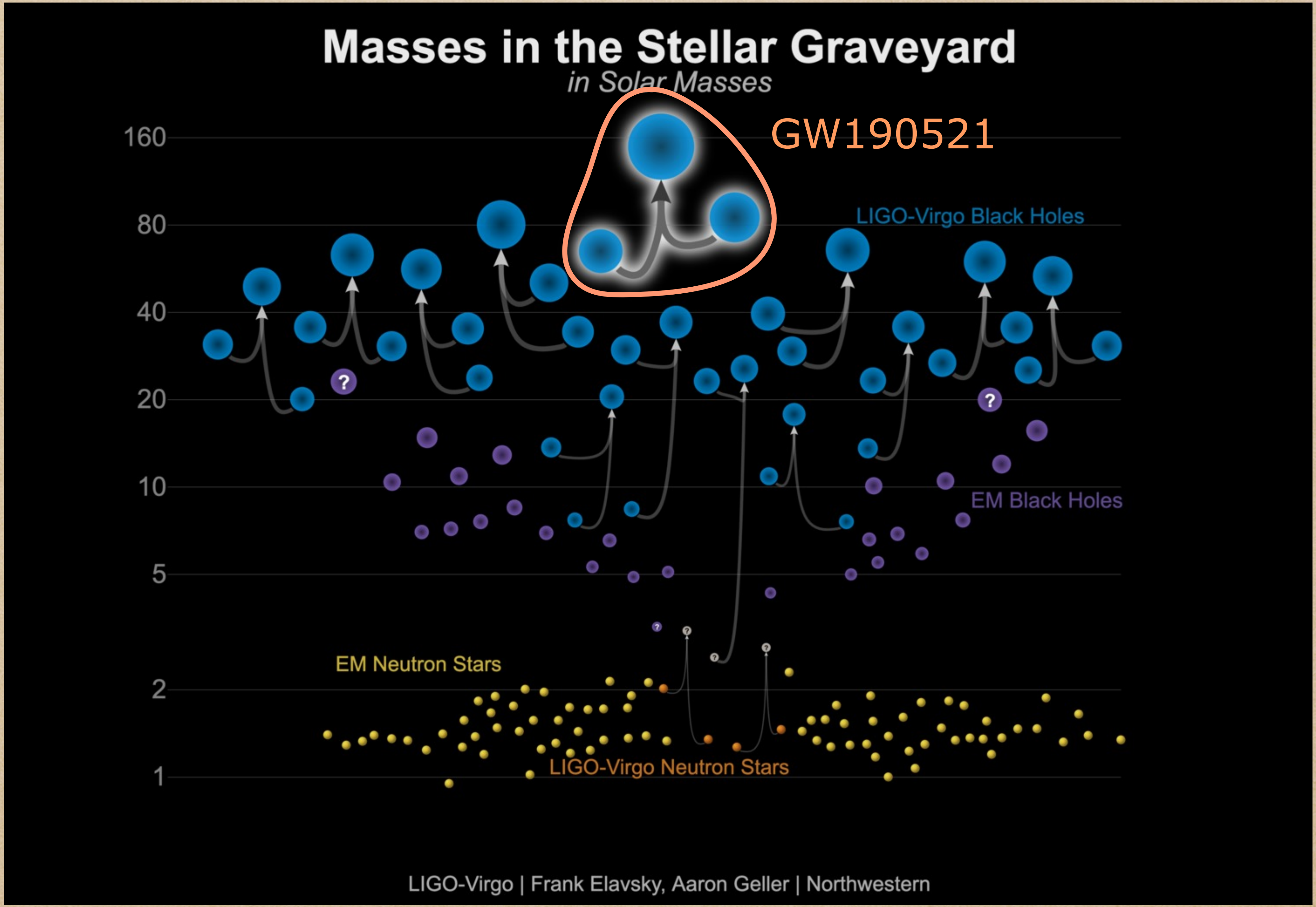


Some things we have learned

- GRB associated with Binary Neutron Star Merger
- Kilonova (infrared transient) due to decay of heavy elements
- Independent measurement of Hubble constant:

  $H_0 = 70^{+12}_{-8}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$
- Binary NS merger rate: $1540^{+3200}_{-1220}\ \mathrm{Gpc^{-3}\,yr^{-1}}$
- First insight into EOS at supernuclear densities through tidal effects
- Results will improve with more NS events

GW190521



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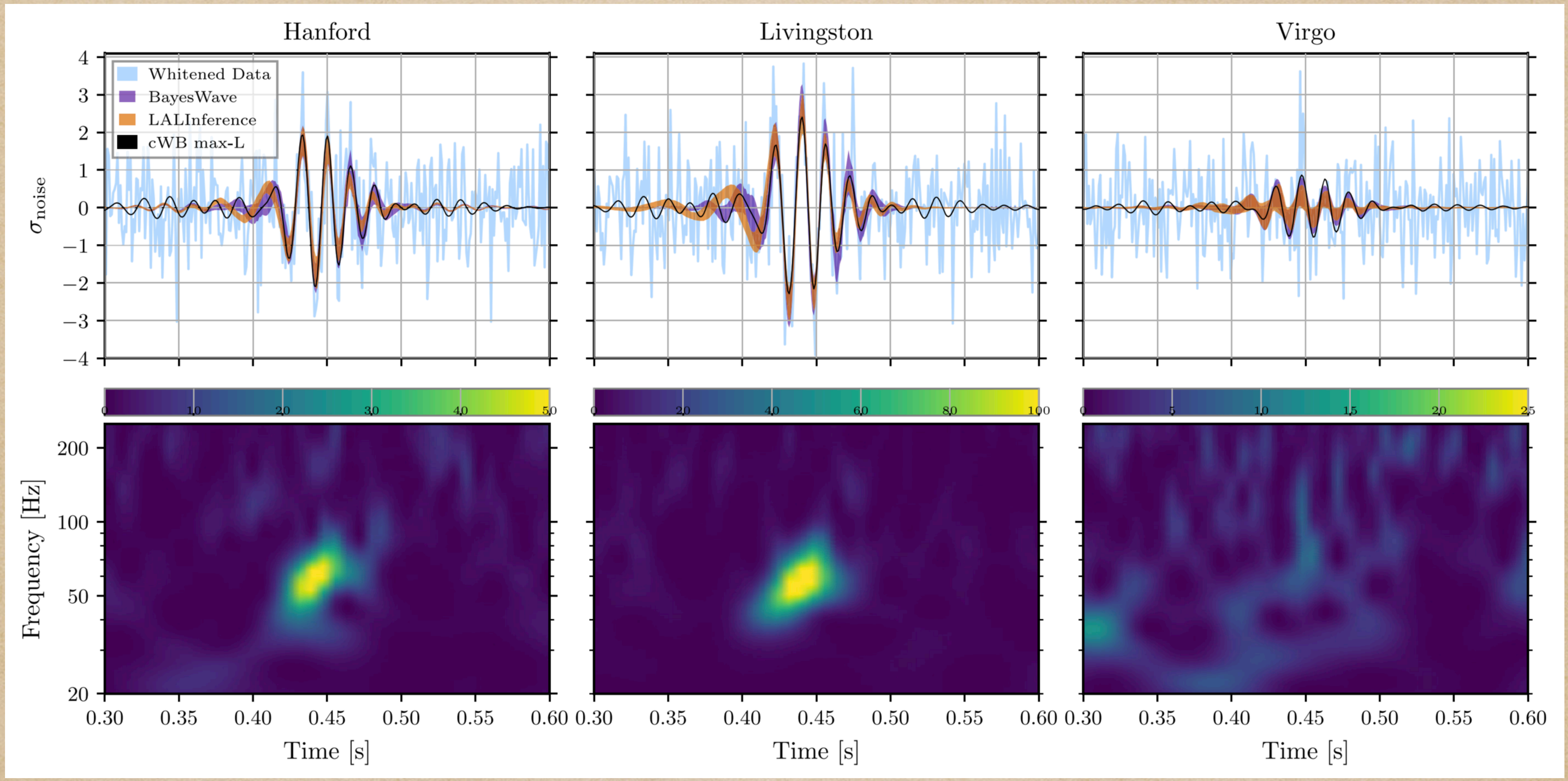
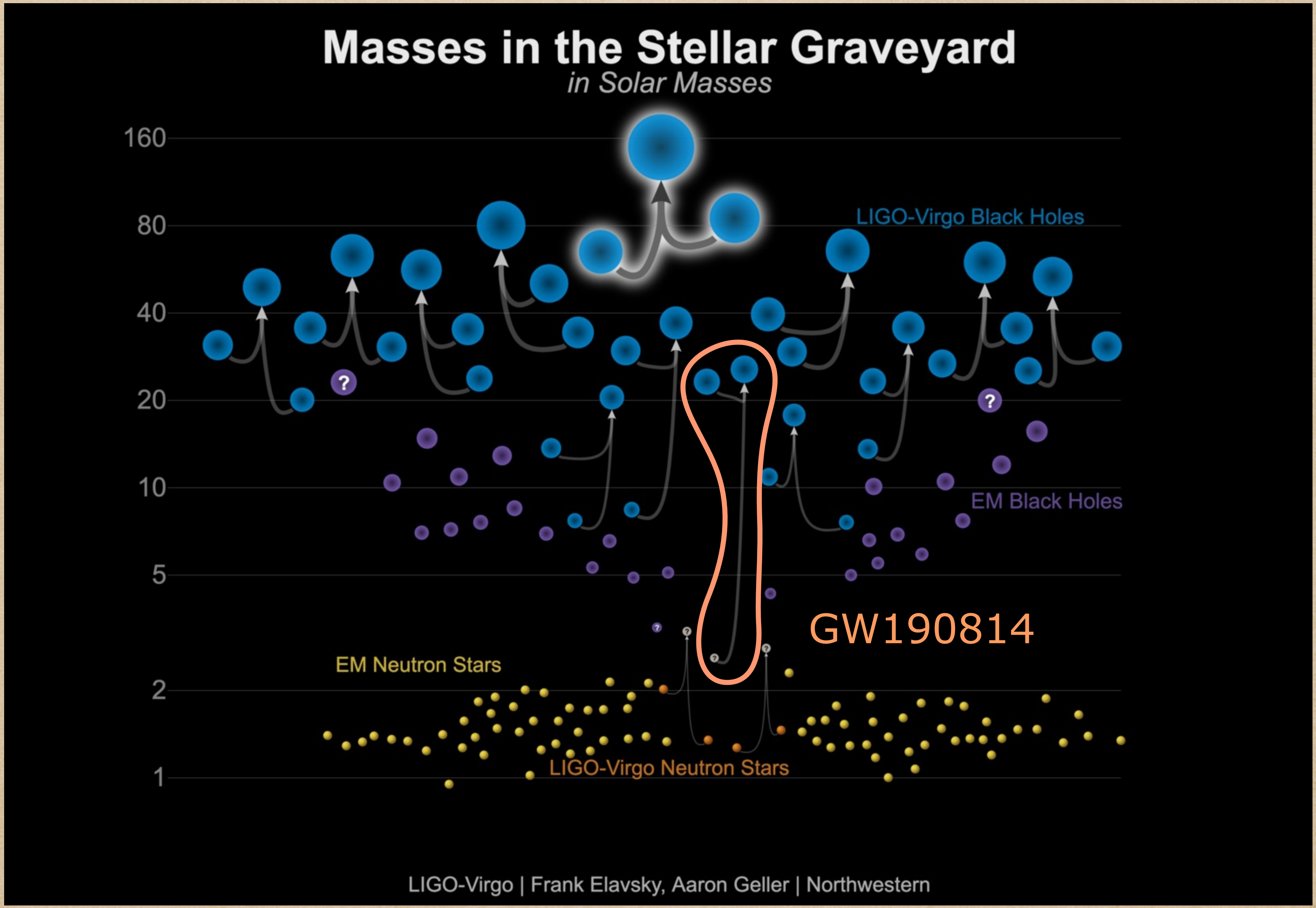


- Progenitor masses: $m_1 = 91\,M_\odot\,, \quad m_2 = 67\,M_\odot$
- Remnant mass: $M = 150\,M_\odot \overset{!}{=} m_1 + m_2$ Abbott et al. 2010.14527
- Mass defect $\Delta M \approx 8\,M_\odot$; cf. Tsar Bomba: $\Delta M \approx 2.65\,\mathrm{kg}$
- Open questions: How did the progenitor BHs form? Is it a BBH?

GW190814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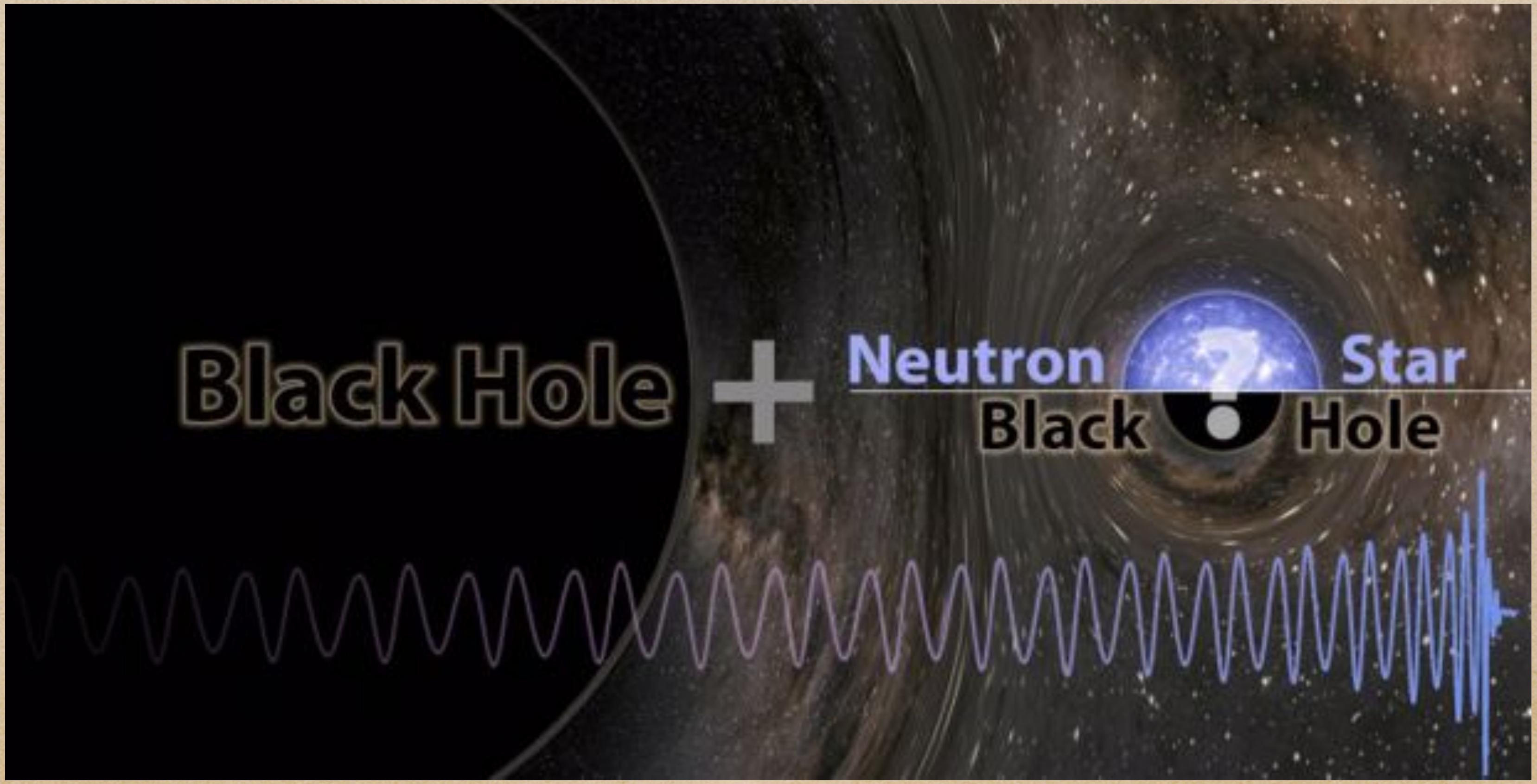
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

GW190521

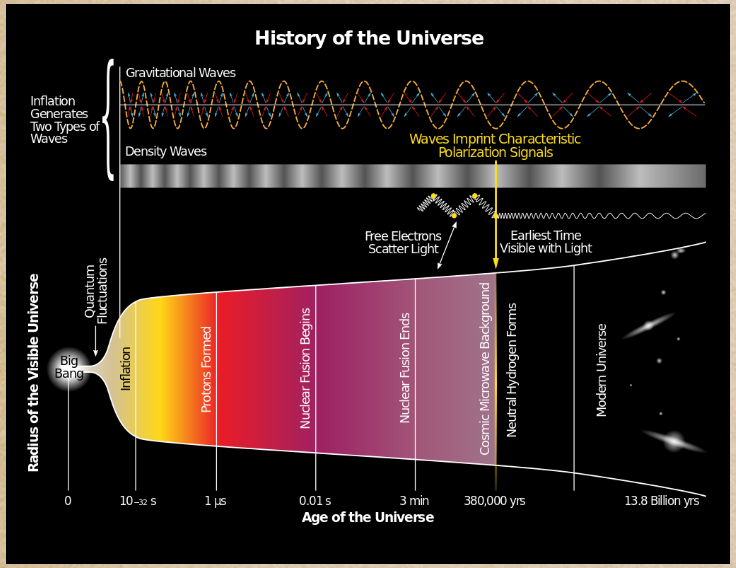
- Progenitor masses: $m_1 = 23.2\, M_\odot\,, \quad m_2 = 2.59\, M_\odot$
- Remnant mass: $M = 25.6\, M_\odot$ Abbott et al. 2010.14527
- Key question: What is the secondary?
  - The heaviest neutron star ever seen?
  - The lightest black hole ever seen?
  - An exotic compact object? Wormhole or Boson Star or …



(Selected) Future applications

Overview

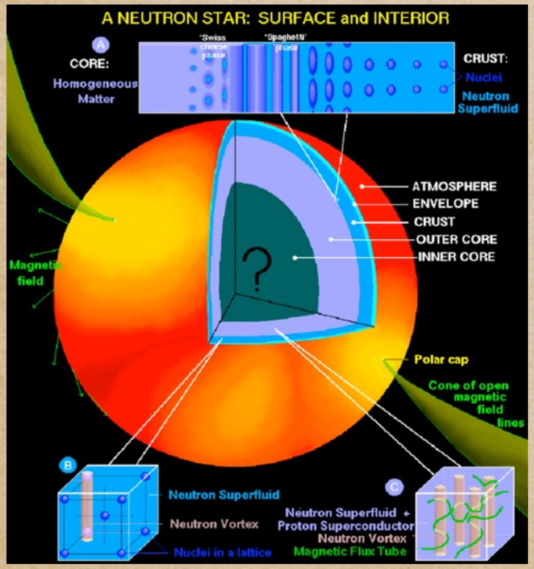
Early Universe



Testing Einstein's theory

Galaxy history

Equation of state



BH populations

The unknown…

Testing GR with GW170817: Graviton mass

- Phenomenological model
- Massive graviton ⇒ Compton wavelength $\lambda_g = \dfrac{h}{m_g c}$
- Dispersion relation: $\dfrac{v_g^2}{c^2} = 1 - \dfrac{h^2 c^2}{\lambda_g^2 E^2}$

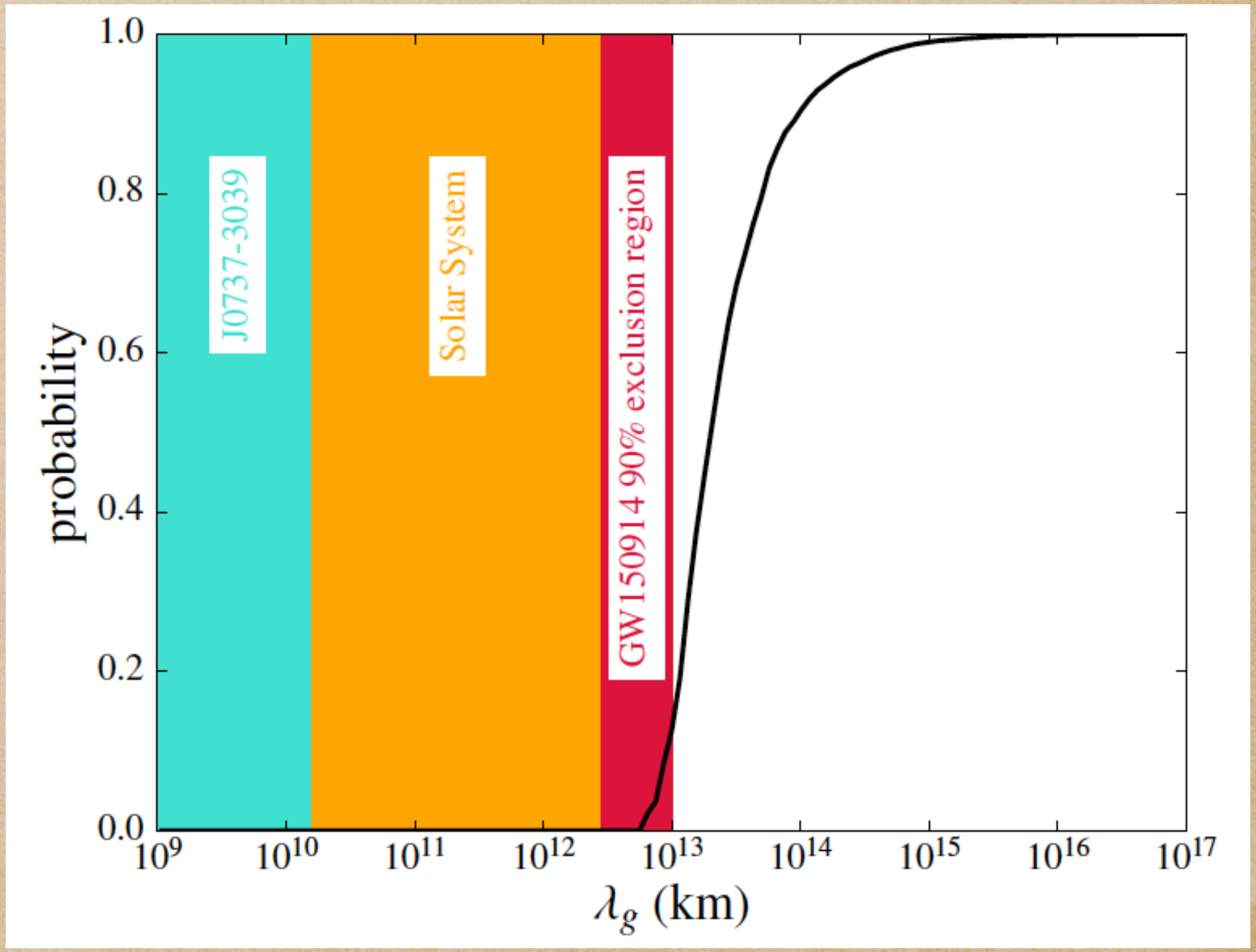
⇒ Quasi-1PN phase term

$$\phi_{\rm MG}(f) = -\frac{\pi D c}{\lambda_g^2 (1+z) f}$$

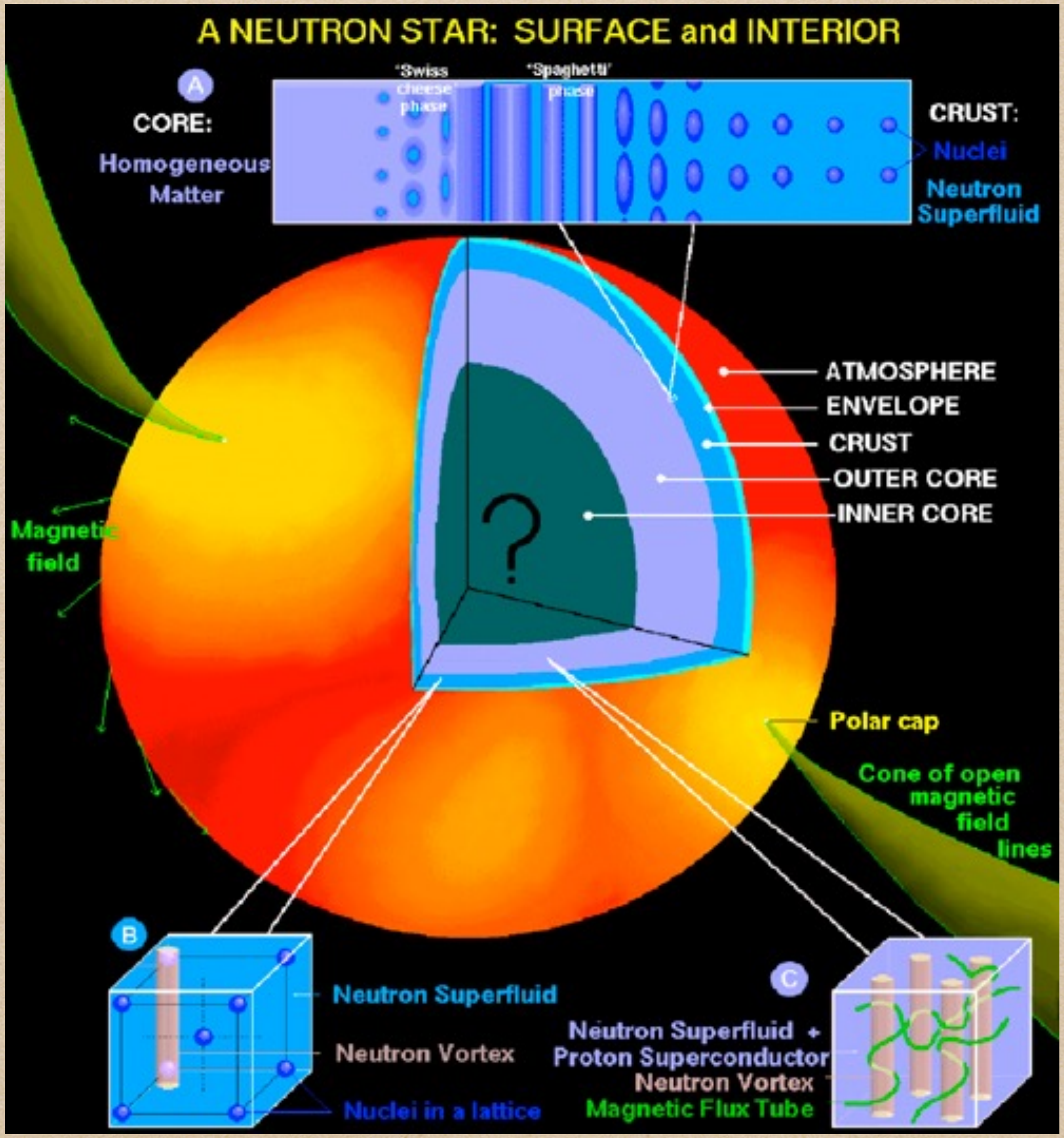
Will 1998 PRD

$$m_g \leq 1.2 \times 10^{-22}\ \mathrm{eV}/c^2$$

Abbott et al 1602.03841

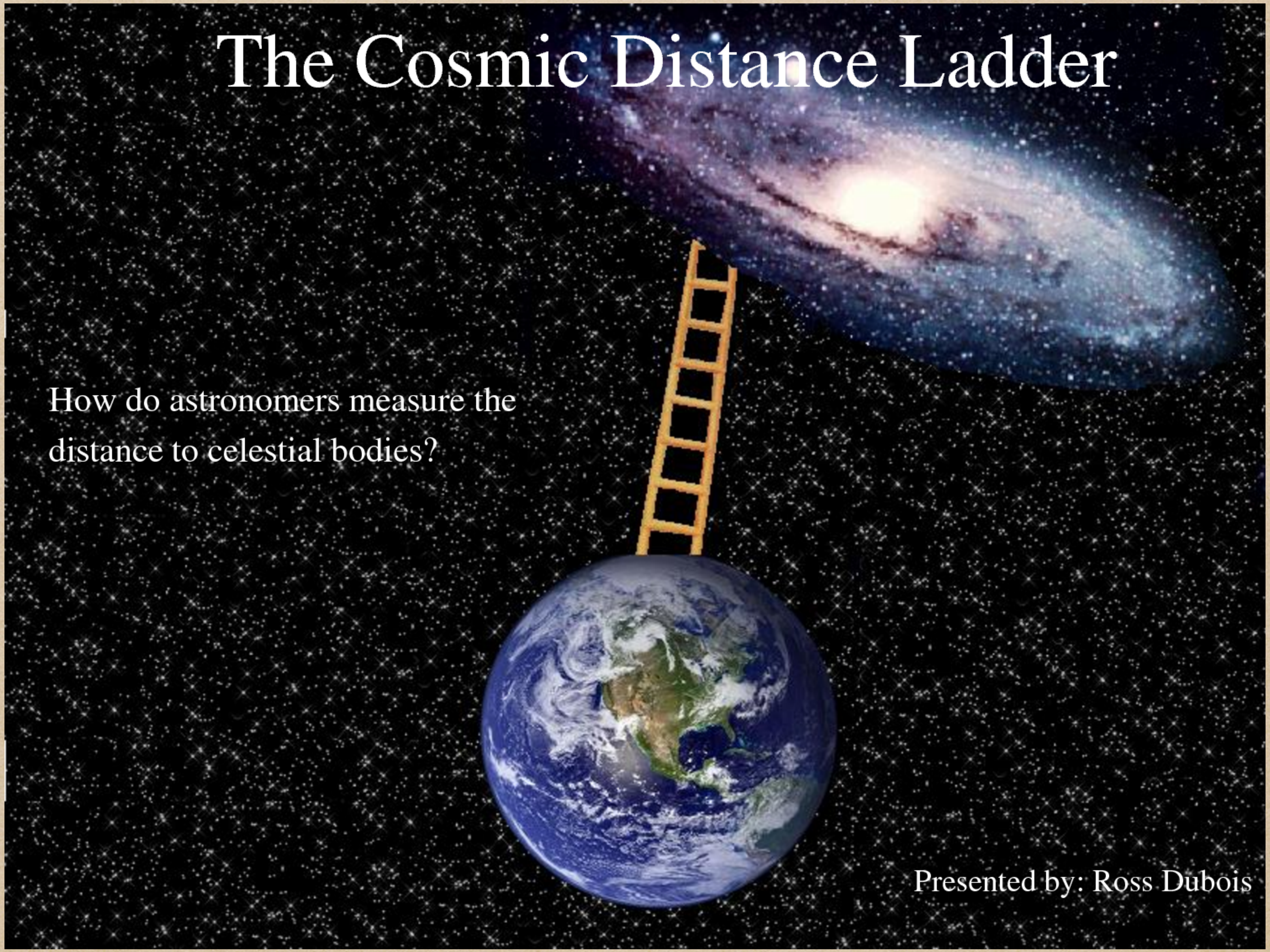


Equation of state of matter



- E.g. through tidal effects

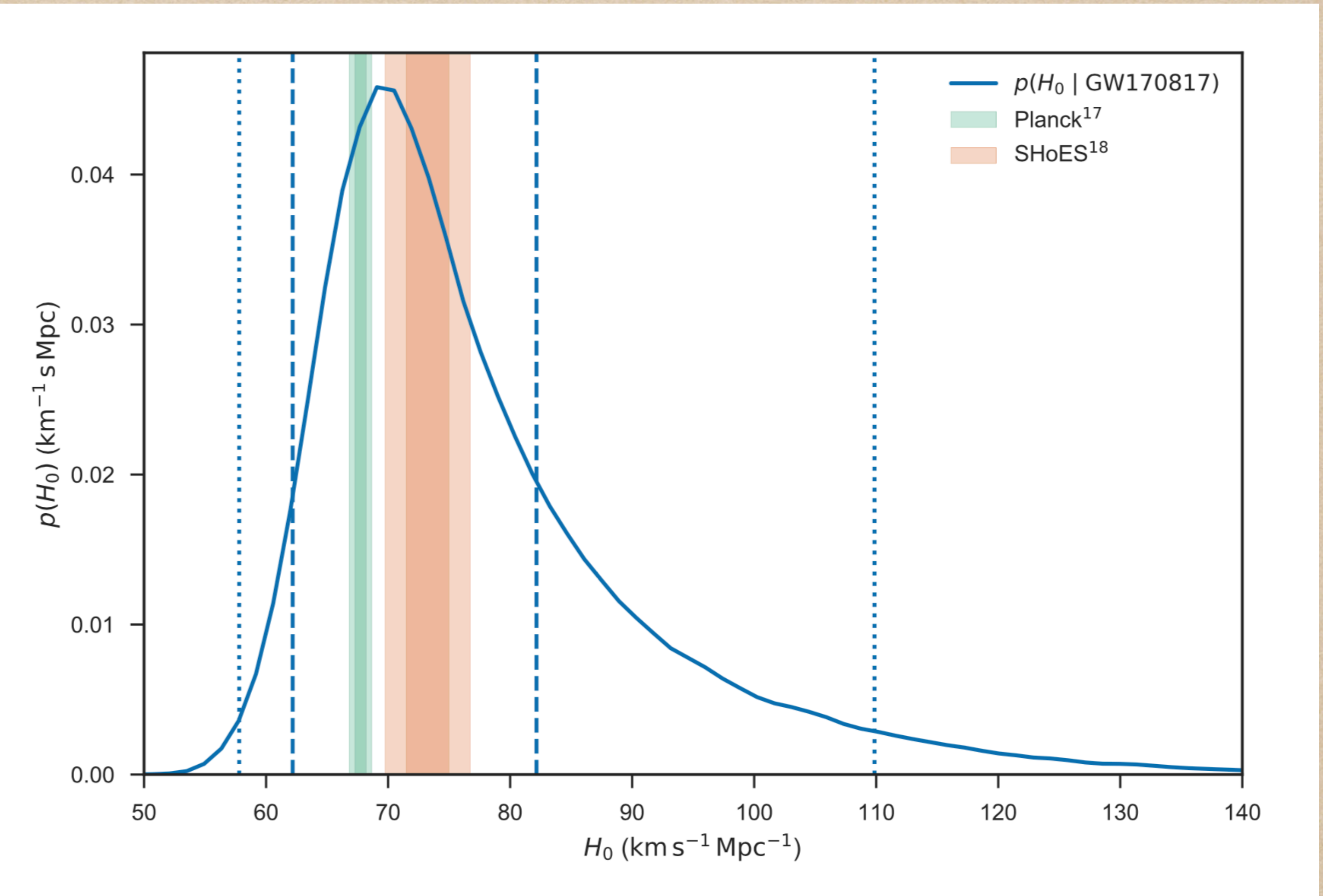
Cosmological distance ladder



- Need electromagnetic counterpart or large number of events !

Standard Sirens: H0 from GW170817

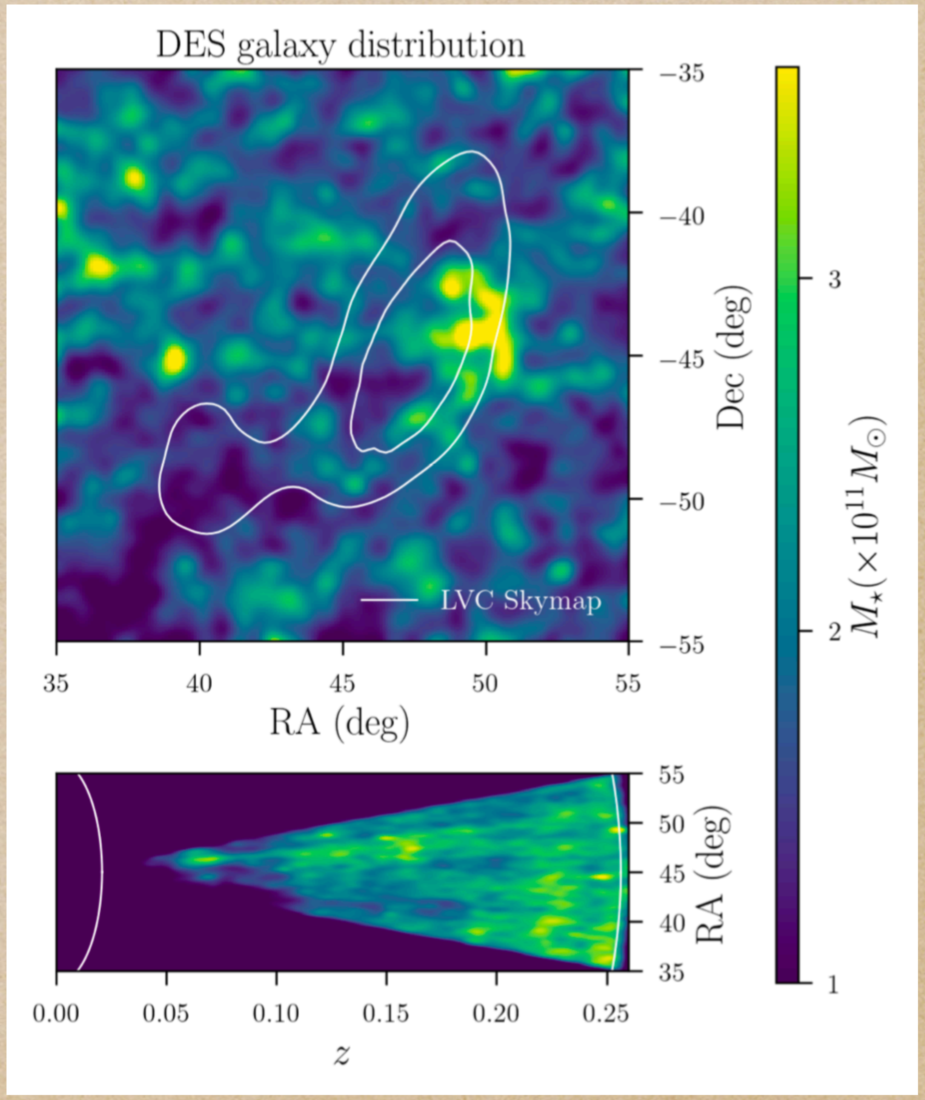
Abbott et al 1710.05835



- Redshift z from EM, Luminosity distance D_L from GWs
- $H_0 = 70^{+12.0}_{-8.0}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$
- More sirens needed to resolve tension

Dark Sirens: H0 from BH binaries

- No EM signal $\Rightarrow$ redshift unknown
- Solution: Statistical analysis using galaxies in sky window
  Combine sky localization from LIGO with galaxy surveys
- Here for GW170814
  - Rather well localized!
  - Dark Energy Survey "DES"
  - Not yet competitive, but will improve with many events



Soares-Santos et al 1901.01540

Conclusions

Conclusions

- GW150914 marks the dawn of GW astronomy
- “We” measured the change in length by a fraction of an atomic nucleus caused by sth. 1 Gyr away!
- >1 BBH! Not merely a lucky shot.
- First surprises: BHs heavier than expected GW190521

  Compact objects in the mass gap GW190814
- GW170817 marks the dawn of multi messenger astronomy!
- Applications: Test GR, BH census, History of universe, EOS,…
- A new window to the universe reveals interesting things…